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ESTIMATING OPTIMIZATION ERROR STATISTICS VIA OPTIMIZATION RUNS FROM MULTIPLE STARTING POINTS

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Abstract

Characteristics of the convergence error in a HSCT structural optimization were investigated. A probabilistic model is used to model the errors in optimal objective function values of poorly converged runs and the Weibull distribution was identified as a reasonable error model. Once the probabilistic error model is identified, we demonstrate that it can be used to estimate average errors from a set of pairs of runs. In particular, by performing pairs of optimization runs from two starting points, we can obtain accurate estimates of the mean and standard deviation of the convergence errors. Positive correlations were identified between the magnitude of the differences of paired optimization runs and the average errors. The results show that finding the error distribution model is a key to estimating the convergence error of optimization runs.

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1. Introduction

Optimization is an iterative procedure, which is subject to convergence errors. It is often hard to find the proper convergence criteria setting in practical engineering applications. Furthermore the high computational cost of using tight convergence criteria often prevents us from finding an optimum to high precision. Numerical optimization errors are deterministic in that computer simulation gives the same output for the same input for repeated runs. However, an optimization procedure can be very sensitive to small changes of input parameters. For example, either the convergence criteria or the starting point can affect optimization results. Therefore, a probabilistic model is useful in characterizing the error in computed optima.

In design optimization of a complex system, sub-optimization problems are often solved within the system level optimization. When a single optimization is flawed, it may be difficult to tell. However, when many optimization results are available, we demonstrated that statistical methods can be used to identify cases with very large errors[1] and estimate the average error of the multiple optimization runs[2].

A structural optimization procedure adopted to obtain the wing structural weight (W_s) of a high-speed civil transport (HSCT) suffered from convergence error, and resulted in noisy W_s in terms of the aircraft configuration variables[1]. The structural optimization was performed *a priori* on a carefully selected set of HSCT configurations to build a response surface model (*cf.* Ref. 3) of W_s. In a previous work[2], the authors applied probabilistic models to the optimization errors and found that the Weibull distribution describes the error characteristics well. The structural optimization had substantial errors because it was difficult to find a proper set of convergence criteria. The objective of the present paper is to demonstrate that statististics of convergence errors can be estimated by performing the optimization runs in pairs.

The present work investigates the effects of change of computer platform and initial design point on the optimization error. The structural optimization results can be substantially different depending on the computer platform, although the errors are highly correlated between computer platforms. The effect of the initial design point is of particular interest because one can easily change the initial design point and repeat the optimization to improve a possibly erroneous run due to convergence difficulties or local optima. The authors[2] successfully estimated average errors by using two sets of optimization runs with different convergence criteria. Here we show that optimization results of two different sets of initial design points can also serve to estimate the average optimization errors. It is reasonable to assume that the uncertainty in the optimization procedure is related to differences in the results due to change of optimization parameters. A positive correlation between differences of paired optimization runs and the average error is identified and the possibility of using the correlation in estimation of average optimization error is investigated.

2. Error in Structural Optimization Results

The application problem in this paper is a HSCT design model developed by the Multidisciplinary Analysis and Design (MAD) Center for Advanced Vehicles at Virginia Tech. A simplified version of the problem is used following Knill et al.,[4] with five configuration design variables including wing root chord, wing tip chord, inboard leading edge sweep angle, airfoil thickness ratio, and fuel weight. Takeoff gross weight is minimized at the system level as a function of the five configuration variables. To improve wing weight equations based on historical data, the GENESIS[5] structural optimization software based on finite element models is used. The finite element model has 1127 elements at 226 nodes with a total number of 1242 degrees of freedom. The structural optimization is a sub-optimization within the system level configuration optimization, and the wing structural weight (W_s) is minimized in terms of 40 structural design variables, including 26 to control skin panel thickness, 12 to control spar cap areas, and two for the rib cap areas[6]. The structural optimization is performed *a priori* for many aircraft configurations and a response surface model of W_s is constructed for use in the HSCT configuration design optimization. For the response surface construction, the five design variables are coded so that each ranges between –1 and +1.

The structural optimization resulted in a noisy W_s response in terms of the HSCT configuration variables[1, 2]. Figure 1 shows the W_s response for 21 HSCT configurations generated by a linear interpolation between two extreme designs. Design 1 corresponds to (-1, -1, -1, -1, -1) (all configuration variables at their lower bounds) and Design 21 corresponds to (1, 1, 1, 1, 1) (all configuration variables at their upper bounds) in a coded form of the HSCT configuration variables. The original GENESIS runs with the default convergence criteria were poorly converged and W_s contained artificial noise errors. Design 6 and Design 15 are seen to have particularly large errors.

Efforts have been made to reduce the error of the HSCT structural optimization. Papila and Haftka[7] repaired erroneous optimizations by changing optimization algorithms or trying different initial designs. After extensive experiments with convergence criteria, it was found that the most effective way to improve the optimization was to tighten one of the convergence criteria[1]. W_s repaired by repeated high-fidelity runs show much smoother response in Figure 1. One important observation is that the noise error tends to be one-sided (greater W_s than the true). That is because the noise error comes from incomplete minimization due to convergence difficulties. However, it was not trivial to choose the right convergence tolerances, and the tightened convergence tolerances more than doubled the cost of the optimization.

