The science of elasticity

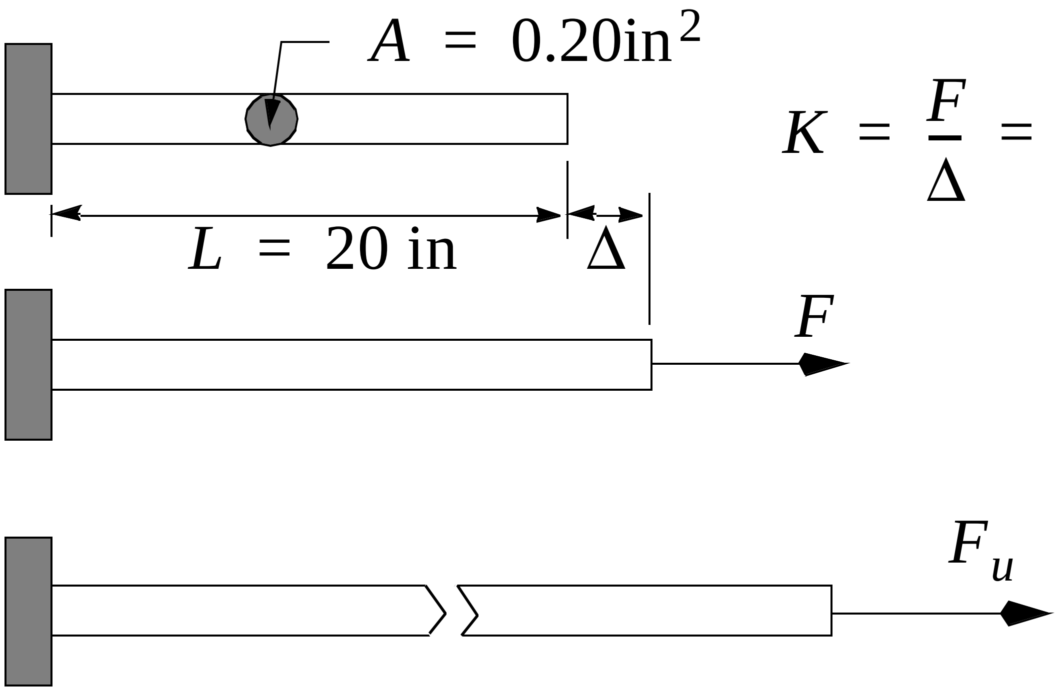
In 1676 Hooke realized that

1. Every kind of solid changes shape when a mechanical force acts on it.
2. It is this change of shape which enables the solid to supply the reaction force to the applied force
3. If the force is not too large, then the solid returns to its unloaded shape on removal of the load — this is elastic behavior

The science of elasticity is about the interactions between forces and deflections in materials and structures.

