Airfoils and Wings

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1 Introduction

The primary purpose of these notes is to supplement the text material related to aerodynamic forces. We are mainly interested in the forces on wings and complete aircraft, including an understanding of drag and related nomeclature.

2 Airfoil Properties

2.1 Equivalent Force Systems

In some cases it's convenient to decompose the forces acting on an airfoil into components along the chord (chordwise) and normal to it. These forces are related to lift and drag through the geometry shown in Figure 1. From

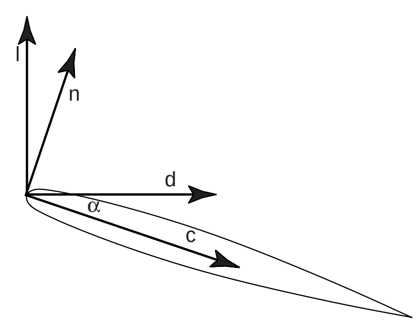

Figure 1: Force Systems

the figure we have

$$c_\ell(\alpha) = c_n(\alpha)\cos\alpha - c_c(\alpha)\sin\alpha$$
$$c_d(\alpha) = c_n(\alpha)\sin\alpha + c_c(\alpha)\cos\alpha$$

Obviously, we can also express the normal and chordwise forces in terms of section lift and drag.

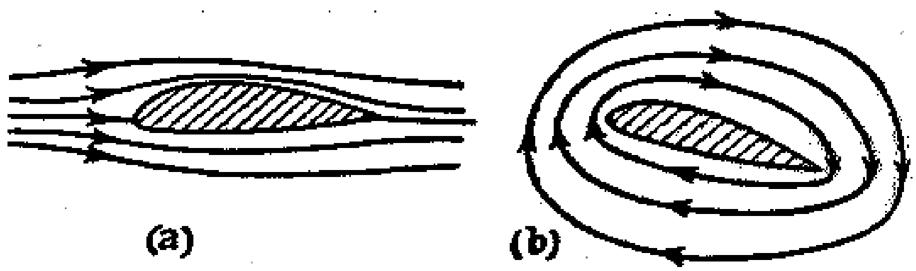

Figure 2: Flow Decomposition

2.2 Circulation Theory of Lift

A typical flow about a lift-producing airfoil can be decomposed into a sum of two flows, as shown in Figure 2. The first flow (a) is 'symmetric' flow and so produces no lift. The 'circulatory' flow (b) is responsible for the net higher speed (and hence lower pressure) on the top of the airfoil (the suction side). This can be quantified by introducing the following line integral

$$\Gamma_{\mathcal{C}} = \int_{\mathcal{C}} \overrightarrow{u} \cdot d\overrightarrow{s}$$

This is the *circulation* of the flow about the path $\mathcal{C}$. It turns out that as long as $\mathcal{C}$ surrounds the airfoil (and doesn't get too close to it), the the value of Γ is independent of $\mathcal{C}$. The circulation is a property of the flow (the airfoil at the given angle of attack). Such a circulation can be produced by imagining a cyclone like flow with circular streamlines. The speed along any streamline varies inversely with radial distance r from the center. This last feature will make Γ the same along any streamline, and, it turns out, along any contour that simply encircles the center of the cyclone. We use the term *vortex* to describe such a flow.

The net result of this view is that we can reproduce the lift properties of the airfoil by replacing it with a vortex at the center of pressure. Additional analysis shows that the lift (per unit span) is related to the circulation by

$$L' = \rho_\infty V_\infty \Gamma,$$

where ρ_∞, V_∞ are the free-stream values of air density and velocity, respectively.

