

- Application of the M.P. leads to boundary-value problem(s) including
  - 2n state/adjoint differential

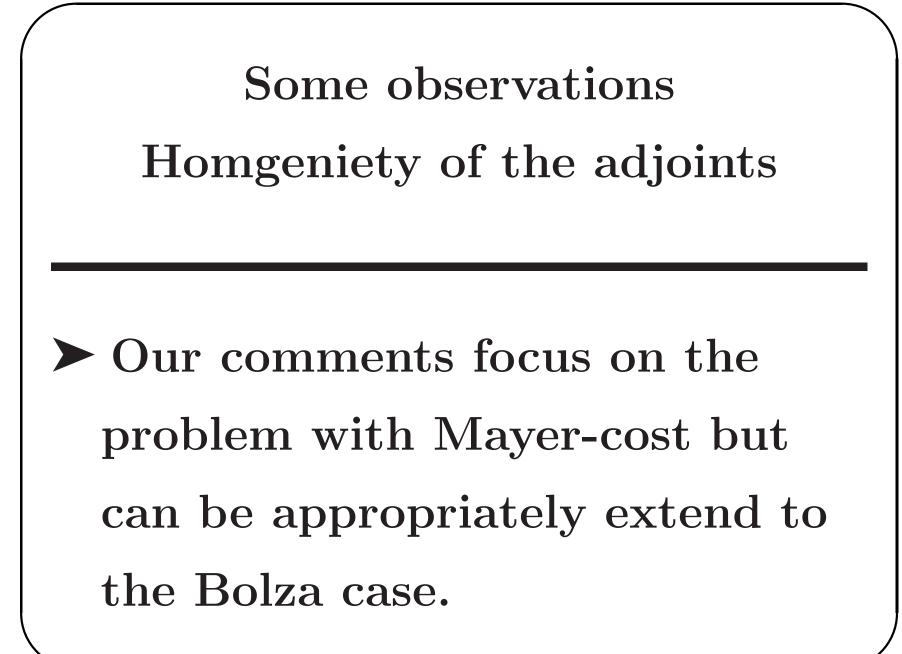
## equations

- extremal control choice from  $\min_{u\in\Omega} H$
- initial state/transversality boundary conditions
- final state/transversality boundary conditions

• possible first-integral from  $d H \quad \partial H$  $\frac{1}{dt} = \frac{1}{\partial t}$ ► Numerical solution is accomplished by formulating a Newton root-finding problem, wherein missing state/adjoint boundary values are the

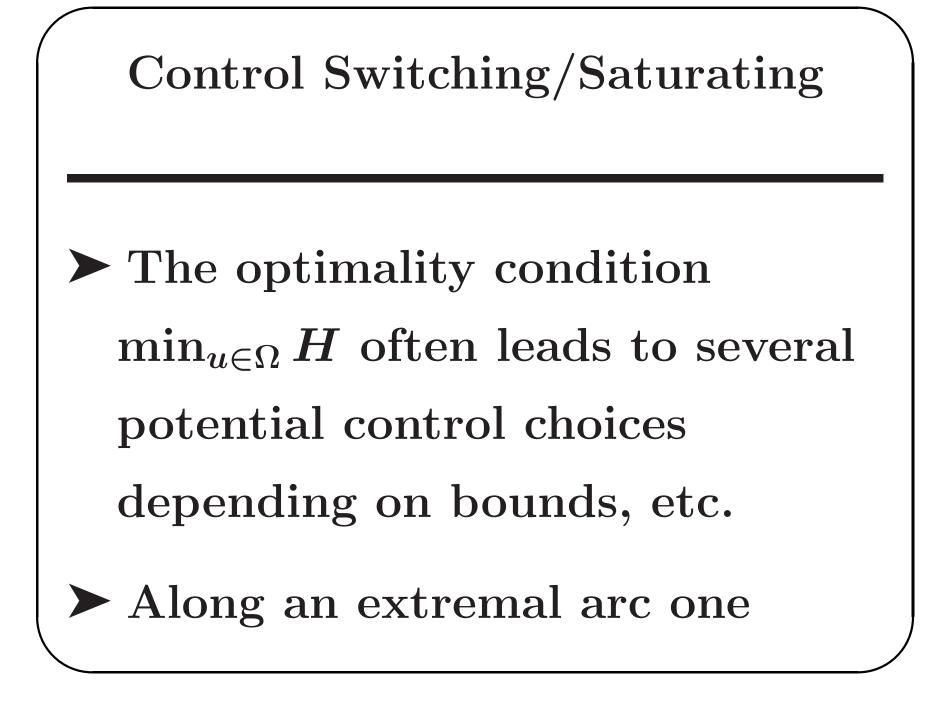
unknown parameters. Given estimates of these parameters, one can *solve* the initial-value problem and test the specified end-conditions.