Stiffness and strength

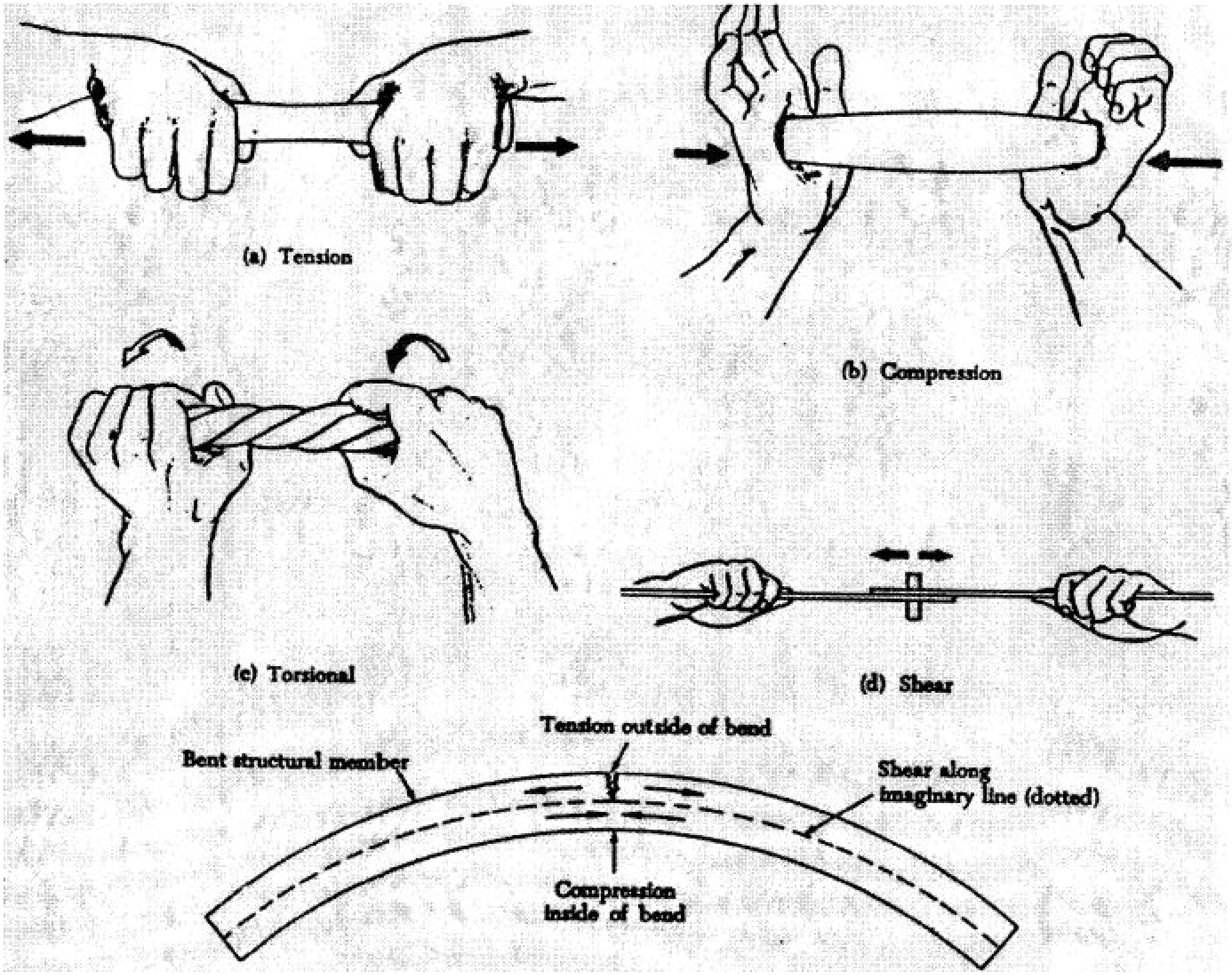
Stiffness is a measure of the force to produce a given deflection. Often denoted by K.



$$K = \frac{F}{\Delta} = \frac{4000\text{ lb}}{0.04\text{ in}} = 100000\text{ lb/in}$$

Strength is the force to cause failure (fracture)

Deformation states of aircraft structure



Concept of stress

The concept of elastic conditions specified at a point inside a material is the concept of stress and strain.

Equilibrium— If an elastic body is in equilibrium every possible section of it is also in equilibrium

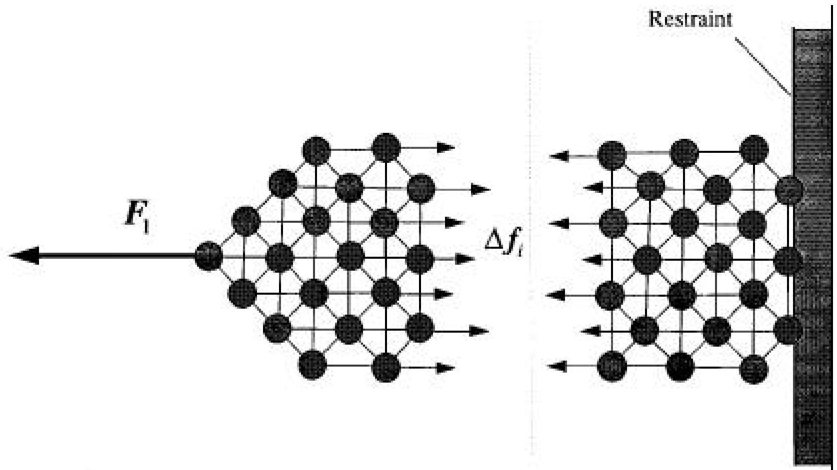


Fig. 7.1 Changes in intermolecular forces as a result of applied load.