To characterize the error we observe in Figure 1, we define optimization error as

$$e = W_s - W_s^t, \tag{1}$$

where W_s is the calculated optimum and W_s^t is the true optimum, which is unknown for many practical engineering optimization problems. Note that we are mainly interested in the convergence error and W_s^t represents the true optimum of the computer model of the optimization problem. Another source of error can be inaccurate computational simulation models, which we do not consider here.

To estimate W_s^t, we need to perform fully converged optimization runs with properly tightened convergence criteria, which can be expensive. In practice, we estimated W_s^t by taking the best of repeated GENESIS runs: two runs with different initial designs and six runs with different sets of convergence criteria[2]. To study the error in W_s from the structural optimization, we used a mixed experimental design of 126 HSCT configurations, intended to permit fitting a quadratic or cubic polynomial of the five-variable HSCT design problem to create a W_s response surface approximation[8]. The optimization error, e, was calculated for each of the 126 HSCT configurations. When the optimization error, e, is calculated for each of the runs, the mean and standard deviation of e can be estimated by

$$\hat{m}_{data} = \frac{\sum_{i=1}^{n} e_i}{n} = \bar{e}\,, \quad \hat{s}_{data} = \sqrt{\frac{\sum_{i=1}^{n} (e_i - \bar{e})^2}{n-1}}\,, \qquad (2)$$

where n is the sample size.

3. Effects of Computer Platform and Initial Design on the Error

As expected from the noise in the low-fidelity results, the optimization procedure produced substantially different W_s values for essentially the same HSCT configurations. Although numerical optimization procedures are deterministic in that the output is the same for the same input, ill-conditioning may cause unpredictable behavior of the results. Previous results[2] showed that a change of the convergence criteria had a large effect on the optimization results. This section shows two additional examples of such behavior of the optimization error.

<u>Change of Computer Platform</u>

It turned out that the HSCT structural optimization could produce substantially different results depending on computer platforms used[9]. We compare the poorly converged structural optimization runs between a DEC Alpha workstation and a Pentium PC for each of 43 HSCT configurations from a face centered central composite (FCCC) design of the five HSCT configuration design variables. In addition, highly converged runs with the tightened convergence criteria were performed to estimate errors of the poorly converged runs. The average error in W_s using Eq. 2 was 2870 *lb*. (3.69% of the average of true W_s) on the DEC computer whereas the average error was 3603 *lb*. (4.63% of the average of true W_s) on the PC (Table 1). Figure 2 shows that the DEC and PC gave different W_s for the majority of the 43 runs, and the differences are noisy. The maximum difference was 18264 *lb*., which is 23.5% of the average of true W_s. However, the estimated errors had a strong positive correlation between the DEC and PC as shown in Figure 3. The correlation coefficient was as high as 0.90. That means that if a run on the DEC has a large error, the corresponding run on the PC also tends to have a large error, and vice versa. As a result, the average of magnitude of difference in W_s was only 1.25%.

<u>Change of Initial Design Point</u>

When one suspects convergence problems or local optima in optimization results, one is likely to change the initial design point and repeat the optimization run. We performed optimization runs with default convergence criteria with two different initial design points for each of the 126 HSCT configurations from a mixture of FCCC and orthogonal array design, originally intended for a quadratic or cubic response surface model. Again, highly converged runs with tightened convergence criteria were performed to estimate errors of each of the poorly converged runs. Table 2 compares the two cases with different initial design points. Case 1 corresponds to poorly converged runs using the default convergence criteria, where a conservative structural design from a previous study is used as a common initial design point for all runs. For Case 2, an initial design point perturbed from that of Case 1 was obtained by multiplying each of the 40 structural design variables by uniform random factors between 0.1 – 1.9.

The average errors were not very different between Case 1 and Case 2, 5.51% and 5.34% (Table 2), respectively, which is understandable since the only difference between the two cases was the initial design point. The differences in W_s between Cases 1 and 2 are plotted in Figure 4, and the largest magnitude of the difference is 58715 *lb*., 72.1% of the average of true W_s. The average magnitude of the difference was 7.3% of the average W_s., much greater than 1.25%, obtained with different computer platforms. The correlation of error was low between Cases 1 and 2 as shown in Figure 5 (correlation coefficient of 0.057).

