An Initial Transversality Condition

- **Our original problem has $x(0) = x_o \in R^n$ (specified)**
- **Now generalize to allow the initial state to belong to a**

prescribed set $\Theta^0 \subset R^n$ given by the intersection of p smooth surfaces. That is,

$$\Theta^0 \equiv \{x \in R^n | \theta_i^0(x) = 0, i = 1, \ldots, p\}$$

➤ **Our modified optimal control problem is to find an admissible control $u^*(\cdot)$ and the**

corresponding state–trajectory $x^*(\cdot)$ to yield a minimum value to the cost.

A Minimum Principle

Theorem: Pontryagin Minimum Principle If the pair $x^*(\cdot),\ u^*(\cdot)$ are an optimal state–control pair for our problem, then there exists a real number λ_o, and a vector–valued function

$\boldsymbol{\lambda}(\cdot) : [0, t_1] \mapsto \boldsymbol{R}^n$, such that:

- a) $\boldsymbol{\lambda}_o \geq 0$
- b) $\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial x}^T$
- c0) $\boldsymbol{\lambda}(t_0) \perp \Theta^0|_{x(t_0)}$
- c0) $\boldsymbol{\lambda}(t_1) \perp \Theta^1|_{x(t_1)}$
- d) $H(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}(t), x^*(t), u) \geq$
  $H(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}(t), x^*(t), u^*(t)) = 0$

for all $v \in \Omega$

- **An analytic statement of condition (c0) is that there are scalars** $\nu_i,\ i = 1, \ldots, p$ **such that**

$$\lambda(t_0) =$$
$$\nu_1 \nabla\theta_1^0(x) + \ldots \nu_p \nabla\theta_p^0(x).$$