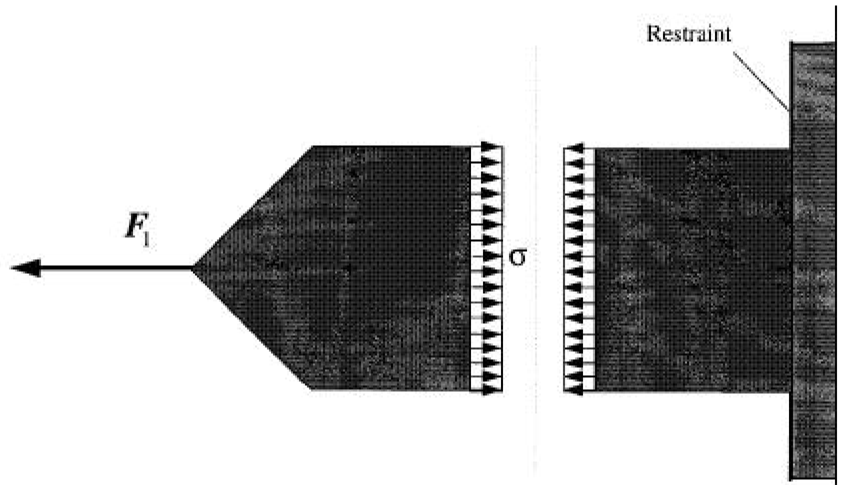


Fig. 7.2 Internal stresses in a solid object as a result of an external force.

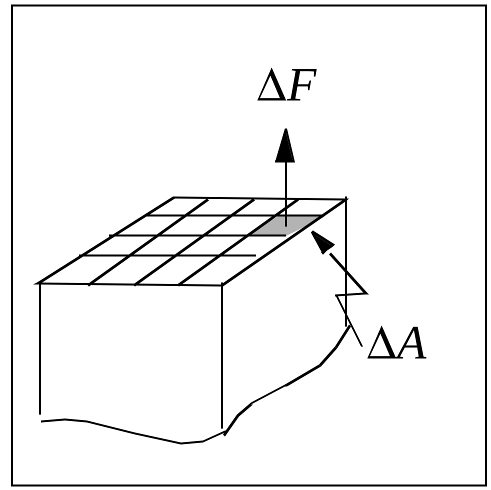
Uniform stress state

If internal force is equal over n equal areas *DA*.

Normal Stress: σ

$$DF = \frac{N}{n} \qquad DA = \frac{A}{n}$$

$$\text{stress} = \sigma = \frac{\text{force}}{\text{area}} = \frac{N}{A}$$



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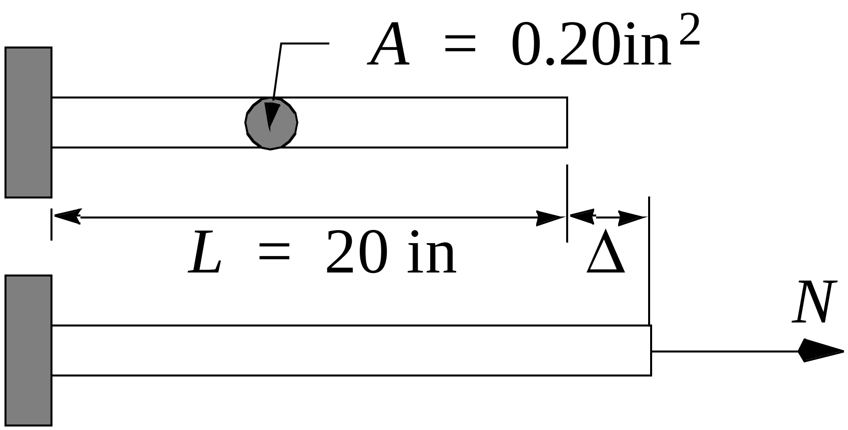