4. Applying MLE to Establish Probabilistic Model of Optimization Error

The unpredictable error in the HSCT structural optimization led us to use a probabilistic model for the error. With multiple optimization runs available, we can obtain a data driven model of the error by fitting a probability distribution to the actual error obtained from Eq. 1. The approach is denoted as *error fit*. This approach requires fully converged high-fidelity optimization runs to calculate error data, which are not always available. However, once the probabilistic distribution of the error is known, as we discuss later in the next sections, the model can be used to estimate the average errors even when fully converged results are not available. We use the maximum likelihood estimation (MLE) method for the distribution fit[10]. In MLE, we find a vector of distribution parameters, b, to maximize the likelihood function, l(b), which is a product of the probability density function, f, over the sample data x_i (i = 1, …, n),

$$l(\mathrm{b}) = \prod_{i=1}^{n} f(x_i;\mathrm{b}). \qquad (3)$$

The quality of fit is checked via the c^2 goodness-of-fit test[10], which is essentially a comparison of histograms between the data and the fit. The test results will be given in terms of the *p*-value. A *p*-value near one implies a good fit and a small chance that the data is inconsistent with the distribution. Conversely, a small *p*-value implies a poor fit and a high chance that the data is inconsistent with the distribution.

Considering the one-sidedness of the optimization error, we selected the Weibull distribution[10, 11], which is defined by a shape parameter, a, and a scale parameter b. The probability density function (PDF) of the Weibull distribution is

$$f(x) = \begin{cases} a b^{-a} x^{a-1} \exp\left(-\left(\dfrac{x}{b} \right)^{a} \right) & if\ x \geq 0 \\ 0 & if\ \ x < 0 \end{cases} . \tag{4}$$

Once we obtain the parameters, a and b, estimates of mean and standard deviation of *e* can be calculated from

$$\hat{m}_{fit} = \frac{b}{a} \Gamma\left(\frac{1}{a} \right), \hat{s}_{fit} = \sqrt{\frac{b^2}{a} \left\{ 2\Gamma(\frac{2}{a}) - \frac{1}{a} \left[\Gamma(\frac{1}{a}) \right]^2 \right\}}, \tag{5}$$

where Γ is the gamma function.

The Weibull model was fitted to Case 1 and Case 2, the poorly converged runs of two different initial design points, and the results are summarized in Table 3. *p*-values of the c^2 test indicated a poor fit for Case 1, while the fit was acceptable for Case 2 with a 5% confidence level. Figure 6 compares histograms of the optimization error, *e*, with the predicted frequencies from the fitted Weibull models. It is seen that the error distribution has a mode near zero and decreases rapidly for large error. The Weibull model of the error fit gives reasonable descriptions of the error distribution for both Case 1 and Case 2, although the c^2 test implied an unsatisfactory fit for Case 1. The dash-dot lines denoted difference fit will be discussed in the next section.

The average errors estimated from the error fit, $\hat{m}_{fit}$, were in reasonable agreements with $\hat{m}_{data}$: -5.63% and –8.54% discrepancies for Case 1 and Case 2, respectively. The estimates of standard deviation from the fits, $\hat{s}_{fit}$, were less accurate particularly for Case 2, with a discrepancy of –14.6% and –23.4%, for Case 1 and Case 2, respectively. Figure 6, comparing histograms of *e* between the error data (bars) and the error fit (solid line), indicates that the Weibull model is suited for the optimization errors for both Case 1 and Case 2.

5. Difference Fit of the Weibull Model

When fully converged results are available, estimating errors in the poorly converged results is of use in that it can provide information for the more common case where converged results are not available. In particular, the probabilistic model identified from the error fit using fully converged data can be used to estimate error statistics for poorly converged optimization runs. Indeed, using the knowledge that the errors can be fit well by a Weibull distribution, we estimated the distribution parameters from the differences of optimal values from two different convergence settings[2]. Here, we propose to use different initial points (*e.g.*, Cases 1 and 2 of the HSCT structural optimization) instead of convergence criteria. Changing convergence settings may require expert level knowledge depending on the optimization software, whereas it is much simpler to change initial designs to generate other sets of optimization results.

For two optimization results, W_s^1 with optimization parameter setting #1 and W_s^2 with optimization parameters setting #2, model the optimization errors as random variables *s* and *t*,

$$s = W_s^1 - W_s^t$$
$$t = W_s^2 - W_s^t. \tag{6}$$

Note that the true optimum, W_s^t, is not random although it may be unknown for many practical engineering optimization problems. Random properties of the errors are due to noisy W_s^1 and W_s^2. Since we want to avoid the expensive calculation of W_s^t, the difference of *s* and *t* is defined as the *optimization difference x*,

$$x = s - t = (W_s^1 - W_s^t) - (W_s^2 - W_s^t) = W_s^1 - W_s^2. \tag{7}$$

If *s* and *t* are independent, the probability density function (PDF) of *x* can be obtained by a combined integration of the PDF functions $g(s; b_1)$ and $h(t; b_2)$,

$$f(x; b_1, b_2) = \int_{-\infty}^{\infty} g(s; b_1) h(s - x; b_2) ds. \tag{8}$$

Note that the optimization difference *x* is easily calculated from W_s^1 and W_s^2 that are readily available. Then, we can fit Eq. 8 to the optimization differences via MLE. This *difference fit* does not require estimation of the true optimum, and the error distributions of the two cases involved are obtained simultaneously.

