# *The Truth About Elliptic Spanloads*
# or
# Optimum Spanloads Incorporating Wing Structural Weight

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# The Context

- In MDO studies, we often input spanloads to wing weight routines.
- We always put in the aerodynamic optimum!
- Is this right?
- Today: A way to pick the best spanload - an inner “loop” in an MDO problem for fixed span and *t/c*.
- No aeroelastics: just the basic problem for a cantilever wing.

# Outline

- Introduction and previous work.
- Methodology, aerodynamic and structural modeling.
- B-777 class study, maximum range configuration.
- B-777 class study, reduced range configurations.
- Conclusions.

# Introduction to Spanload Optimization with Wing Structural Considerations

- Everyone, including our work, considers ***cantilever wings***.
- It is “well known” that finding the optimum spanload should include both aerodynamics and structures.
- Previous studies: Prandtl, R. T. Jones, Klein and Viswanathan, Ilan Kroo, Sean Wakayama, McGeer and Craig and McLean.
- Previous studies found optimum spanloads and minimum induced drag with an applied wing structural constraint.
- *Key result:* Significant drag savings can be obtained if wing span is increased while keeping the wing weight fixed (by, say, keeping the wing root bending moment fixed).

# Previous Application of Wing Structural Constraints

- The classical wing structural constraints are:
  - Root bending moment constraint:

$$RBM = \int_{s_{root}}^{s_{tip}} L \cdot \left(s - s_{root}\right) ds$$

  - Integrated bending moment constraint:

$$IBM = \int_{s_{root}}^{s_{tip}} M(s_0)\, ds_0, \quad M(s_0) = \int_{s_0}^{s_{tip}} L \cdot (s - s_0)\, ds$$

- More advanced structural constraints have also been applied.
- Previously, wing weight was fixed while letting the span vary. Comparisons were made for different wing planforms.
- Induced drag savings were not related to aircraft performance benefits in terms of fuel or take-off weights.

# Example: R.T. Jones

TECHNICAL NOTE 2249

THE SPANWISE DISTRIBUTION OF LIFT FOR MINIMUM
INDUCED DRAG OF WINGS HAVING A GIVEN LIFT AND
A GIVEN BENDING MOMENT

Robert T. Jones

Ames Aeronautical Laboratory

December 1950

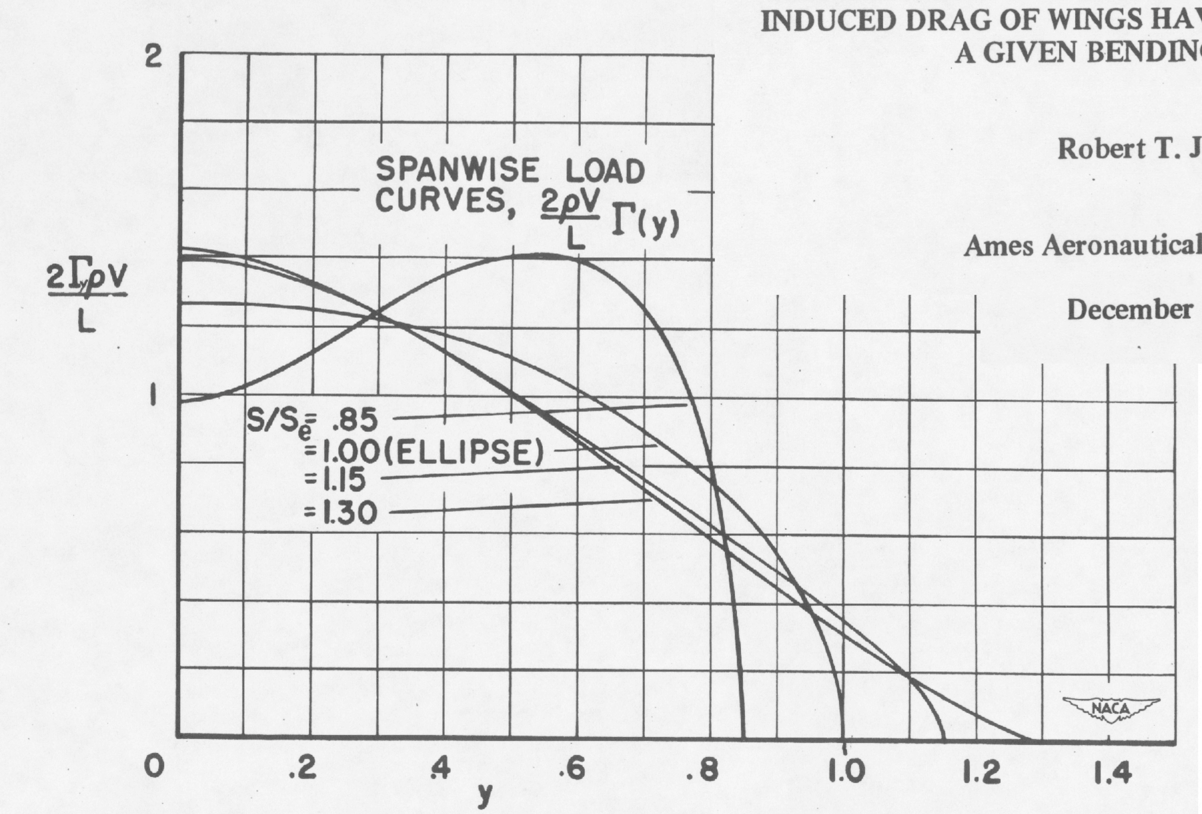

FIGURE 5.- VARIATION OF SHAPE OF THE SPANWISE LOADING CURVE WITH SEMISPAN RATIO S/S$_e$, FOR WINGS HAVING A FIXED TOTAL LIFT AND A FIXED BENDING MOMENT

