Backtracking

- Newton-like methods suggest a step of unit length
- We decide to accept $\alpha_c = 1$ if

$$F(x_k + \alpha_c p_k) < F(x_k) + \alpha_c \mu_1 g_k^T p_k$$

where $0 < \mu_1$ (Goldstein).

- **If the test fails we reduce α by considering a model quadratic function that approximates the α dependence.**

$$m_q(\alpha) \approx \hat{F}(\alpha) \equiv F(x_k + \alpha p_k)$$

- **Initially we have**

$$\begin{aligned} m_q(0) &= \hat{F}(0), \\ m_q'(0) &= \hat{F}'(0) = g_k^T p_k, \\ m_q(1) &= \hat{F}(1) \end{aligned}$$

- **With this data we can generate**

a quadratic.

$$m_q(\alpha) = \left[\hat{F}(1) - \hat{F}(0) - \hat{F}'(0)\right]\alpha^2 + \hat{F}'(0)\alpha + \hat{F}(0)$$

- **It can be shown that**

$$\alpha^* = \frac{-\hat{F}'(0)}{2\left[\hat{F}(1) - \hat{F}(0) - \hat{F}'(0)\right]}$$

minimizes m_q and that

$$\alpha^* < \frac{1}{2(1-\mu_1)}$$

- **We actually minimize m_q subject to bounds**

$$0 < \ell \leq \alpha \leq u < 1$$

Typically $u = 1/2$ and $\ell = 1/10$.

- **If this new α is not acceptable**

we then use a cubic approximation m_c based on

$$m_q(0)$$

$$m_q'(0)$$

$$m_q(\alpha_{\text{prev}})$$

$$m_q(\alpha_{\text{2prev}})$$