The difference fit would not be applicable to the runs with different computer platforms in our case because the structural optimization errors are highly correlated between DEC and PC. Therefore, the difference fit was performed for the pair of Cases 1 and 2 using the Weibull distribution. Recall that a relatively large perturbation (multiplication factors between 0.1 – 1.9) of the initial design point was used to get Case 2 from Case 1 to reduce the possible correlation of W_s errors between Case 1 and Case 2.

The results of the poorly converged data fit using the Weibull model are shown in Table 4. Note that the distribution parameters a and b of Case 1 and Case 2 are simultaneously estimated. Because there is no closed form of the probability density function of the difference for the Weibull model, Eq. 8 was numerically integrated using Gaussian quadrature. The c^2 test on the optimization difference indicated a reasonable fit with a *p*-value of 0.5494. From the difference fit, we estimate the mean and standard deviation of the optimization error of each of the two cases involved. Table 4 shows that the estimates of mean error by the difference fit, $\hat{m}_{fit}$, have reasonable agreements with $\hat{m}_{data}$: -14.7% and –19.4% discrepancies for Case 1 and Case 2, respectively. The estimates of standard deviation, $\hat{s}_{fit}$, are also in a reasonable match with $\hat{s}_{data}$: 12.0% and 0.704% discrepancies for Case 1 and Case 2, respectively. Figure 6 compares the histogram predicted by the difference fit with the data and we observe that the difference fit is comparable to the error fit.

In summary, the Weibull model was useful for estimating average convergence errors causing noisy optimization results. This information about the error distribution family helped us to use the difference fit effectively to estimate errors of poorly converged optimizations. In practice, preliminary information about the error distribution family may be sufficient for the difference fit, because we can use statistical test and graphical examination of the difference fit to identify a good distribution model.

6. Relation of Average Difference of Optimization Runs to Average Error

The difference fit uses a MLE fit to the difference data to estimate the statistics of the optimization errors. For example, the difference data of Cases 1 and 2 is shown in Figure 4 and a reasonable assumption would be that the magnitude of the differences is somehow related to the error level. If we can find a functional relationship, we may estimate the error statistics without the MLE fit.

The expected value of the magnitude of differences of two random variables *s* and *t* is

$$E(|s-t|) = E(|x|) = \int_{-\infty}^{\infty} |x| \, \underline{f}(|x|;\, \mathrm{b}_1, \mathrm{b}_2)\, dx, \qquad (9)$$

where *f* is the PDF of the magnitude of the difference, |x|= |*s* – *t*|. Often we can assume that *s* and *t*, errors of the two sets of optimizations, can be described by the same probabilistic model, because the errors are from the same source and of the same magnitude such as when we have pairs of poorly converged optimization runs with different initial design points. Then the expected value of the difference can be calculated as

$$E(|s-t|) = 2\int_{0}^{\infty} x f(x;\, \mathrm{b}_1, \mathrm{b}_1)\, dx, \qquad (10)$$

where *f* is the PDF of *x*.

To show the relationship between the mean and the mean difference, we calculated the expected values of the difference for 25 variants of the Weibull distribution from a combination of the shape parameter a in {0.4, 0.6, 0.8, 1.0, 1.2} and the scale parameter b in {1000, 2000, 3000, 4000, 5000}. Note that we want to relate the average difference to the average error, which can be calculated from Eq. 5. Figure 7 shows the relationship between the average absolute difference and the average error for the 25 variants of the Weibull distribution. We can see a strong positive correlation and a linear relationship appears to be reasonable for a given shape parameter a. However, the slope of the linear relationship changes along with a. The overall behavior of the relationship was approximated by a least squares fit to the 25 data points assuming a linear model,

$$E(s) = 1134 + 0.5653\, E(|s-t|). \qquad (11)$$

In this case the average error is approximately half of the average difference. The R^2 value of the least squares fit was 0.976. The high R^2 indicates that Eq. 11 is useful for estimating the magnitude of the error for a wide range of parameters of the distribution.

To see how the relationship depends on the distribution model, the same calculation was performed assuming that the errors follow the gamma distribution. The PDF of the gamma distribution defined by a shape parameter a and a scale parameter b is

$$f(x) = \begin{cases} \dfrac{1}{b^{a}\,\Gamma(a)}\, x^{a-1} \exp\left(-\dfrac{x}{b}\right) & if\ x \geq 0 \\ 0 & if\ \ x < 0 \end{cases}. \qquad (12)$$

Again, we used 25 variants of the gamma distribution from a combination of a in {0.4, 0.6, 0.8, 1.0, 1.2} and b in {1000, 2000, 3000, 4000, 5000}. Figure 8 shows the relationship between the average absolute difference and the average error for the 25 variants of the gamma distribution. Again we observe a strong positive correlation and the slope of the linear relationship appears to change slightly along a. Note that the overall slope of the linear relationship is steeper than that of the Weibull.