# Jones’s Induced Drag Results

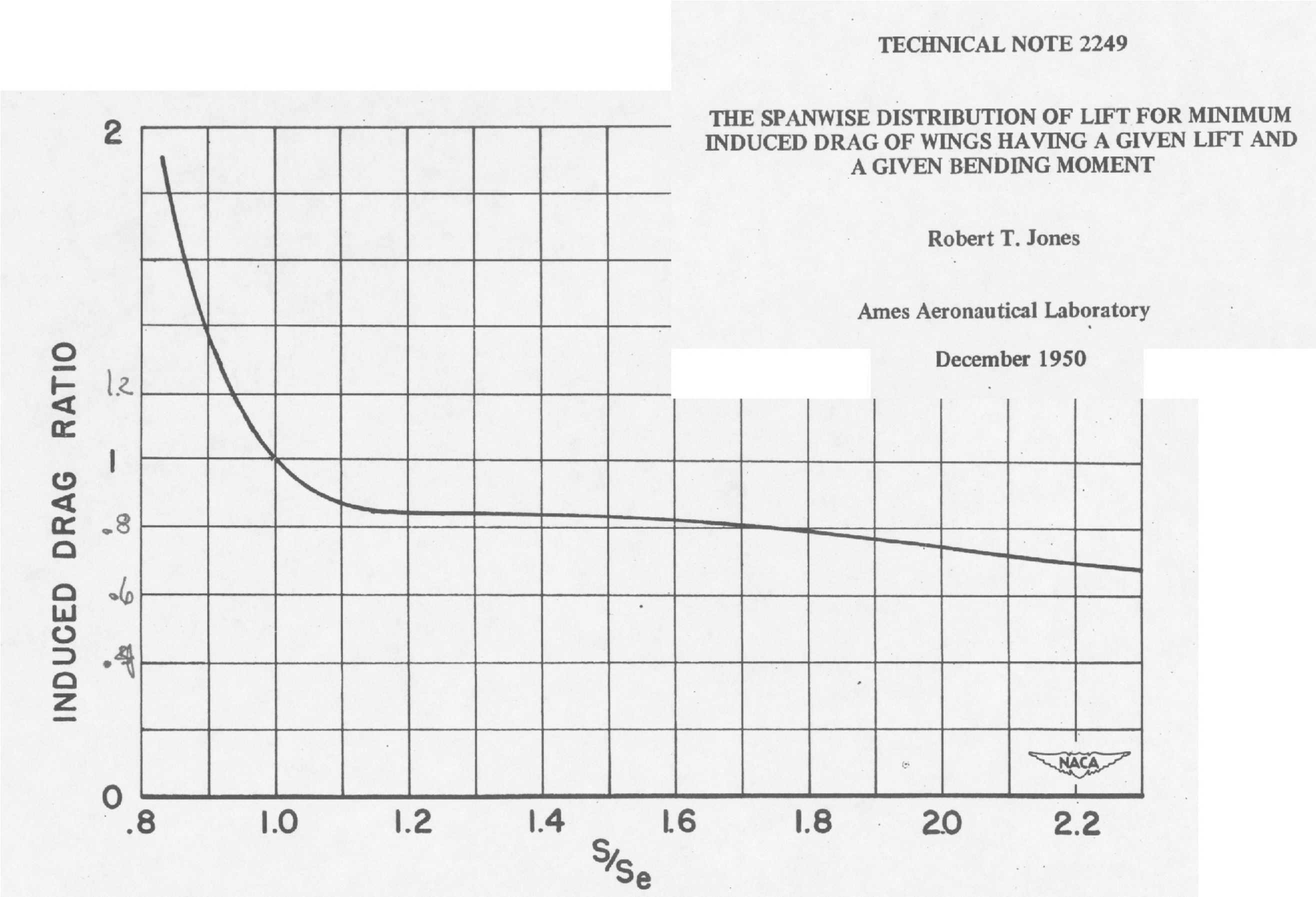

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FIGURE 4.- VARIATION OF INDUCED DRAG WITH SEMISPAN RATIO $S/S_e$, FOR WINGS HAVING A FIXED TOTAL LIFT AND A FIXED BENDING MOMENT.

# Our Approach

- Minimum induced drag spanloads subject to a wing root bending moment (WRBM) constraint are calculated.
- The wing planform and thickness are held constant.
- The structural constraint (WRBM) is only used to generate spanloads.
- The actual wing weight is calculated using a general structural model where the spanload is one of the inputs.
- Changes in induced drag and wing weight are related to changes in fuel and take-off weights with the help of the Breguet Range equation.