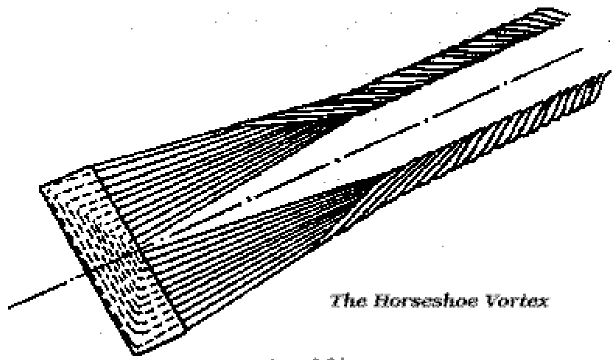

Figure 3: Horseshoe Vortex System

3 Three-dimensional Aerodynamics

The typical geometry of wings has been introduced earlier. Here we explore the implications of the circulation theory introduced above. Since the 3-D wing can be thought of as a distribution of 2-D sections, when we replace each section by its vortex we end up with a line of vorticies. At the wing tip(s) something has to happen, because the vortex line cannot simply end. From another point of view, we expect relatively high pressure on the bottom of the wing and low pressure on the top. It's clear that at the end of the wing there would be a pressure field inducing a flow around the wing-tip. We can combine these observations by suggesting that the vortex along the wing turns sharply and proceeds along the direction of the flow. This produces the *horseshoe* system shown in the Figure 3. An idealized version is shown in Figure 4. The ideal system consists of the *bound* vortex carried in the wing and the two trailing vorticies (counter-rotating). In theory, the system is closed off by the *starting* vortex, left back at the airport when the aircraft started to fly. The effects of the trailing vortices can be commonly seen in the 'contrails' of high flying jet aircraft. Some of the water vapor in the jet exhaust is entrained in the trailing vorticies. The sunlight makes the condensed vapor visible.

3.1 Induced Downwash Angle

The circulatory flow from the trailing vorticies add a downward component of the flow. Thus, at a given spanwise location, the flow, instead of being along the $\overrightarrow{V}_\infty$ direction is rotated downward through an angle ϵ, the downwash angle. Since $C_L \propto \Gamma$ and $\epsilon \propto \Gamma$ we have a similar relation between the

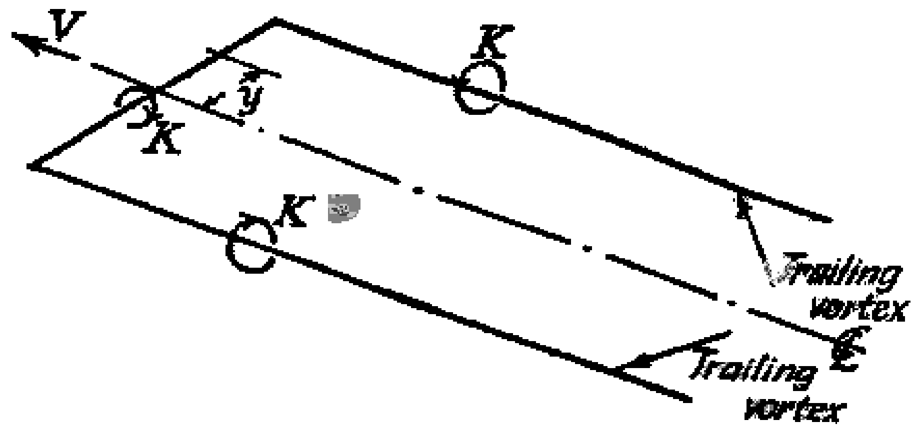

Figure 4: Idealized Horseshoe Vortex System

lift coefficient and the downwash angle. For the simplest case it turns out that ϵ is constant along the span and is given by

$$\epsilon = \frac{C_L}{\pi AR}.$$

This idealization corresponds to a case where, for example, the section properties are the same along the span and the wing planform is elliptic. The Supermarine Spitfire, so famous in the Battle of Britain, was built with such a planform design.

Because of this rotation we have

$$\alpha = \alpha_\infty + \frac{C_L}{\pi AR}.$$

This, in turn implies that

$$\frac{d\,\alpha}{d\,C_L} = \frac{d\,\alpha_\infty}{d\,C_L} + \frac{1}{\pi AR}\ ,$$

or, upon solving for the 3-D lift-curve slope

$$C_{L\alpha} = \frac{a_\infty}{1 + a_\infty/\pi AR}\ ,$$

where $a_\infty = \frac{d\,C_L}{d\alpha_\infty}$ is the lift-curve slope for the airfoil section. Thus, one important effect of the finite wing-span (3-D) is that the lift-curve slope is diminished from the section value (a_∞). The effect is more pronounced for short-stubby wings. Note that the a_∞ term in the denominator must be in radian measure.

A second, even more important effect, appears in the drag evaluation. Note that since the local section flow has been rotated, the local lift is slightly tilted so that a component of it is aligned with the free-stream velocity. For small angles we have

$$C_{D\imath} = C_L \cdot \epsilon = \frac{C_L^2}{\pi AR}.$$

This *lift-induced* drag plays an important role in the overall aerodynamic characteristics of the vehicle.

