AOE 5244
Optimization Techniques
HW Set 2

1. Let X be the inner-product (Hilbert) space of real n-tuples with inner-product $< x^1,\ x^2 >_X = (x^1)^T Q x^2$, where Q is a symmetric, positive-definite $(n \times n)$ matrix and let Y be the inner-product (Hilbert) space of real m-tuples with inner-product $< y^1,\ y^2 >_Y = (y^1)^T R y^2$, where R is a symmetric, positive-definite $(m \times m)$ matrix. Suppose that an operator $T : X \mapsto Y$ is represented by a matrix A. What is the matrix representation of the adjoint operator T^* (same bases)?

2. Consider the sequence given by $\{y_k\}_{k=0,1,\ldots} \equiv c^{2^{-k}}$, where $c > 0$ (each term is thus the square-root of the previous term). Show that $\lim_{k \leadsto \infty} = 1$ and establish the rate of convergence. Construct a few terms of the sequence for $c = 10$.

3. Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 6 & 11 \end{bmatrix}$$

Find the Q-R decomposition of A. Find $\mathcal{N}(A)$, $\mathcal{R}(A)$, $\mathcal{N}(A^*)$, and $\mathcal{R}(A^*)$. Verify that $\mathcal{N}(A) \bigoplus \mathcal{R}(A^*) = I\!R^3$.

4. Consider the Zermelo problem described in class and used in Prob 2 of H.W. Set 1. Write (a) Matlab .m file(s) to evaluate the cost functional for the transcribed problem with the time interal divided into N uniform intervals and the control piecewise constant on this grid.

Use the `fmins` procedure to solve this problem for $\kappa = 1,$ and $T = 8$, and several values of $N = 4, 8, 16, 32$. Generate a graph of the optimal controls. Also, include on the graph the control given by $\tan \beta^*(t) = \kappa(T - t)$.