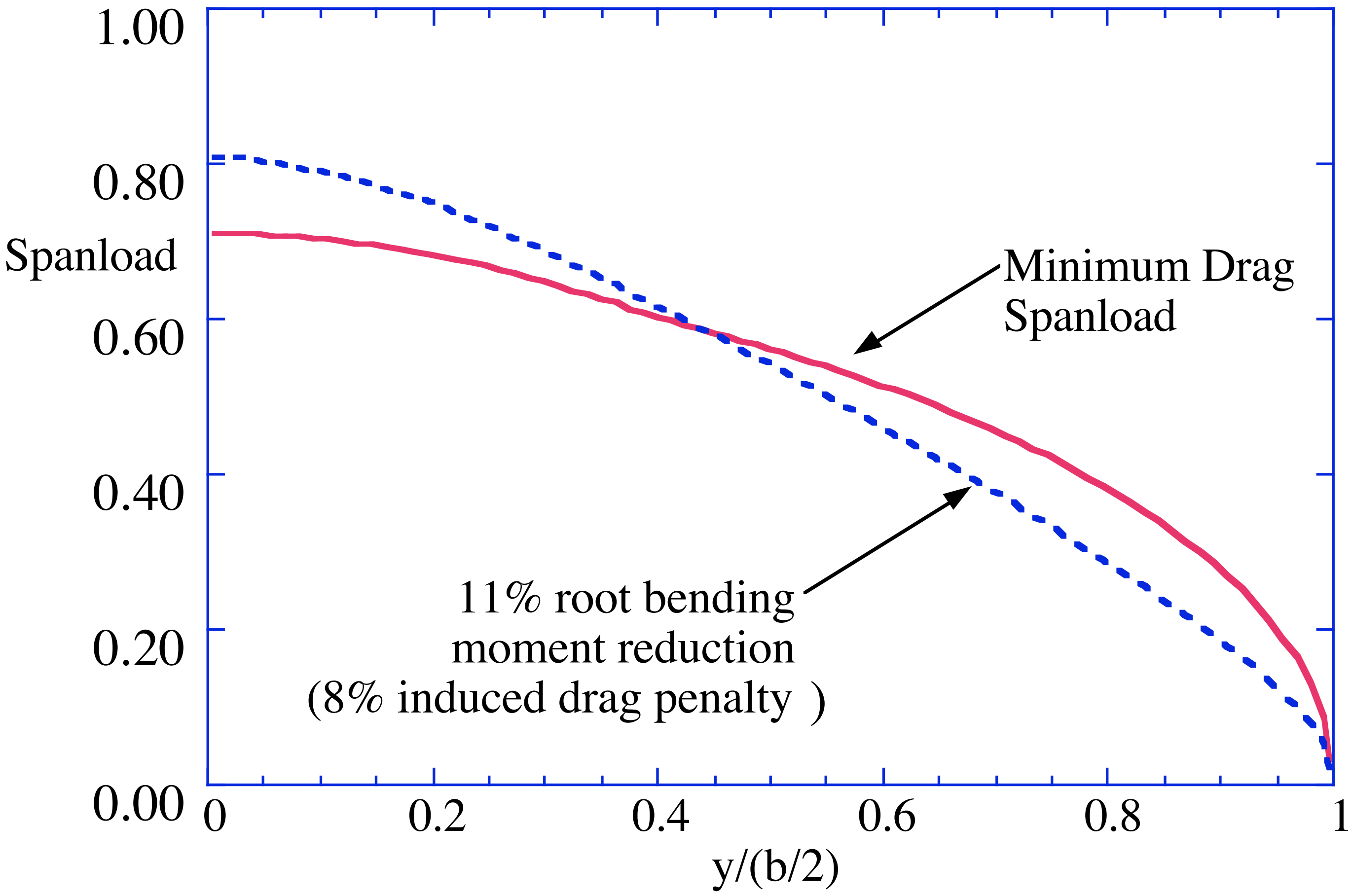
# Aerodynamic Model

- Uses a discrete vortex model with the calculations performed in the Trefftz plane.
- The particular aerodynamic theory was developed by Mickey Blackwell, assuming a flat wake.
- Our code is based on Joel Grasmeyer's implementation at Virginia Tech.
- Optimum spanloads are calculated using the method of Lagrange multipliers with constraints applied for lift, pitching moment and root bending moment coefficients.
- For a planar wing without a root bending moment constraint: *elliptic spanload*!

# Implementation of the Root Bending Moment Constraint

- The structural constraint is implemented while maintaining a constant planform shape and thickness distribution:
  - The spanload for minimum drag is found first *without* taking into account any bending constraints.
  - The root bending moment that this spanload produces is calculated.
  - The root bending moment is reduced from this minimum drag value.
  - A new spanload is calculated with the same lift coefficient and a reduced root bending moment constraint.
- The wing root bending moment constraint is implemented using the method of Lagrange multipliers.

# Example: Reduced Wing Root Bending Moment Result for a Typical Transport Wing Case



- A root bending moment reduction shifts the load curve inwards.
- The lift coefficient remains the same.
- Induced drag increases and wing weight decreases.

# Wing Weight Calculations

- The required bending material weight along a variable box beam is calculated by integrating the area under the bending moment curve. The bending material weight code was developed at Virginia Tech by Amir Naghshineh-Pour.
- The structural analysis uses a maximum load factor of 2.5 with a safety margin of 1.5. *Aeroelastic effects are neglected*.
- With the bending material weight ($w_1$) obtained, final wing weight calculations are performed with the equation from FLOPS (replacing the FLOPS bending material weight with results from Naghshineh-Pour's code):

$$W_{wing} = \frac{GW \cdot w_1 + w_2 + w_3}{1 + w_1}$$

*Note Gross weight (GW) is required!*

# Breguet Range Equation Implementation

- An average design lift coefficient and total drag coefficient are found at the cruise condition.
- At this design lift coefficient, the minimum induced drag spanload is found. It is then assumed that:

$$C_{D\_total} = C_{D\_rest} + C_{D\_induced(C_{L\_design})}$$

- The lift coefficient corresponding to the maximum allowable load factor is also calculated.
- At this lift coefficient, wing weight is found. It is then assumed that:

$$TOGW = W_{rest} + W_{wing} + W_{fuel}$$

- From the initial, minimum drag spanload, the root bending moment is reduced, producing more triangularly loaded lift distributions with an increased value of induced drag.

# Breguet Range Equation Implementation

- New loads corresponding to a root bending moment reduction, with maximum load conditions, are used in the structural model for wing weight calculations.
- The wing weight calculation requires knowledge of take-off gross weight:

$$W_{wing} = \frac{GW \cdot w_1 + w_2 + w_3}{1 + w_1}$$

- The Breguet Range equation solved for the take-off weight gives:

$$TOGW = (TOGW - W_{FUEL})\exp\left(\frac{Range \cdot sfc_{cruise} \cdot C_D\_total}{Speed \cdot C_L\_design}\right)$$

$$TOGW = (W_{Wing} + W_{rest})\exp\left(\frac{Range \cdot sfc_{cruise} \cdot (C_{D\_rest} + C_{D\_induced})}{Speed \cdot C_L\_design}\right)$$

- Iteration gives take-off gross weight and wing weight simultaneously.
- Fuel weight for the corresponding root bending moment reduction is:

