Constrained Optimization
Second-Order Conditions

➤ **For the *linearly constrained* case we have**

$$\hat{H} = \left(\nabla^2 \hat{f}\right) = Z^T \left(\nabla^2 f\right) Z,$$

where the columns of Z are a

basis for the null space of $\boldsymbol{A}$.

- **If x^* is a minimizer then $\hat{H} \geq 0$**
- **If**

$$\nabla F = A^T \lambda, \quad \text{and}$$
$$\hat{H} > 0,$$

then x^* is a (local) minimizer.

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➤ **For the general constrained case we have**

$$\hat{H} = Z^T \left(\nabla^2 \mathcal{L}\right) Z,$$

where the columns of Z are a

basis for the null space of $[c_x]$.

- **Note we consider the *reduced* Hessian of the *Lagrange* function.**

- **If x^* is a *regular* minimizer then $\hat{H} \geq 0$**

- **If x^* is a regular point for c and**

$$\nabla\mathcal{L} = 0 \quad \textbf{and}$$

$$\hat{H} > 0,$$

then x^* is a (local) minimizer.

An Example

➤ **Find the largest (volume) box that can be inscribed in the ellipse**

$$\left(\frac{x_1}{a}\right)^2 + \left(\frac{x_2}{b}\right)^2 + \left(\frac{x_3}{c}\right)^2 - 1 = 0$$