If $N = 4000$ lb, then

$s = 20{,}000 \text{ lb/in}^2$

Internal stress distribution

Stress distribution may be nonuniform

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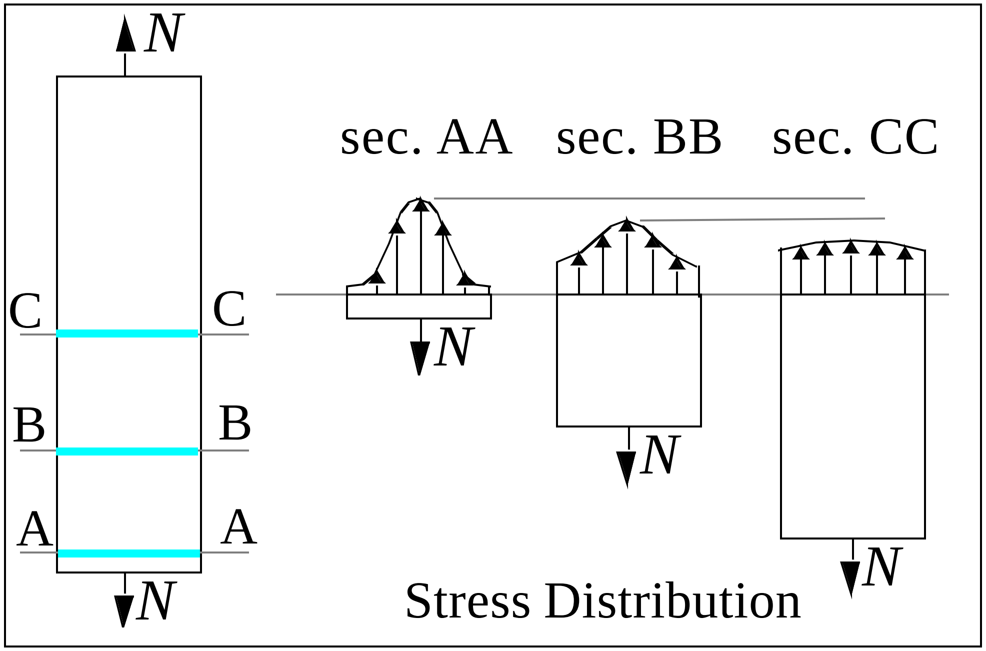
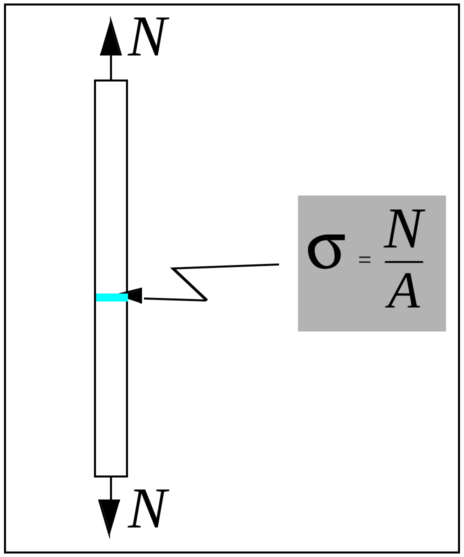
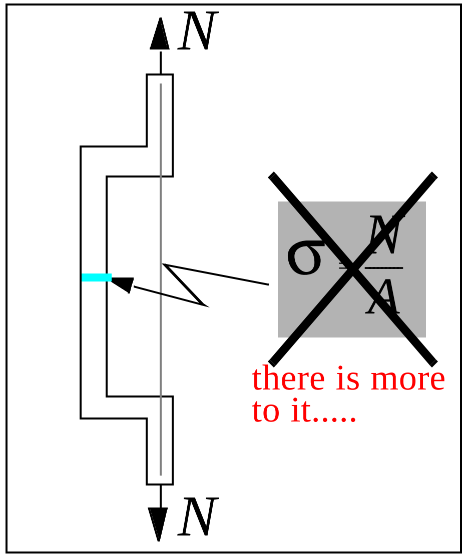


Other types of stresses

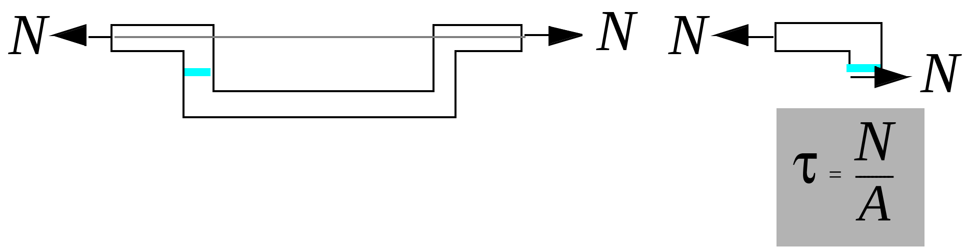
Nonuniform normal stress distribution may not be easily computed





Other types of stresses

Shearing Stress:



Bearing Stress:

Concept of stress (continued)

Stress expresses how hard (i.e. how much force) the atoms at a point within a solid are being pulled apart or pushed together by the load.