$$W_{FUEL} = TOGW - W_{wing} - W_{rest}$$

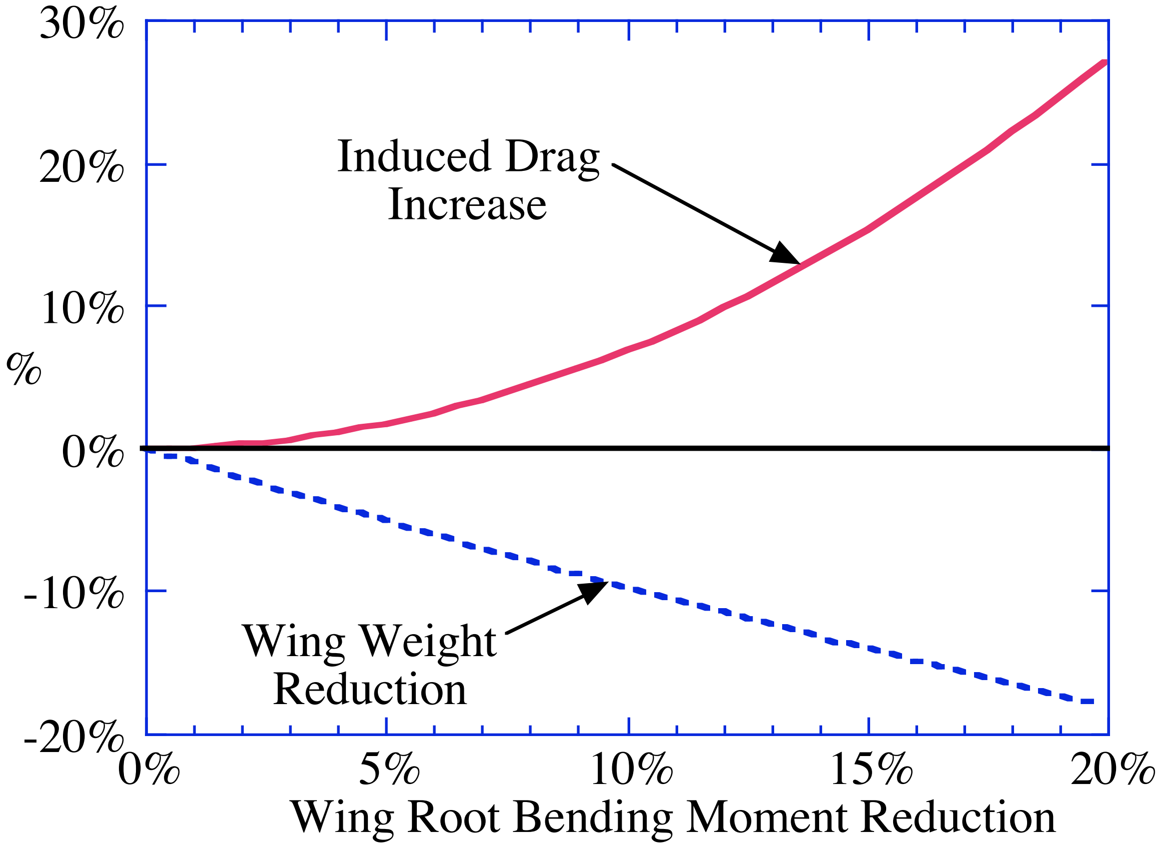
# B-777 Class Maximum Range Configuration

- Wing and tail planform geometry are given. Trimmed flight in pitch is assumed. Wing is composed of two lifting surfaces.
- Wing thickness to chord distribution is given for wing weight calculations.
- Engine data is used for engine inertia relief factors in weight calculations.
- Other inputs such as maximum load factor and center of gravity location are required.
- Performance specifications are assumed to correspond to aerodynamically optimum spanloads, that is, to elliptical load distributions.

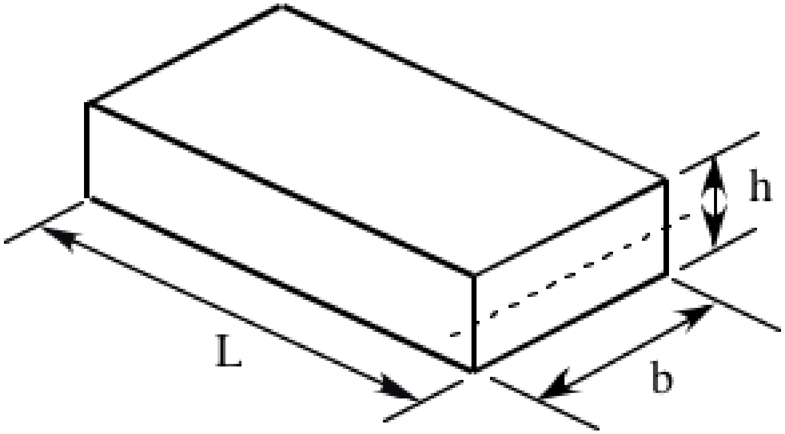
| | **PERFORMANCE SPECIFICATIONS** | |
|---|---|---:|
| 1 | Maximum Gross-Weight (lbs) | 588893 |
| 2 | Fuel Weight (lbs) | 215000 |
| 3 | Maximum Range (nm) | 7600 + 500 reserve range |
| 4 | Cruise Mach Number | 0.85 |
| 5 | Cruise Altitude (ft) | 40000 |
| 6 | Static Specific Fuel Consumption (lb/hr/lb) | 0.29 |

# Wing Weight Reduction and Induced Drag Increase. B-777 type aircraft.

- Induced drag increases parabolically from aero optimum.
- Wing weight decrease is nearly linear.
- ***Note!*** Therefore, a small root bending moment reduction will always be beneficial