Several ideas/insights are useful in this task.



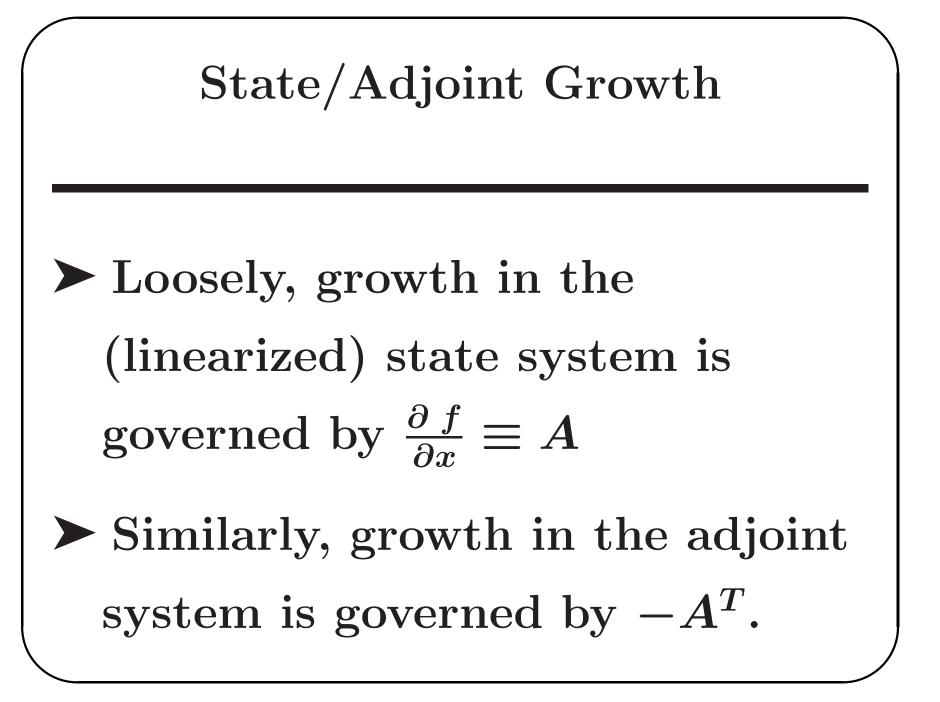
► Mulitplying the cost-functional g by a positive constant (say a) amounts to a change in J-units and has no effect on the problem. The adjoint differential equations are linear in the adjoint variables

► The optimality condition  $\min_{u \in \Omega} H$  is unchanged if all adjoints are multiplied by a positive constant. > If  $(\lambda_0, \vec{\lambda}(\cdot))$  leads to an extremal state/control pair  $(x^*(\cdot), u^*(\cdot), u^*(\cdot),$ then  $(a\lambda_0, a\vec{\lambda}(\cdot))$  leads to the same state/control pair.



encounters times at which the control (switching) or its time-derivative (saturation) is discontinuous. ► Numerical IVP solvers commonly use some form of smooth extrapolation, eq representing the solution as a

