Smooth function of 1 - variable

- Consider a smooth function on the interval $[a, b]$

$$f(x^* + p) = f(x^*) \; + \; f'(x^*) \; p \; + \; R(x^*, p)$$

where $\lim_{p \to 0} R(x^*, p)/p = 0$

Smooth function of 1 - variable

- Suppose $a < x^* < b$ minimizes f and suppose $f'(x^*) \neq 0$.
- Let $p = -Kf'(x^*)$ then

$$f(x^* + p) - f(x^*) = -Kf'(x^*)^2 + R(x^*, p)$$

- **the first term on the *rhs* is negative**
- **by choosing K (and hence $|p|$) sufficiently small we guarantee the *rhs* is negative**
- **this leads to $f(x^* + p) - f(x^*) < 0$ a contradiction**

Smooth function of 1 - variable

- **Suppose $a < x^* < b$ minimizes f then $f'(x^*) = 0$.**
- **By a similar argument, if $a(b)$ minimizes f then**
  $f'(a) \geq 0 \quad (f'(b) \leq 0)$;

f twice continuously differentiable

- **Suppose $a < x^* < b$ minimizes f then $f'(x^*) = 0$ and $f''(x^*) \geq 0$.**

f twice continuously differentiable

- Suppose $a < x^* < b$ and suppose that $f'(x^*) = 0$ and that $f''(x^*) > 0$ then there is a $\delta > 0$ such that $f(x^*) < f(x), \quad \forall\, x^* - \delta < x < x^* + \delta$