# Wing weight is linearly proportional to the wing root bending moment?



$$\sigma = \frac{My}{I}, \quad I = \frac{bh^3}{12}$$

$$\sigma_{allowable} = \frac{M\frac{h}{2}}{\frac{bh^3}{12}} = \frac{6M}{bh^2}$$

$$W \propto bhL \Rightarrow \quad bh \propto \frac{W}{L}$$

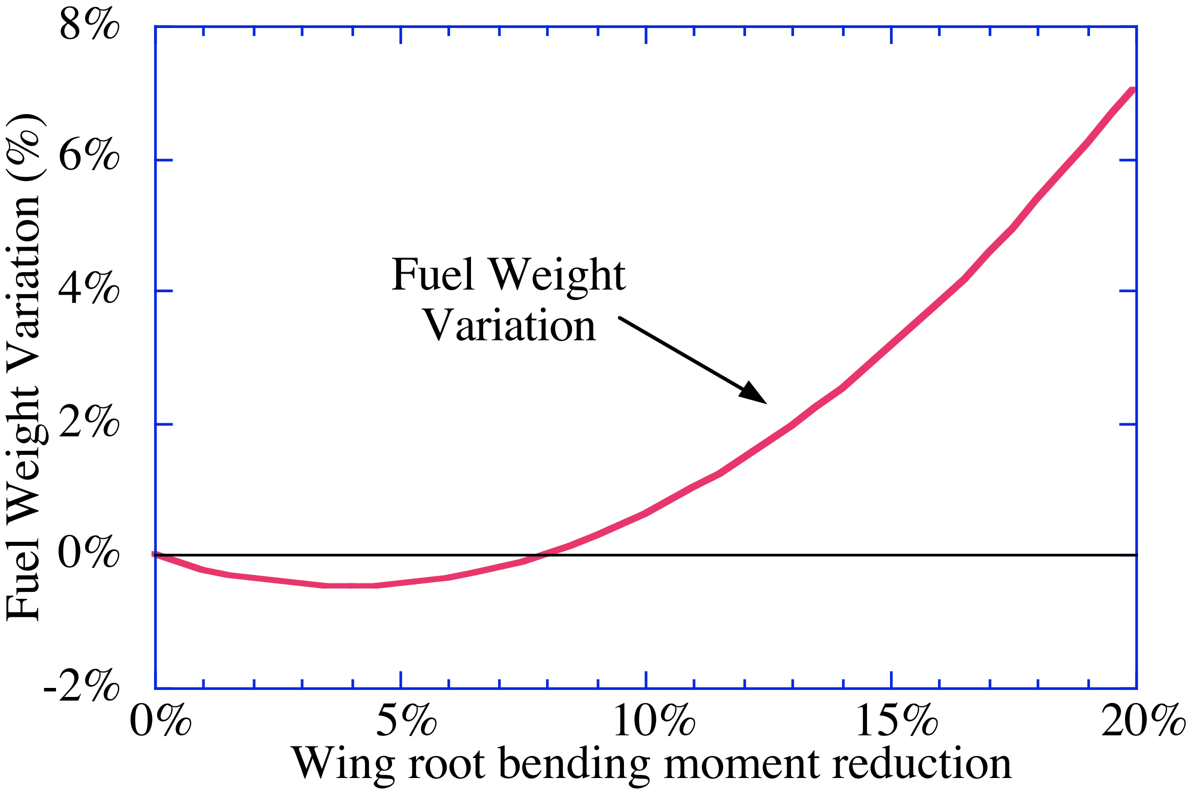
$$or \quad bhh = \frac{6M}{\sigma_{allowable}}$$

And combining yields: $\boxed{W \propto \frac{L}{h}\frac{M}{\sigma_{allowable}}}$

Thanks to Prof. Eric Johnson!

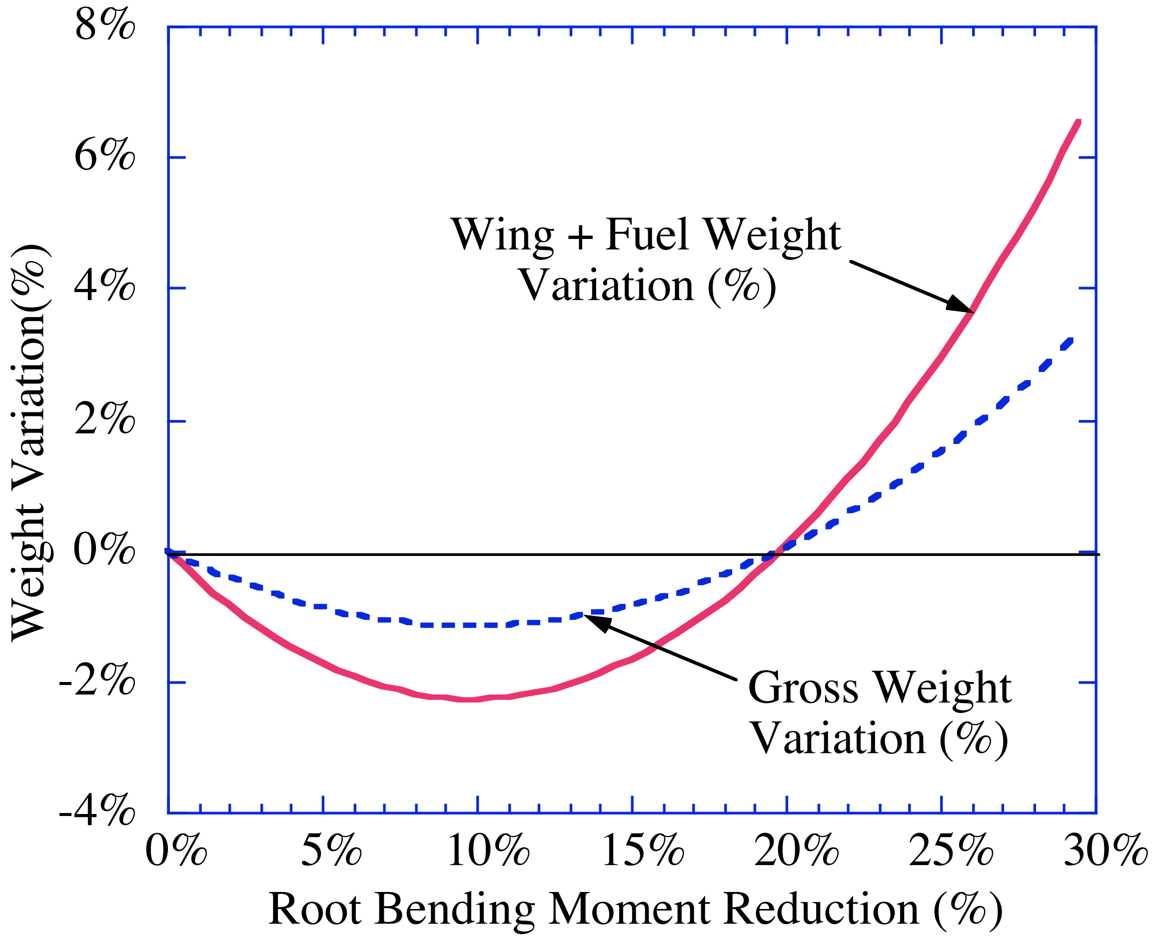
# Fuel Weight Variation.
# B-777 type, maximum range configuration.

- For low root bending moment reductions fuel weight is reduced due to the slow induced drag increase in this range.
- Large fuel weight penalties are obtained for high bending moment reductions, corresponding to very triangular load distributions with high load values at the wing root.