Units of stress are

- lb/in^2 or psi; 1000 psi = 1ksi
- N/m^2 = Pa; 1MPa = 10^6Pa

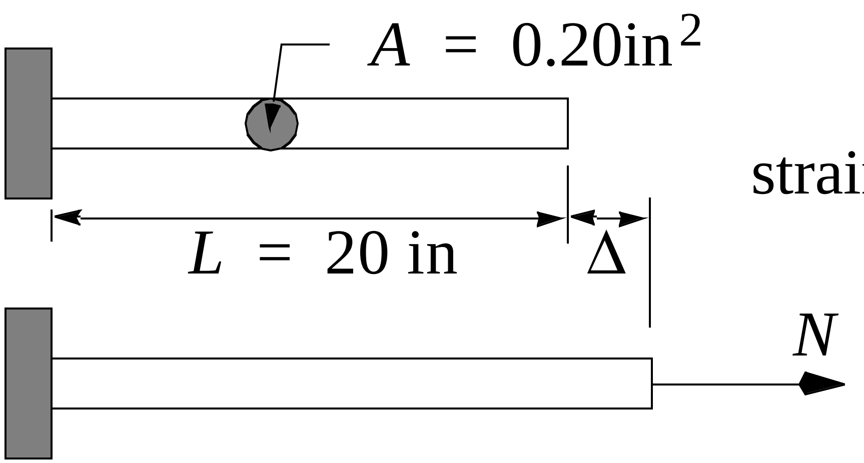
Conversion

$$1\text{ksi} = 1000\frac{\text{lb}}{\text{in}^2} \times 4.448\frac{\text{N}}{\text{lb}} \times \left(\frac{1\text{in}}{0.0254\text{m}}\right)^2 = 6.894\text{MPa}$$

Strain in bars

Deformations <u>at a point inside a material</u> is generally represented by strain.

The amount of change in the dimension of a unit length material point is called strain in the direction of the change (change in dimension per unit length.



$$\text{strain} = \varepsilon = \frac{\text{extension}}{\text{original length}} = \frac{\Delta}{L}$$

If $\Delta = 4000\ \text{lb}$, then $\varepsilon = 0.0020\ \text{in/in}$.

Concept of strain (concluded)

Strain expresses how far the atoms at a point within a solid are being dragged apart or pushed together.

Strain is dimensionless, but is often reported as

- in/in or mm/mm; e.g., 0.002 in/in
- in percent by the definition 100ε
  e.g., 0.2% = 0.002
- microstrain using the definition $1\mu\varepsilon = 1 \times 10^{-6}$
  e.g., $0.002 = 2000\mu\varepsilon$

Introduction to Mechanics of Materials

Material law: The relation between stress and strain. Obtained from tests (**ASTM E8** – *Standard Test Methods for Tension Testing of Metallic Materials*).

In the tensile test a slender member is pulled parallel to its axis. Metallic tensile test specimens are usually circular section bars that are designed to achieve as nearly as possible a uniform state of axial normal stress and strain in portion of the specimen called the gage length