An overall relationship approximated via least squares fit to the 25 data points is

$$E(s) = -277.8 + 1.066\, E(|s - t|). \tag{13}$$

The R^2 value of the least squares fit was 0.967 and the average error is approximately equal to the average difference in this case. Note that this slope is almost doubled that of the Weibull results. The results demonstrate that the relationship between average difference and average of random variables depends on the distribution family. Therefore, it is important to know the error distribution to estimate the average error from differences of optimization data. This result illustrates the value of using the probabilistic modeling of error via the distribution fit. Since we know that the convergence optimization error is well modeled via the Weibull, we applied Eq. 11 to the HSCT structural optimization data. Table 5 shows that the average of differences between Cases 1 and 2 is 5942 *lb*., and the corresponding estimate of the average error is 4493 *lb*. This is in good agreements with the average errors of Case 1 and Case 2, 4458 *lb*. and 4321 *lb*., respectively.

7. Concluding Remarks

Characteristics of convergence error in a HSCT structural optimization were investigated. The structural optimization procedure was very sensitive to changes of convergence criteria and initial design point, and it could produce substantially different results on different computer platforms. We showed that a probabilistic model can be used to obtain error statistics of two sets of optimization runs with different initial design points. Both the previous results and results presented in this paper indicated that the Weibull distribution is a reasonable model for the convergence error.

The approach of fitting the differences of pairs of poorly converged optimization runs could estimate the averages and standard deviations of the errors. We showed that the Weibull model allowed us to estimate the error statistics in poorly converged optimization runs without requiring any fully converged optimization runs. The difference fit can be applied by changing the convergence criteria or by changing the initial design points. Since initial design points are simple and straightforward to change, one may easily apply the difference fit to estimate errors of various optimization runs.

Strong positive correlations were identified between the average difference of random variables and their average. It was possible to apply an approximate linear relationship to estimate average optimization error. Although this approach does not require fitting the distribution model, it is important to know that the error distribution is Weibull, because the relationship depends on distribution models. The results show that finding the error distribution model is a key to estimating errors of optimization runs.

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References

1. Kim, H., Papila, M., Mason, W. H., Haftka, R. T., Watson, L. T, and Grossman, B., "Detection and Repair of Poorly Converged Optimization Runs," *AIAA Journal*, Vol. 39, No. 12, 2001, pp. 2242-2249.
2. Kim, H., Haftka, R. T., Mason, W. H., Watson, L. T, and Grossman, B., "A Study of the Statistical Description of Errors from Structural Optimization," AIAA 2000-4840, 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, CA, September, 2000.
3. Myers, R. H. and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, John Wiley and Sons, New York, N. Y., 1995.
4. Knill, D. L., Giunta, A. A., Baker, C. A., Grossman, B., Mason, W. H., Haftka, R. T., and Watson, L. T., "Response Surface Models Combining Linear and Euler Aerodynamics for Supersonic Transport Design," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 75-86.
5. VMA Engineering, *GENESIS User Manual*, Version 5.0, Colorado Springs, CO, 1998.
6. Balabanov, V., Giunta, A. A., Golovidov, O., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Reasonable Design Space Approach to Response Surface Approximation," *Journal of Aircraft*, Vol. 36, 1999, pp. 308-315.
7. Papila, M. and Haftka, R. T., "Uncertainty and Wing Structural Weight Approximations," AIAA Paper 99-1312, 40th AIAA/ASME/ASCE/ASC Structures, Structural Dynamics, and Material Conference, St. Louis, MO, April, 1999.
8. Papila, M. and Haftka, R. T., "Response Surface Approximations: Noise, Error Repair, and Modeling Errors," *AIAA Journal*, Vol. 38, No. 12, pp. 2336-2343, 2000.
9. Papila, M., *Accuracy of Response Surface Approximations for Weight Equations Based on Structural Optimization*, PhD Dissertation, University of Florida, 2001.
10. Law, A. M. and Kelton, W. D., *Simulation Modeling and Analysis*, McGraw Hill, New York, 1982.
11. Hahn, G. J. and Shapiro, S. S., *Statistical Models in Engineering*, John Wiley and Sons, Inc., New York, 1994.

Table 1: Effect of computer platforms on the errors of the structural optimization (based on 43 HSCT configurations).

Set of optimization runs	Case DEC	Case PC
Description	DEC Alpha workstation	Pentium PC
Average W_s	80643 *lb.*	81376 *lb.*
Average error (% compared to the average of true W_s)	2870 *lb.* (3.69%)	3603 *lb.* (4.63%)
Correlation coefficient of errors between DEC and PC	0.90	
Average of absolute differences of W_s between DEC and PC (% compared to the average of true W_s)	974 *lb.* (1.25%)	
Maximum of absolute differences of W_s between DEC and PC (% compared to the average of true W_s)	18264 *lb.* (23.5%)	

Table 2: Effect of initial design point on the errors of the structural optimization (based on 126 HSCT configurations using a SGI Origin workstation).

