Unconstrained Optimization
Model Algorithm

- Test for convergence
- Compute a search direction ($\vec{p}$)
- Compute a step from the

current point

$$\vec{\boldsymbol{x}}(\boldsymbol{\alpha}) = \vec{\boldsymbol{x}}_{\boldsymbol{k}} + \boldsymbol{\alpha}\vec{\boldsymbol{p}}$$

➤ **Update the estimate**

$$\vec{\boldsymbol{x}}_{\boldsymbol{k}+1} \leftarrow \vec{\boldsymbol{x}}_{\boldsymbol{k}} + \boldsymbol{\alpha}\vec{\boldsymbol{p}}$$

Search Direction

- Clearly, we want to choose $\vec{p}$ so that F will decrease along the line from $\vec{x}_k$ in the $\vec{p}$ direction.
-

$$\hat{F}_k(\alpha) \equiv F(\vec{x}_k + \alpha\vec{p})$$

- $$\frac{d\hat{F}_k(\alpha)}{d\,\alpha}\Big|_{\alpha=0} = g_k^T p$$
  where $\vec{g}_k = \nabla F\big|_k$.
- One choice is
  $$\vec{p} = -\vec{g}_k$$
- This is the *steepest descent* direction.

- **Note that**

$$\frac{d\hat{F}_k(\alpha)}{d\,\alpha}\Big|_{\alpha=0} = -\|\nabla F|_k\|^2.$$

- **For *generalized* steepest descent**

$$\|\vec{p}\| = \left[p^T C p\right]^{1/2},\; C > 0$$

$$C\vec{p} = -\vec{g}_k$$

$$\frac{d\hat{F}_k(\alpha)}{d\,\alpha}\Big|_{\alpha=0} = -\|\nabla F|_k\|_C^2.$$

- **For Newton's method**

$$G_k \vec{p} = -\vec{g}_k$$

where G_k is the Hessian of F at $\vec{x}_k$.

- **In this case**

$$\frac{d\hat{F}_k(\alpha)}{d\,\alpha}\Big|_{\alpha=0} = -g_k^T \left[G_k\right]^{-1} g_k$$

which is guaranteed to be

negative only if $G_k > 0$.

- **Using a eigenvalue we write**

$$G_k = U \Lambda U^T$$

- **Modify G_k by changing entries in Λ.**

Step Length

- We know $F_k = F(\vec{x}_k)$ and $g_k = \nabla F(\vec{x}_k)$.
- With p chosen we can calculate $\frac{d\hat{F}_k(\alpha)}{d\,\alpha}\Big|_{\alpha=0}$
- For Newton's method $\alpha = 1$ is a

natural choice.

- **For the *Goldstein - Armijo* rule we select $0 < \mu_1 < \mu_2 < 1$ and require**

$$\mu_1(\alpha g_k^T p) \geq (F_k - F_{k+1}) > \mu_2(\alpha g_k^T p)$$