Lower Triangular Matrices

Forward Solution

A matrix equation of the form

$$\boldsymbol{A\,x \;=\; b}$$

can be solved by *forward substitution* if the matrix A is in lower triangular form. This means we can solve for $x(1)$ by simple arithmetic, then solve for $x(2)$ using the known $x(1)$ and so on. Suppose we have an $n \times n$ Matlab array a in lower

triangular form and a $n \times 1$ Matlab array `b`. Write a Matlab code segment to solve for the unknown x using the forward substitution procedure.

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Forward Solution

➤ **The form of the matrix is**

$$\begin{pmatrix} a(1,1) & 0 & \dots & 0 \\ a(2,1) & a(2,2) & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ a(n,1) & a(n,2) & \dots & a(n,n) \end{pmatrix}$$

- **The first equation reads**

$$a(1,1)\,x(1) + 0\,x(2) + \ldots + 0\,x(n) = b(1)$$

- **which is easily solved as**

$$x(1) = b(1)/a(1,1)$$

- **The k-th equation reads**

$$a(k,1)\,x(1) + a(k,2)\,x(2) + \ldots$$
$$+ a(k,k)\,x(k) + 0\,x(k+1) + \ldots$$
$$+ 0\,x(n) = b(k)$$

- In the forward solution procedure at this stage we have already solved for

$$x(1),\ x(2), \ldots,\ x(k-1)$$

- Move the known quantities to the right hand

side and solve to find

$$x(k) = \left[b(k) - \sum_{i=1}^{k-1} a(k,i)x(i) \right] / a(k,k)$$

- in Matlab this is written

  $\texttt{x(k)} = (\texttt{b(k)} - \texttt{a(k, 1 : k - 1)} * \texttt{x(1 : k - 1)}) / \texttt{a(k, k)}$

- We can avoid ambiguity about whether `x` is a $(1 \times n)$ (row) or a $(n \times 1)$ (column) in several ways.

  1. pre-allocate it $\texttt{x} = \texttt{zeros(n, 1)}$
  2. explicitly label it $\texttt{x(1, 1)},\ \texttt{x(k, 1)}$

Root-finding by Bisection

Apply one step of the *bisection* method to the problem of finding a zero of

$$f(x) = \cos(x) - x * \sin(x)$$

on the interval $[0, \pi/2]$. Note that $f(0) = 1$ and that $f(\pi/2) = -\pi/2$.

Bisection

- We evaluate at the mid point $x_m = \pi/4$

$$f(\pi/4) = 0.1517\ldots$$

- Since $f(x_m)$ has the same sign as $f(0)$ we replace the *lower bound* with x_m.
- We now have the reduced interval $[\pi/4,\ \pi/2]$
- We again find the mid-point and proceed.

Problem 5

Sub-arrays

- **The matrix is**

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}$$

- **The sub array** $\mathtt{a}(:, 2:3)$ **consists of all the rows**

and columns 2 and 3

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \\ 8 & 9 \end{pmatrix}$$

- **Evaluate from the inside to the outside**

  `a(:)` **is a** 12×1 **array (column)**

  `a(:)'` **is a** 1×12 **array (row)**

  `size(a(:)')` **is** (1×12)**.**

- **$a(3, 2 : 4)$ denotes row 3 and columns 2 through 4**

  Since the size of the array is (4×3) the expression cannot be evaluated. It produces an error.

Problem 6

Using the array *a* described in Question 5 examine each of the following statements. If the expression is not legal indicate an error. If the expression is legal indicate its *size*.

i) `a(:,3) > a(3,2:4)'`

ii) `a + a(1,2)`

iii) `a(1,:)  - a(2:4,1)'`