To account for other than the ideal ellipic-loading case, we commonly use correction factors and write:

$$\epsilon = \frac{C_L(1+\tau)}{\pi AR}$$

and

$$C_{D\imath} = \frac{C_L^2(1+\delta)}{\pi AR}.$$

The parameters τ and δ can be estimated for other planforms. Note that the modified lift-curve slope for the wing is now:

$$C_{L\alpha} = \frac{a_\infty}{1 + a_\infty(1+\tau)/\pi AR}.$$

4 Drag Breakdown

For some purposes it's convenient to decompose the drag on an object into its *components*; for example as a guide to estimating the drag on a proposed design. Unfortunately, the nomenclature associated with such drag breakdown is not very universal. At the first level we recall that fluid forces can be transmitted as normal pressure or as shear (tangential friction). This leads us to decompose into:

- Surface friction drag: this is the drag arising from tangential shear stresses operating along the *wetted area* - the surface that the fluid contacts;

- Normal pressure drag: this is the drag arising from normal pressure forces operating at the boundary.

The normal pressure drag is itself decomposed into several contributions. Before enumerating these we observe that in an ideal fluid (*i.e.* frictionless) the normal pressure drag on an object immersed in the flow is zero. This is known as D'Alembert's Paradox. In a real (viscous) fluid the normal pressure will differ from the ideal case, because, in effect the viscosity slows the fluid close to the body, effectively changing its apparent shape. The normal pressure drag is thus decomposed as:

1. form-drag: pressure drag arising from the viscous effects on the normal pressure field;
2. vortex drag: also known as induced drag, this is the drag associated with the effect of trailing vortices on 3-D bodies;
3. wave drag: drag associated with the formation of shock waves in high speed flight

4.1 Form Drag

Figure 5 shows the effect of viscosity on normal pressure in two flows. For the circular cylinder case on the top, the effect of viscosity on the normal pressure field is quite drastic. As expected, the inviscid case produces a pressure profile that is symmetric (left to right) so that the normal pressure force in the flow direction is zero. The real flow (shown dashed) is quite different. In this case one might expect that 90% of the drag is form drag, while 10% is due to tangential shear. For the streamlined body on the bottom, the effect is perhaps less drastic and the form drag and surface friction drag might be about equal. These are two-dimensional, incompressible flows with no vortex drag and no wave drag.

4.2 Wave Drag

This is the drag associated with the formation of shock waves that occur when the flow is supersonic. Our intent here is to give the student some appreciateion of the phenomena. We know that the flow over a lifting surface is generally faster than the free-stream flow. As we increase the free-stream speed, the flow will reach local supersonic conditions along the upper part (suction-side) of the surface. The flight Mach number for which the flow first becomes locally sonic is the called the *critical* Mach number. If the speed is increased further then some wave drag is encountered as the free-stream Mach number increases towards unity. We define the *divergence*

