AOE 5244
Optimization Techniques
HW Set 1

1. Consider the brachistochrone problem of minimizing the time for the bead to go from rest at the origin to a point on the vertical line at $x = X$. See the Figure. Set up an optimal control problem and carefully describe each of the elements.

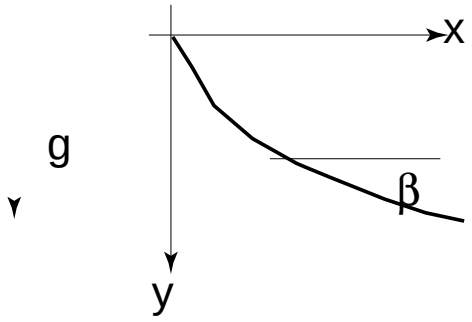

Figure 1: Brachistochrone

2. Consider the Zermelo problem with stream speed κy. You are to approximate this problem as a finite-dimensional problem. Specifically, divide the time interval $[0, T]$ into N equal intervals and approximate the control as piecewise-constant on each interval. In this case you can explicitly find the point $(x(t_i, y(t_i)$ given the point $(x(t_{i-1}, y(t_{i-1})$, and the (constant) control, β_i. Carry out the required explicit integration over a single interval wherein $\beta(\cdot)$ is constant.

For fixed N state the underlying finite-dimensional problem to be solved.