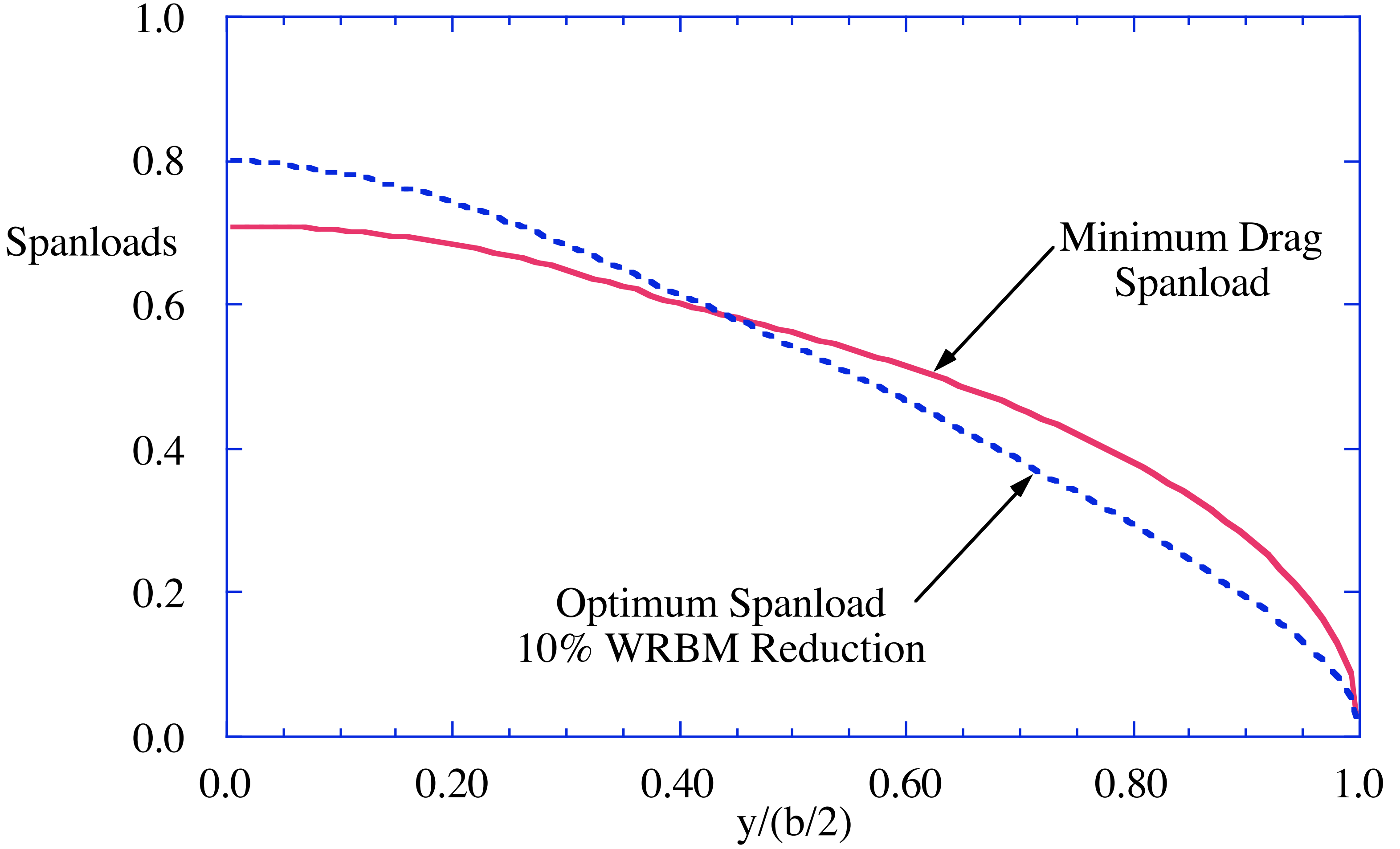


# Wing+Fuel and Gross Weight Variation. B-777 type, maximum range configuration.

- Maximum gross weight reductions of about 1% can be obtained.
- Minimum gross weight found for a root bending moment reduction of 10%.
- Shorter range aircraft are expected to experience higher benefits since they are more driven by structures than by aerodynamics.



# Spanload for Max TOGW Reduction
# B-777 type, maximum range configuration.



# Reduced Mission Range Studies

- The spanloads generated with the root bending moment constraint can affect weights in a different way depending on the mission range performed.
- $W_{rest}$ and $C_{D\_rest}$ still have the same value they had for the maximum range configuration.
- The induced drag coefficient for reduced root bending moments is found with the aerodynamics code.
- Wing weight is taken from the maximum range study.
- Take-off weight is found with the equation:

$$TOW = (W_{Wing} + W_{rest})\exp\left(\frac{Range \cdot sfc_{cruise} \cdot (C_{D\_rest} + C_{D\_induced})}{Speed \cdot C_{L}\_design}\right)$$