Set of optimization runs	Case 1	Case 2
Description	Using the original initial point	Using a perturbed initial point from the original
Average W_s	85340 *lb.*	85202 *lb.*
Average error (% compared to the average of true W_s)	4458 *lb.* (5.51%)	4321 *lb.* (5.34%)
Correlation coefficient of errors between Cases 1 and 2	0.057	
Average of absolute differences of W_s between Cases 1 and 2 (% compared to the average of true W_s)	5942 *lb.* (7.3%)	
Maximum of absolute differences of W_s between Cases 1 and 2 (% compared to the average of true W_s)	58715.3*lb.* (72.1%)	

Table 3: Check of the Weibull model to the errors of the HSCT structural optimization.

Set of optimization runs	Case 1	Case 2
$\hat{m}_{data}$	4458 *lb*. (5.51%)*	4321 *lb*. (5.34%)
$\hat{m}_{fit}$	4207 *lb*. (5.20%)	3952 *lb*. (4.88%)
$(\hat{m}_{fit} - \hat{m}_{data})/\hat{m}_{data}$	-5.63%	-8.54%
$\hat{s}_{data}$	8383 *lb*. (10.4%)	9799 *lb*. (12.1%)
$\hat{s}_{fit}$	7157 *lb*. (8.85%)	7505 *lb*. (9.28%)
$(\hat{s}_{fit} - \hat{s}_{data})/\hat{s}_{data}$	-14.6%	-23.4%
a	0.6161	0.5646
b	2891	2415
p-value of c^2 test	0.0005	0.0925

* Percentage with respect to the average of true W_s

Table 4: Difference fit to the errors of the HSCT structural optimization.

Set of optimization runs	Case 1	Case 2
$\hat{m}_{data}$	4458 *lb*. (5.51%)*	4321 *lb*. (5.34%)
$\hat{m}_{fit}$	3804 *lb*. (4.70%)	3481 *lb*. (4.30%)
$(\hat{m}_{fit} - \hat{m}_{data})/\hat{m}_{data}$	-14.7%	-19.4%
$\hat{s}_{data}$	8383 *lb*. (10.4%)	9799 *lb*. (12.1%)
$\hat{s}_{fit}$	9393 *lb*. (11.6%)	9868 *lb*. (12.2%)
$(\hat{s}_{fit} - \hat{s}_{data})/\hat{s}_{data}$	12.0%	0.704%
a	0.4666	0.4262
b	1659	1236
p-value of c^2 test	0.5494	

* Percentage with respect to the average of true W_s

Table 5: Estimate of average error of the structural optimization of HSCT using the relationship (Eq. 11) between average difference and average error for the Weibull distribution.

	Case 1	Case 2
$\hat{m}_{data}$	4458 *lb*. (5.51%)*	4321 *lb*. (5.34%)
Average of absolute differences of W_s between Cases 1 and 2	5942 *lb*. (7.34%)	
Estimate of average error from Eq. 11	4493 *lb*. (5.55%)	

* Percentage with respect to the average of true W_s

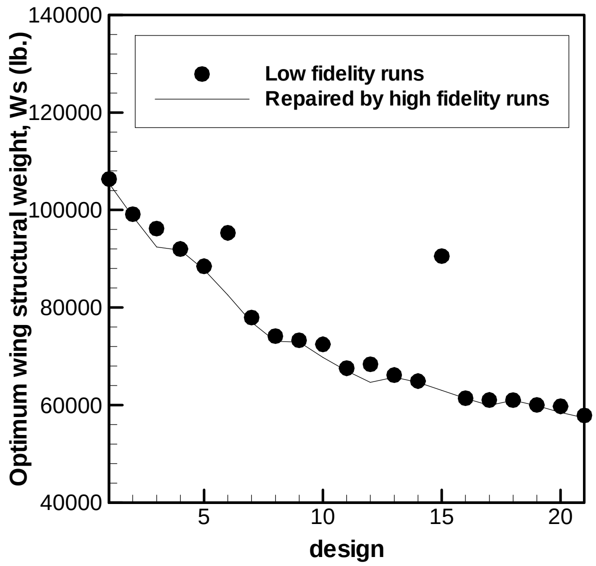


Figure 1: Noisy W_s response from the HSCT structural optimization. Poor results could be repaired by tightening convergence criteria.

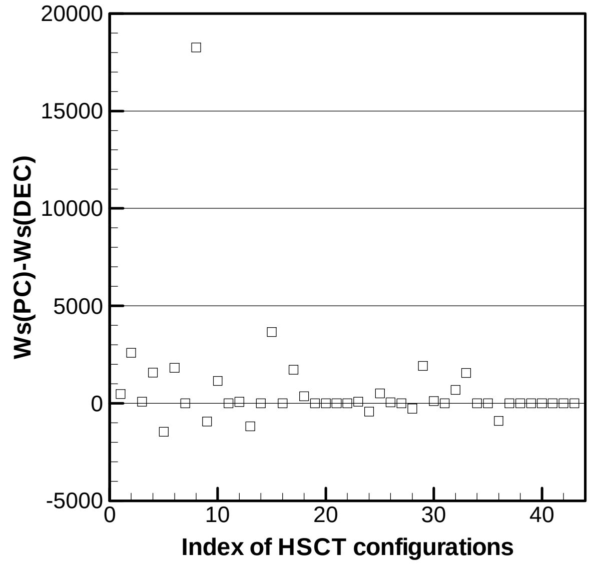


Figure 2: Differences of W_s between a PC and a DEC Alpha workstation.