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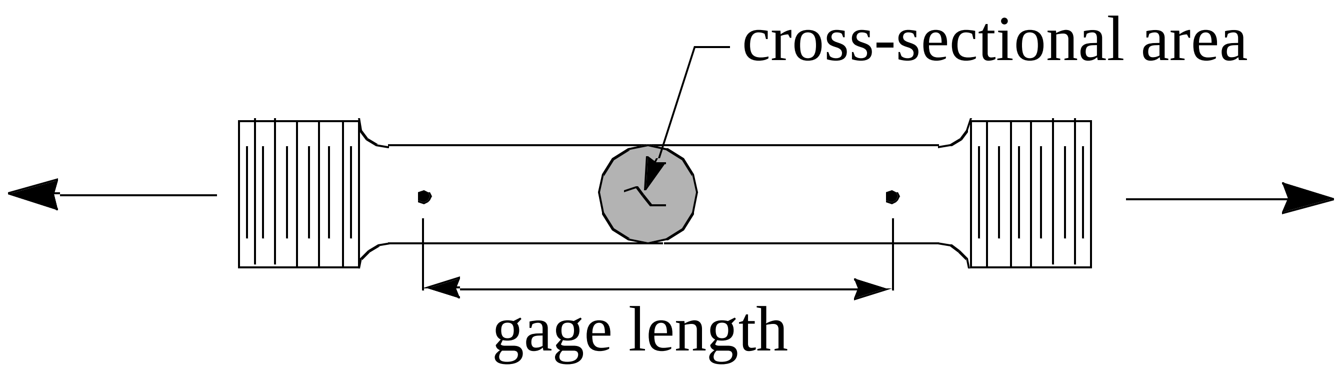
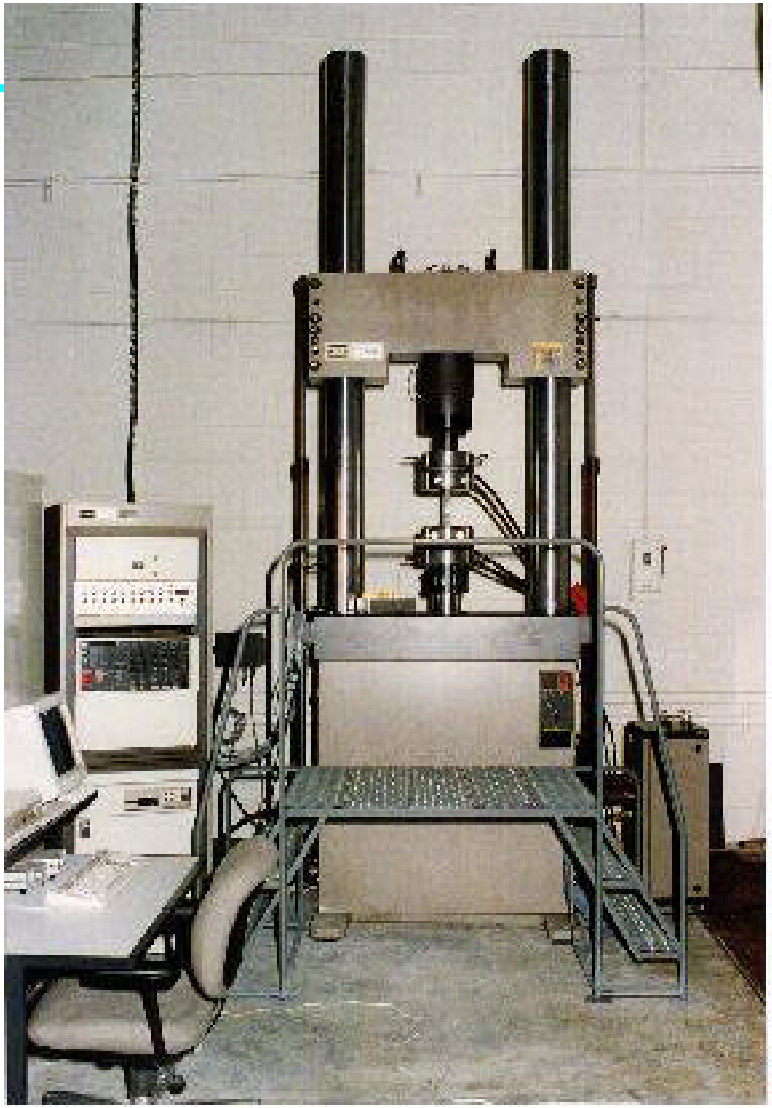


