Nonlinear Optimal Control Problem

- **We have a *dynamic model***

$$\dot{x}(t) = f(x(t), u(t))$$

- **where $f : R^n \times R^m \mapsto R^n$ is smooth.**

- **The (piecewise continuous) control function**

$$u(\cdot) : [0, T] \mapsto \Omega \subset R^m$$

- **An initial condition** $x(0) = x_o \in R^n$ **is specified.**

- **A** ***target set*** $\Theta^1 \subset R^n$

$$\Theta^1 \equiv \{x \in R^n | \theta_i^1(x) = 0, i = 1, \ldots, q\}$$

➤ A real-valued cost functional

$$\boldsymbol{V(u(\cdot), x(\cdot)) \equiv \int_0^{t_1} f_o(x(t), u(t))dt}$$

A Minimum Principle
Preliminaries

- **Define the augmented adjoint vector**

$$\tilde{\boldsymbol{\lambda}} \equiv (\boldsymbol{\lambda}_o, \boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_n)$$

- **Define the augmented *rhs***

function

$$\tilde{f}(x,u) \equiv (f_o, f_1, \ldots, f_n)$$

- **Define the *variational Hamiltonian***

$$H(\tilde{\lambda}, x, u) \equiv < \tilde{\lambda}, \tilde{f}(x,u) >$$

$$= \Sigma_{i=o}^{n} \lambda_i f_i(x,u)$$

A Minimum Principle

Theorem: Pontryagin Minimum Principle If the pair $x^*(\cdot),\ u^*(\cdot)$ are an optimal state–control pair for our problem, then there exists a real number λ_o, and a vector–valued function

$\boldsymbol{\lambda}(\cdot) : [0, t_1] \mapsto R^n$, **such that:**

- **a)** $\boldsymbol{\lambda}_o \geq 0$
- **b)** $\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial x}^T$
- **c)** $\boldsymbol{\lambda}(t_1) \perp \Theta^1|_{x(t_1)}$
- **d)** $H(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}(t), x^*(t), u) \geq$

  $H(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}(t), x^*(t), u^*(t)) = 0$

  for all $v \in \Omega$

Transversality Condition

- **Condition (c) : There exists scalars $\mu_i, \quad i = 1, \ldots, q$ such that:**

$$\lambda(t_1) = \mu_1 \nabla\theta_1(x) + \ldots \mu_q \nabla\theta_q(x)$$