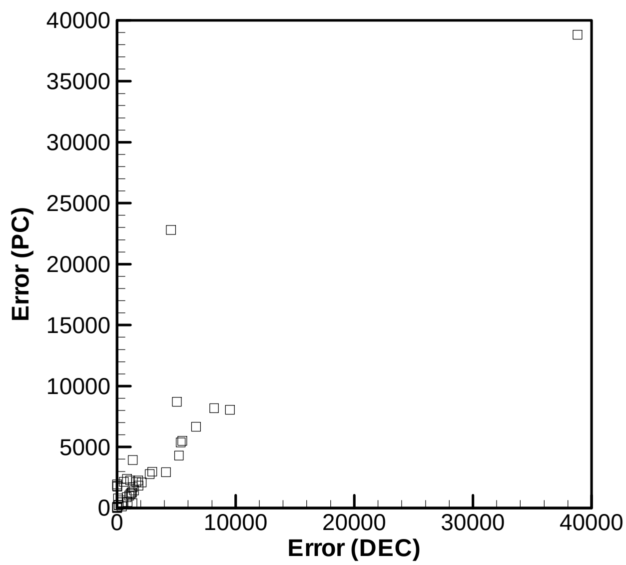


Figure 3: Scatter plot of errors in W_s between a PC and a DEC Alpha workstation.

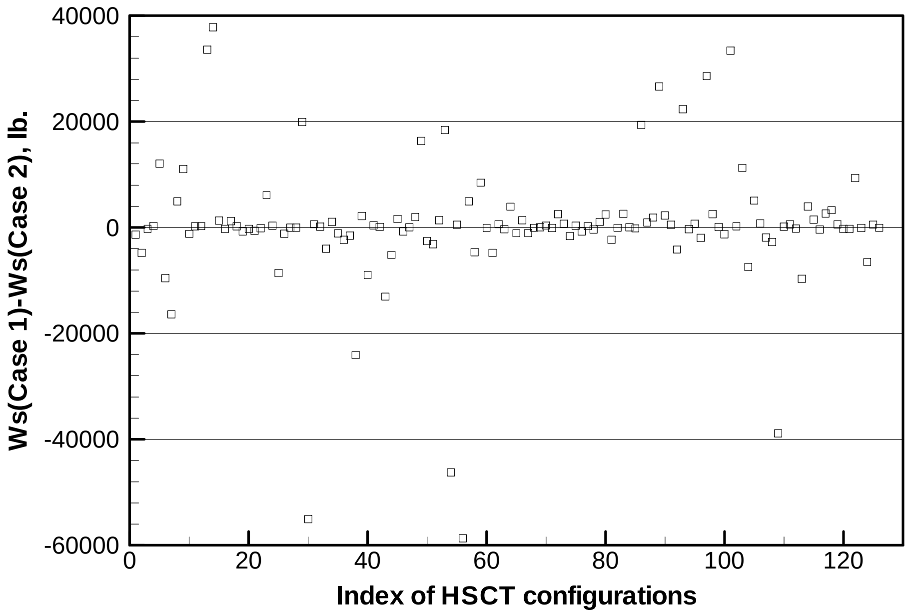


Figure 4: Differences of W_s between two cases of different initial design point.

