AOE 5244
Optimization Techniques
HW Set 4

1) Consider the problem of minimizing the functional

$$F(x) \equiv -(x(1)+x(2)) \quad \text{subject to} \quad c_1(x) = 1-(x(1)^2+x(2))^2 = 0.$$

Use the optimality conditions of Chapter 3 to verify that $x^*(1) = x^*(2) = \frac{1}{\sqrt{2}}$ is a minimizer. Apply the augmented Lagrangian procedure to this problem using the the `fminu.m` procedure from Matlab to perform the *inner* minimization. Start at the point $x_0 = [-1; 0]$ and take the penalty parameter to be 10.

2) Repeat problem 1 using the `fmincon` procedure from the Matlab Optimization Toolbox. Show results (including timing) for the case wherein the gradient of cost/constraints is computed exactly and for the case where the are computed by finite-difference approximation.

3) Consider the problem of maximizing the steady (*i.e.* unaccelerated) rate-of-climb for an aircraft. The problem is discussed as Example 2, Section 1.2 (pp 8 & 9) of the text by Bryson and Ho. Introducing non-dimensional variables, and under certain small-angle approximations the first constraint can be written

$$c_1(V,\gamma,n) = \tau - \frac{1}{2\,(L/D)}\left(V^2 + \left(\frac{n}{V}\right)^2\right) - \sin\gamma = 0\ ,$$

where V is the speed, γ is the inclination of the velocity vector, and n is load-factor (ratio of lift to weight). Here, τ is the (constant) thrust-to-weight ratio, and (L/D) is the (constant) maximum lift-to-drag ratio.

A second constraint imposes force equilibrium normal to the path

$$c_2(V,\gamma,n) = n - \cos\gamma = 0\ .$$

The rate of climb, the vertical component of the velocity, is given by

$$F(V,\gamma,n) = V\sin\gamma\ .$$

Note that we want to *maximize* this quantity.

Use the values $\tau = 0.7$ and $(L/D) = 8$. Solve this problem using the `fmincon` procedure from the Matlab Optimization Toolbox.