# B-777 Class Aircraft. Reduced Ranges

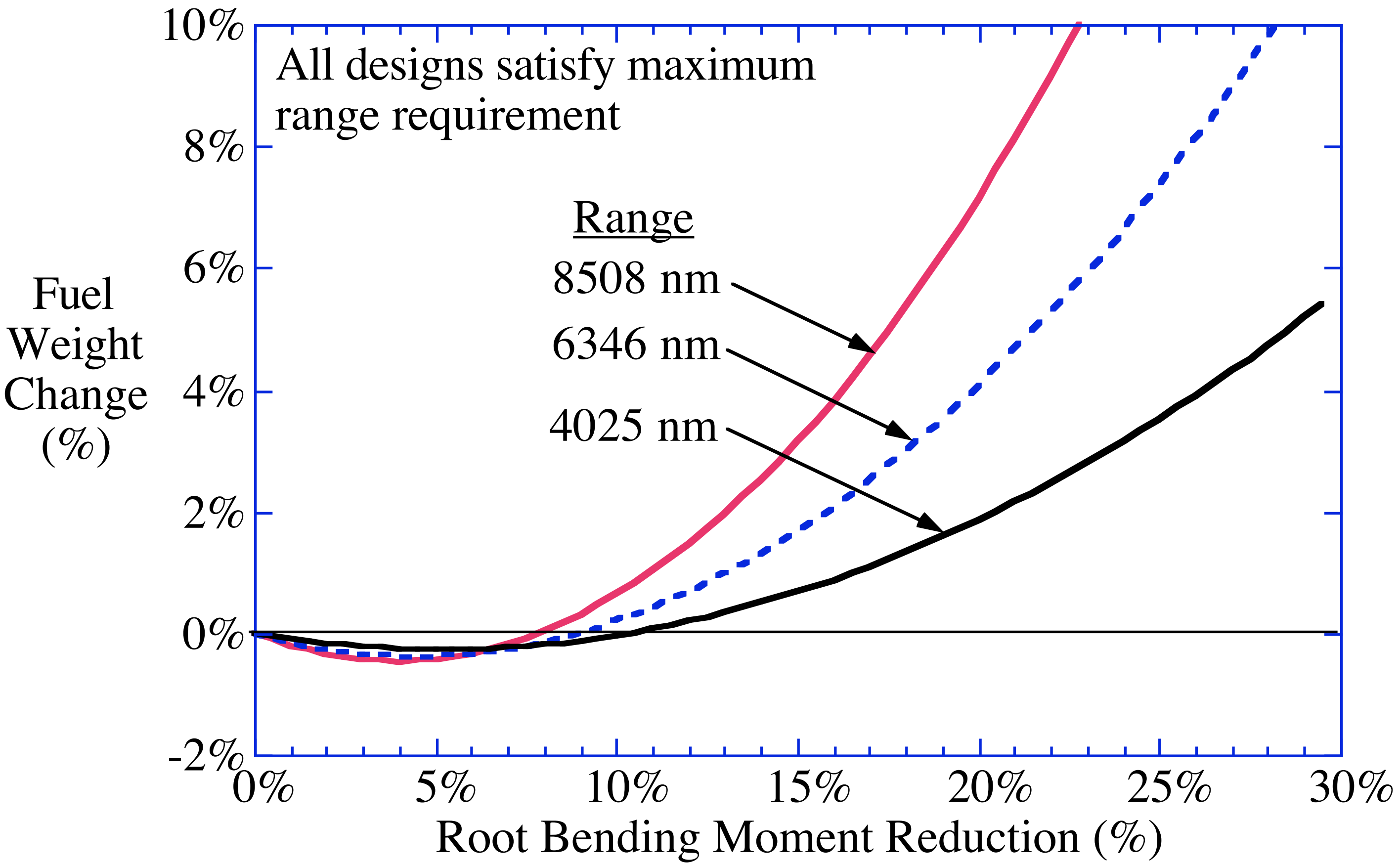
- A B-777 type aircraft configuration is studied with reduced fuel loads corresponding to ranges from about 8000 to 4000 nautical miles, typical mission ranges for this aircraft.

| Case study | Mission Fuel Weight | Mission Range |
|---|---:|---:|
| 1 | 215000 lbs. | 8508 nm |
| 2 | 185000 lbs. | 7446 nm |
| 3 | 155000 lbs. | 6346 nm |
| 4 | 125000 lbs. | 5205 nm |
| 5 | 95000 lbs. | 4025 nm |

- Fuel weight, wing plus fuel weight and take-off weight variations are studied as a function of root bending moment reduction.
- These weight variations are non-dimensionalized by maximum weights:

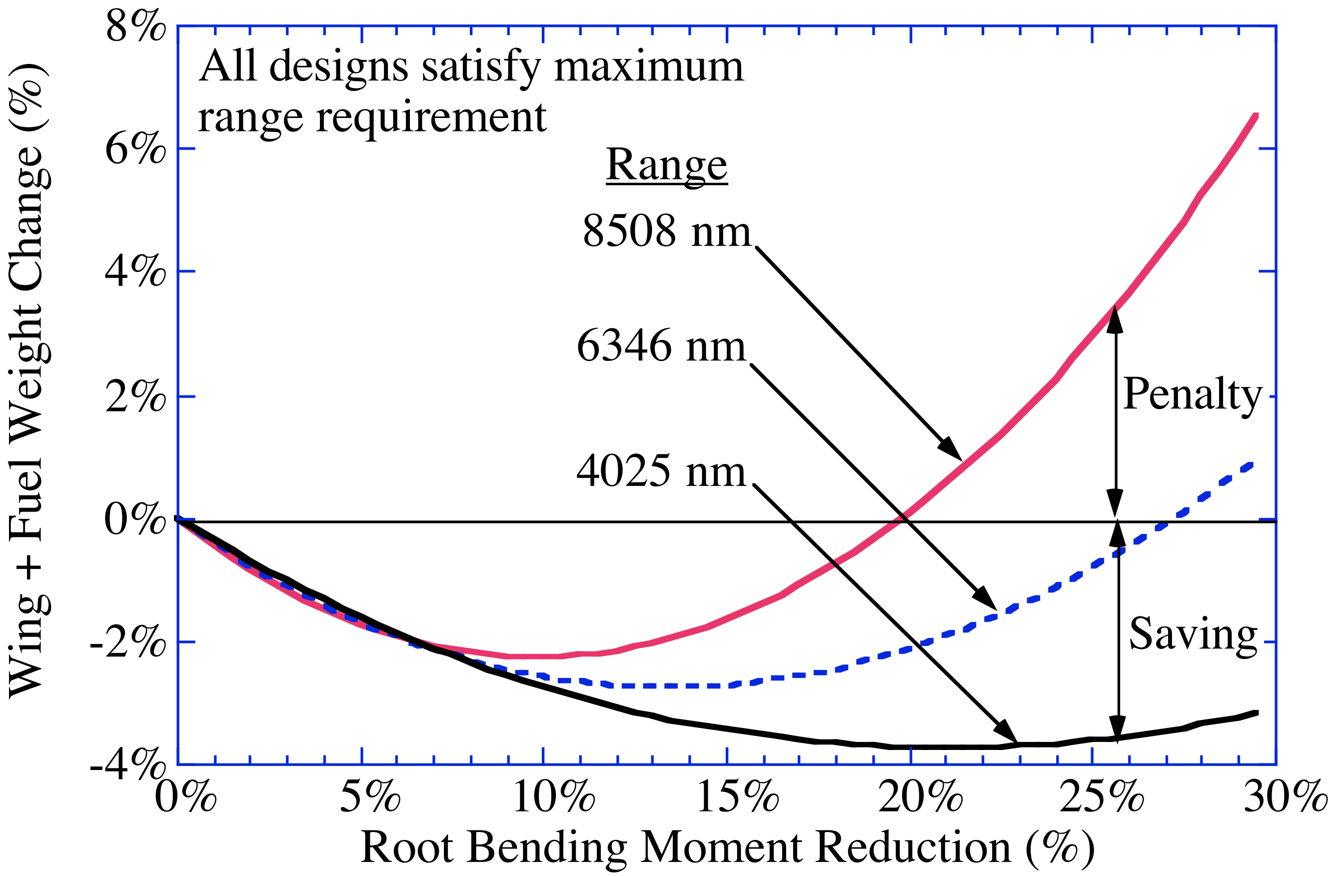
$$Fuel\_Weight\_Variation = \frac{W_{FUEL}(new) - W_{FUEL}(initial)}{W_{FUEL}(\max\_range)}$$