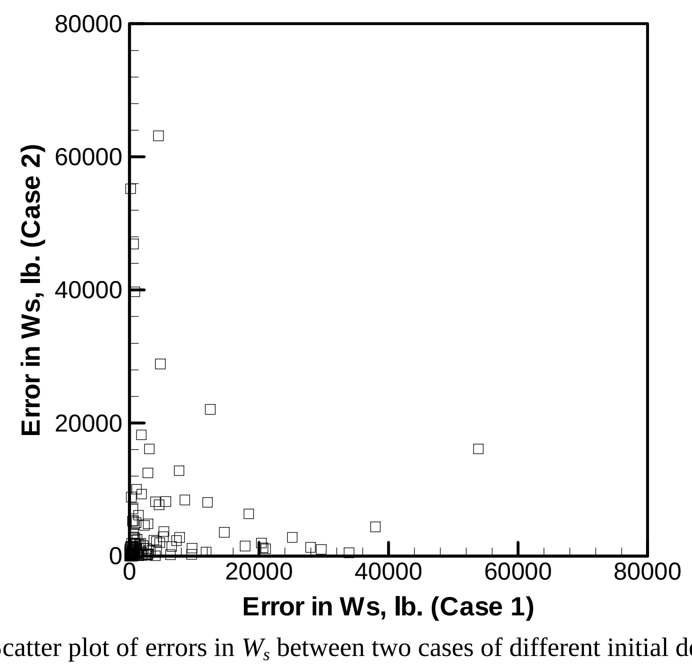
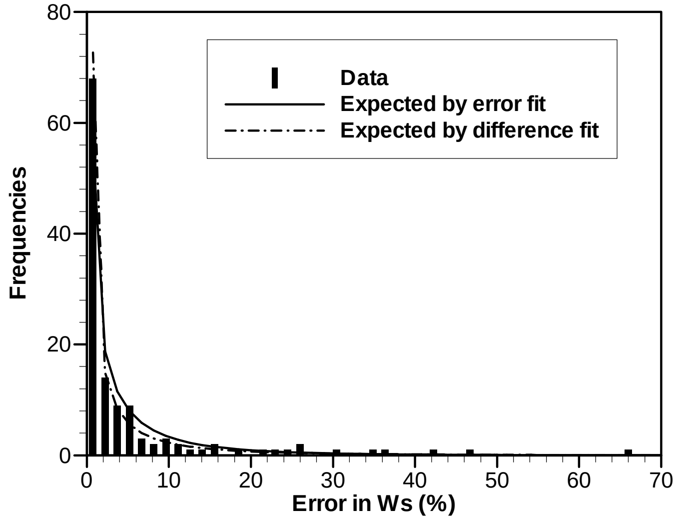
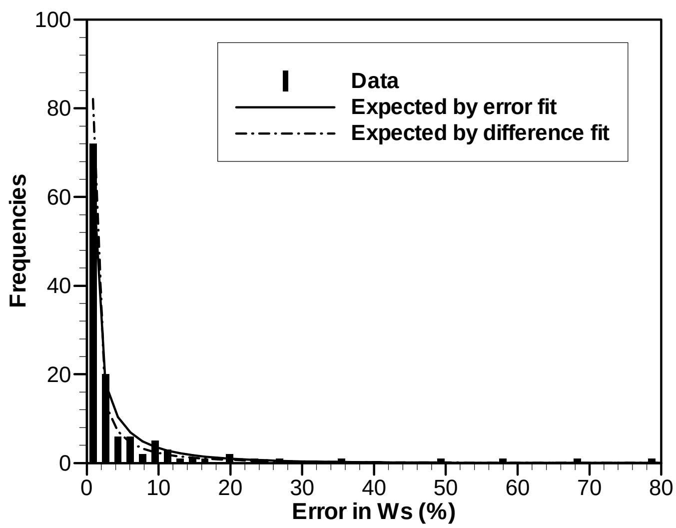


Figure 5: Scatter plot of errors in W_s between two cases of different initial design point.



(a) For Case 1



(b) For Case 2

Figure 6: Comparison of histograms between the optimization error data and the Weibull fits for the HSCT structural optimization problem.

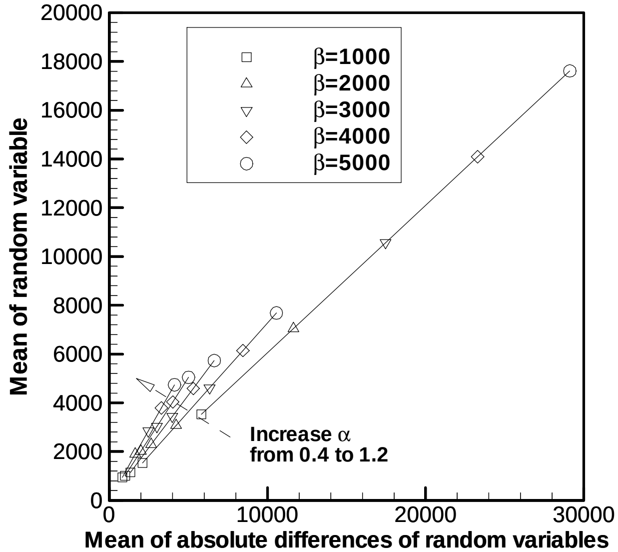


Figure 7: Correlation between mean of random variables and mean of absolute values of differences of the random variables for the Weibull distribution.

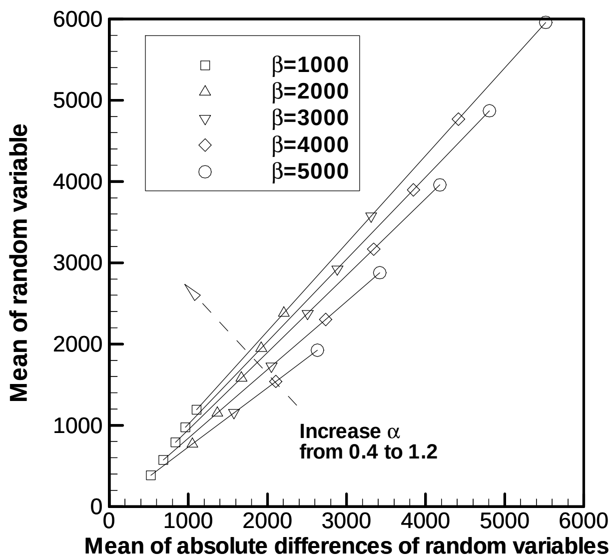


Figure 8: Correlation between mean of random variables and mean of absolute values of differences of the random variables for the gamma distribution.