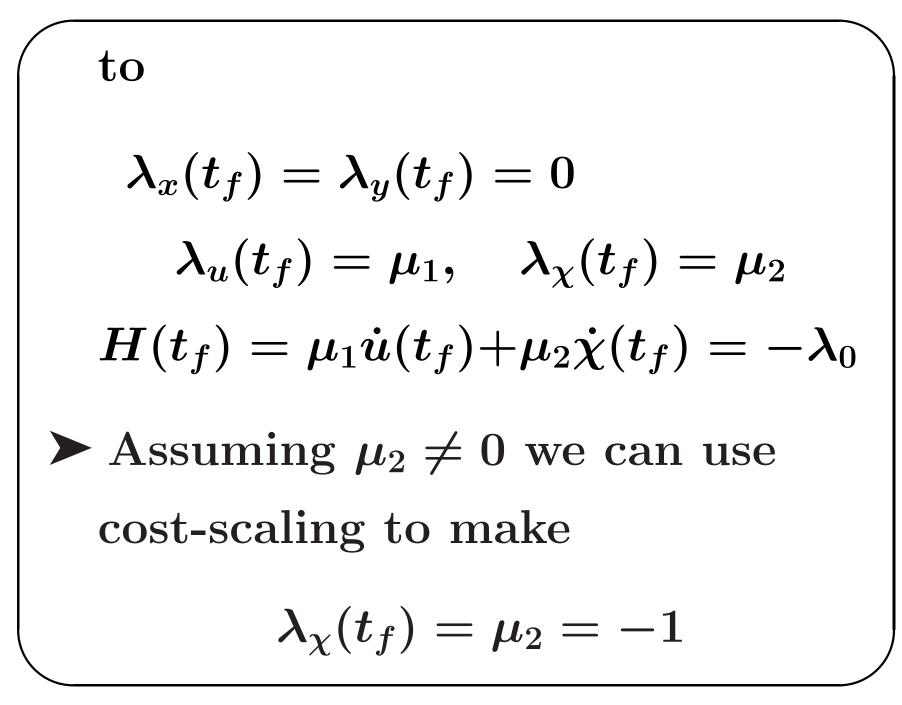
4/5th degree polynomial in t. ► Discontinuities in the control (or its time derivative) in the interior of an integration step degrade solution accuracy ► It's best not to integrate through these points.

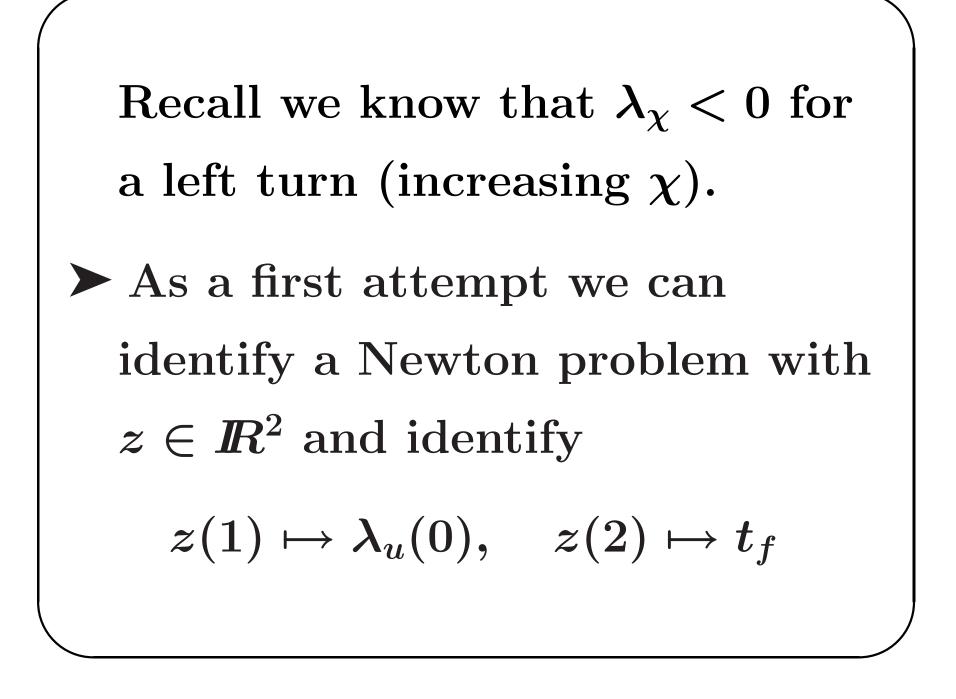


 $\blacktriangleright$  If  $\sigma$  is an eigenvalue of the linearized state/adjoint system, then so is  $-\sigma$ .  $\blacktriangleright$  Over long integration times the components in the state/adjoint system will differ by many orders of magnitude.

## Minimum-Time Horiztonal Plane Turns

## The terminal transversality conditions for this problem lead





▶ We are to find  $z^* \in \mathbb{R}^2$  so that  $u(t_f) = U_f$  and  $\chi(t_f) = \pi$ ► If we consider the family of problems parameterized by  $U_f$ we can obtain a solution for one-member of the family by choosing  $\hat{\lambda}_u(0) < 0$  and

integrating the IVP forward until  $\chi(\hat{t}) = \pi$ . Read out the resulting value of  $u(\hat{t})$  and the pair  $z = (\hat{\lambda}_u(0), \hat{t})$  will solve the problem with  $U_f = u(\hat{t})$ .