# Fuel Weight Variation for Different Ranges.



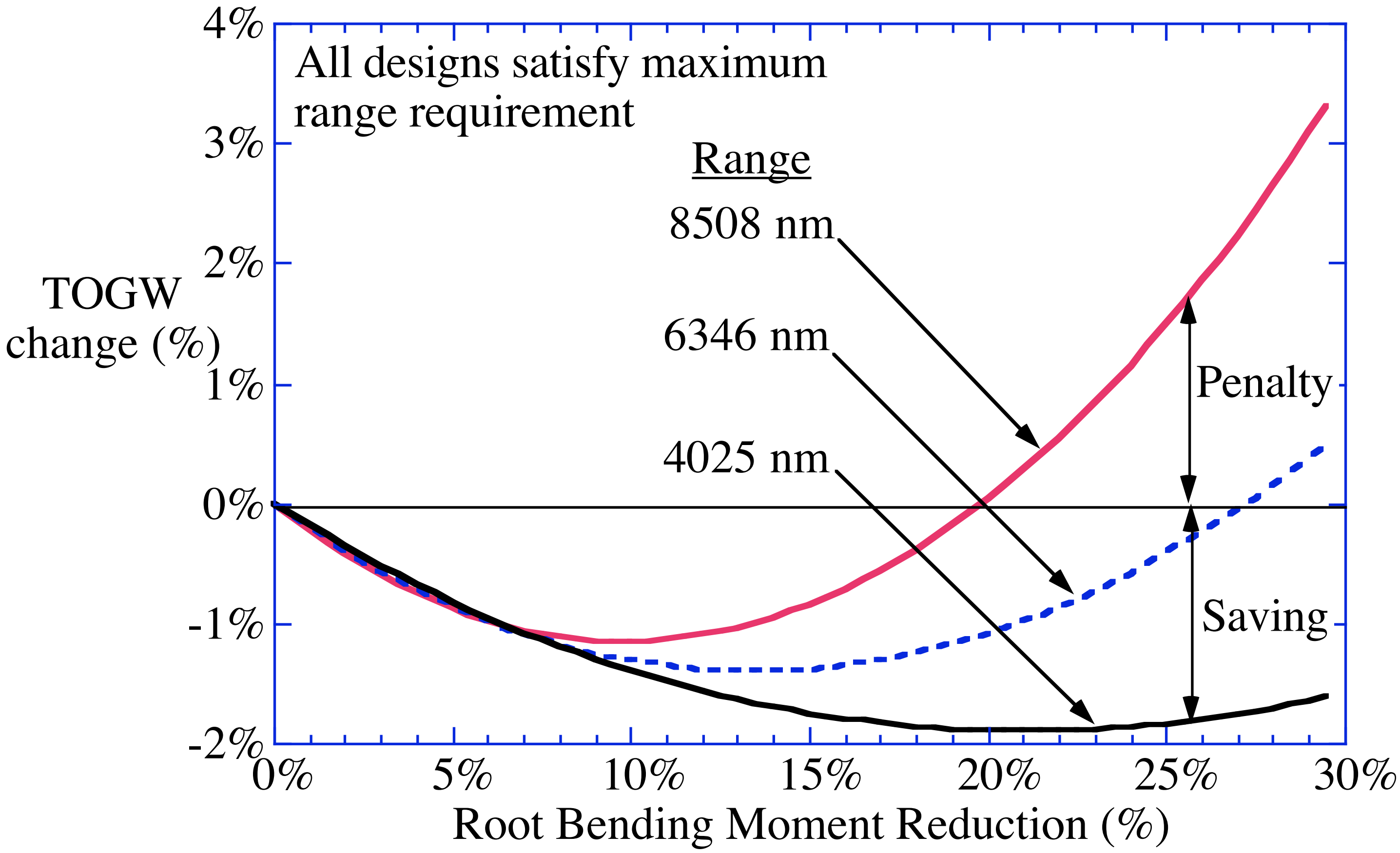
- Low root bending moment reductions: fuel weight variations are similar.
- For high root bending moment reductions, the needed fuel weight to complete the mission increases more sharply for high mission ranges.

# Wing+Fuel Weight Variation for Different Ranges



- Larger weight reductions can be achieved for lower ranges.
- The reduced range optimum corresponds to higher root bending moment reductions.

# Take-off Weight Variation for Different Mission Ranges



- Takeoff weight variations: almost double for reduced ranges.
- A reduced range optimum spanload can result in weight penalties when performing the maximum mission range.

# Conclusions for Spanload Optimization with a Root Bending Moment Constraint

- The system minimum will always occur for a spanload with a lower wing root bending moment than the aerodynamic optimum.
- Larger take-off weight reductions can be achieved for reduced mission ranges with more triangular spanloads.
- This methodology fits naturally in an MDO approach.
- Aircraft configurations must be studied through the range of operating missions, since a specific spanload can give different benefits from mission to mission.

# ?

http://www.thefighterenterprise.com/image_gallery/pr_photos/jsfpr_photos/jsf_1stflight/x350370d.html

courtesy Geoffrey Buescher