Active Set Strategies

- **In the presence of inequality constraints we need to ensure both feasiblity**

$$\boldsymbol{A}_{\mathrm{e}}\boldsymbol{x} = \boldsymbol{b}_{\mathrm{e}}$$

$$\boldsymbol{A}_{\mathrm{i}}\boldsymbol{x} \geq \boldsymbol{b}_{\mathrm{i}}$$

and optimality, including the KKT conditions

$$\boldsymbol{\lambda}_j \geq 0$$

for those $\boldsymbol{\lambda}$ components associated with the inequalities.

- **In most algorithms these are implemented in the form of an**

active set **strategy**

- **Some sub-collection (possibly all) of the inequalities will be satisisfied as equalities at the solution point. At the current iteration we have some conjecture for this set - this is the *active set*.**

- **We use $\hat{A}_k$ for the matrix consisting of the equalities plus additional rows corresponding to the active inequalities. We proceed with a step of our null-space method with this active set. After the step we have $\hat{A}_k x_c = b_k$ and we estimate**

the associated Lagrange multipliers.

- **The current estimate may not be feasible because some row of $A_{\bar{\imath}}$ that was not in the active set may yield**

$$A_{\jmath} x < b_{\jmath}$$

In this case we must add this constraint to the active set

- **The current estimate may not be optimal because some row of $\boldsymbol{A}_{\mathrm{i}}$ that was in the active yield may yield**

$$\boldsymbol{\lambda}_{\jmath} < 0$$

If this occurs for a single constraint then we can argue that the offending constraint must be deleted the active set. If there are several constraints that violate the KKT sign test, then it is less clear what must (should) be done. Some authors

suggest dropping the *worst* violator.

- **The details of this process of modifying the constraints to be included as equalities define the active set strategy. As a general rule, for null-space methods we prefer a large number of active**

constraints so we first add constraints as necessary to guarantee feasiblity, before we consider dropping constraints.