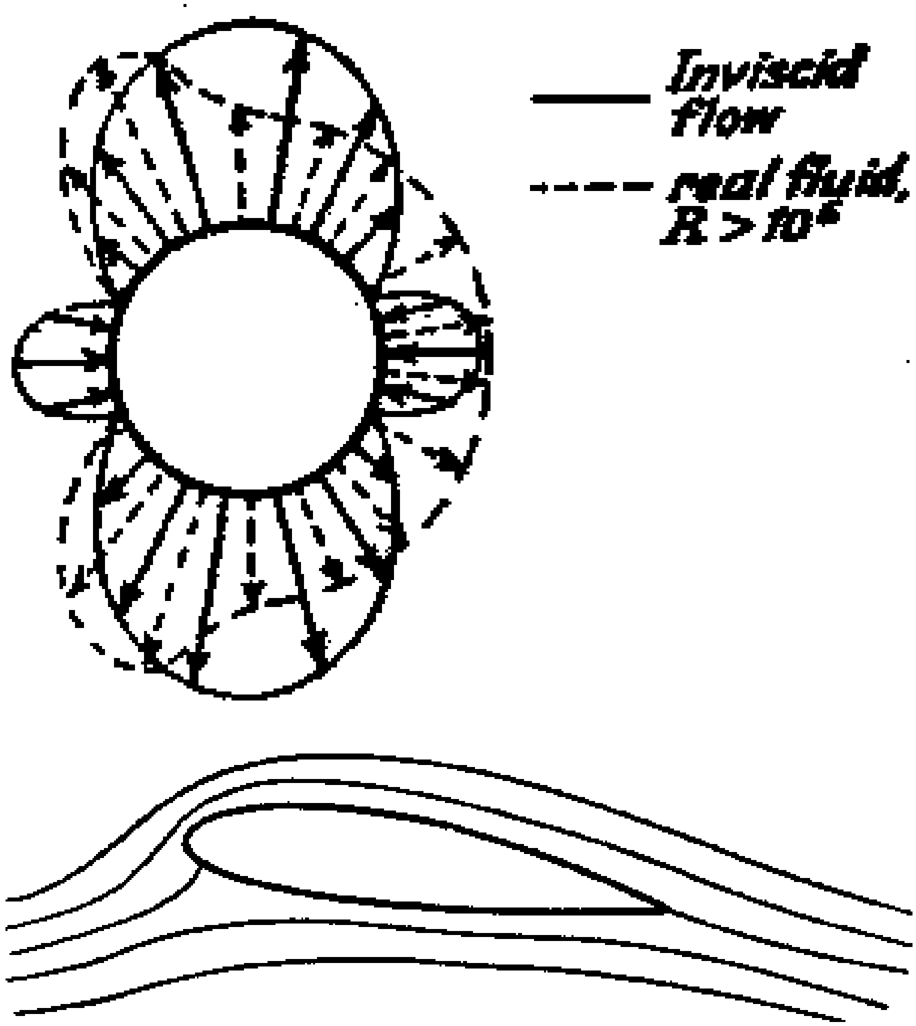

Figure 5: Form Drag in Two Cases

Mach number as the value of M for which the slope, $d\,C_D/d\,M|_{M_c} = .1$. It happens that the important quanitity in the formation of the shock system is the component of velocity normal to the wing's leading edge. This leads to using wing-sweep as a mechanism to delay the drag-rise.

At supersonic speeds we can develop a theory of *slender-body* drag. This is based on the notion that the flow must be turned to go around the body and leads us to consider the distribution of the cross-sectional area as we traverse from the nose to the tail. The essential feature is the slope of this cross-sectional area (say $S'(x)$) and the key is to make the $S(\cdot)$ distribution smooth. R. Whitcomb suggested the *area-rule*; for example, as we reach the values of x where the wing is thick, we should narrow the fuselage to maintain a nice area distribution. This leads to the *coke-bottle* shape of high-performance aircraft. The key to low drag is a smooth distribution of cross-sectional area.

5 Airplane Drag

Since a complete aircraft is a combination of lifting surfaces we find yet another way to separate drag effects as an enumeration of a sum:

$$C_D = C_{Do} + C_{Di} + C_{Dr},$$

where C_{Di} is the vortex drag on the wing, C_{Do} is the remaining drag on the wing, and C_{Dr} is the residual drag on the fuselage, nacelles, empennage, *etc.* The sum of the first and the last is called *parasite* drag; that is,

$$C_{Dp} = C_{Do} + C_{Dr}.$$

It happens that the parasite drag coefficient is weakly dependent on the lift coefficient (angle-of-attack). We use the approximation

$$C_{Dp} \approx C_{Dpe} + \left(\frac{d\,C_{Dp}}{d\,C_L^2}\right) C_L^2.$$

In this expression the C_{Dpe} term and the derivative-term are evaluated at a point - thus these are constants. This type of C_L dependence is pretty natural, since the vortex drag is also quadratic in C_L.

Substituting this into the complete drag coefficient we find:

$$\begin{aligned} C_D &= C_{Dp} + C_{Di} \\ &= C_{Dpe} + \left[\pi A\!\!\!R\left(\frac{d\,C_{Dp}}{d\,C_L^2}\right) + (1+\delta)\right](1/\pi A\!\!\!R)C_L^2. \end{aligned}$$

To simplify we define the Oswald efficiency factor by

$$e \equiv \left[\pi A\!\!R\left(\frac{d\,C_{Dp}}{d\,C_L^2}\right) + (1+\delta)\right]^{-1},$$

so that the drag expression becomes

$$C_D = C_{Do} + \frac{C_L^2}{\pi A\!\!R e}.$$

The efficiency factor is generally in the range $.8 < e < .95$. Note that e can be interpreted as

$$e = \frac{d\,C_{Di}/d\,(C_L^2)}{d\,C_D/d\,(C_L^2)};$$

that is, what fraction of the slope of the C_D *vs* C_L^2 comes from the induced drag term ?

5.1 Trim Drag

The tendency to decompose the aircraft into its pieces, tends to obscure requirements that apply to the complete configuration. For example, in equilibrium flight one must have zero total pitching moment about the *c.g.* The decomposition suggested above will generally lead to a configuration that is not in pitch equilibrium. To achieve this equilibrium one must adjust the pitch control surface(s) (elevator, flaps, cannard) to zero out the moment. This will re-distribute the lift and generally change the drag. The increment in drag is commonly called *trim-drag*. It's common to ignore trim-drag in an early design exercise, but as the design matures and more information is available one should re-visit the drag model with the trim requirement enforced.

6 References

Look at the Webpage for Prof. Mason's Applied and Computational Aero course `http://www.aoe.vt.edu/aoe/faculty/Mason_f/CAtxtTop.html`.