Stress Strain Testing

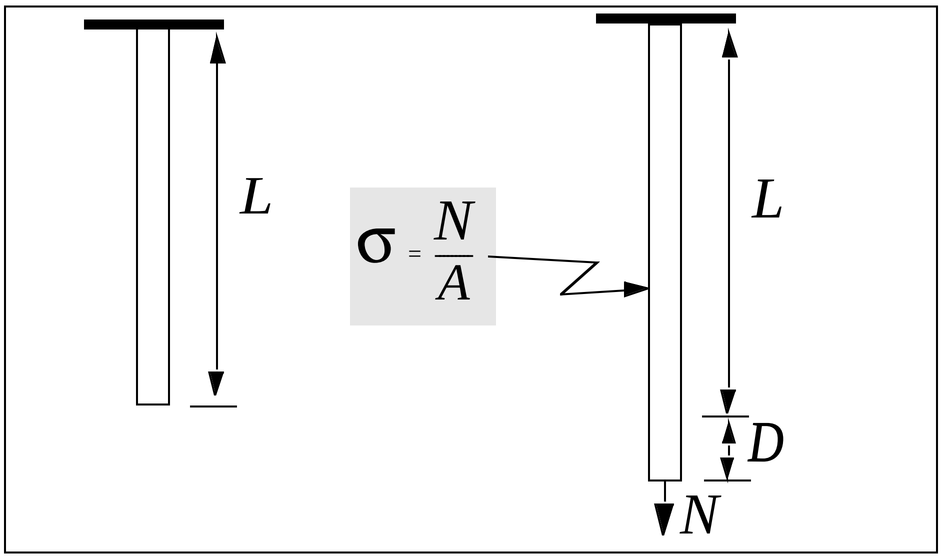
The tensile test is usually conducted in a universal load frame with an attached load cell to measure the axial force applied to the specimen and an extensometer, or electrical resistance strain gage, to measure the elongation or the strain of the gage length

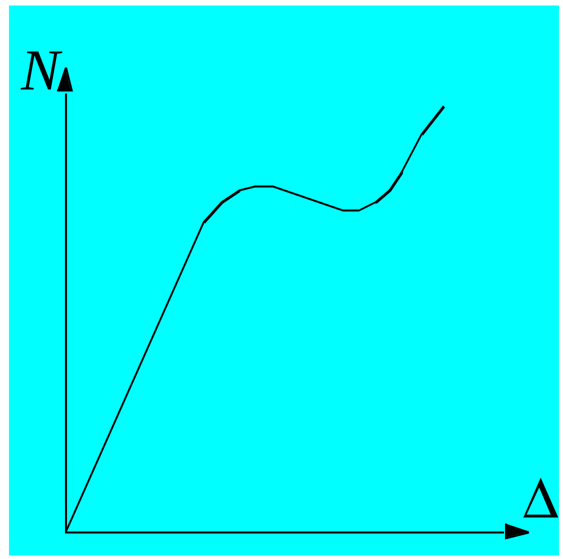
The loading rate is slow in static testing so that inertia effects are negligible, and usually the load is increased monotonically in time until fracture of the specimen.



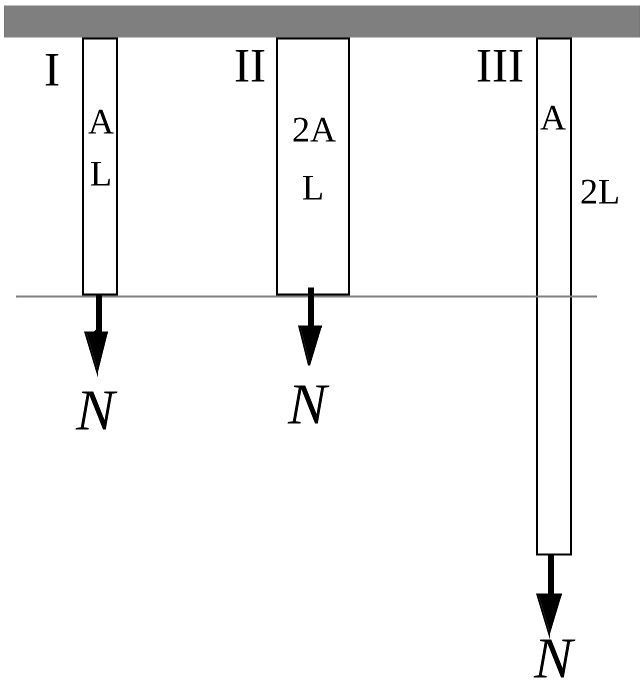
Load deflection experiment

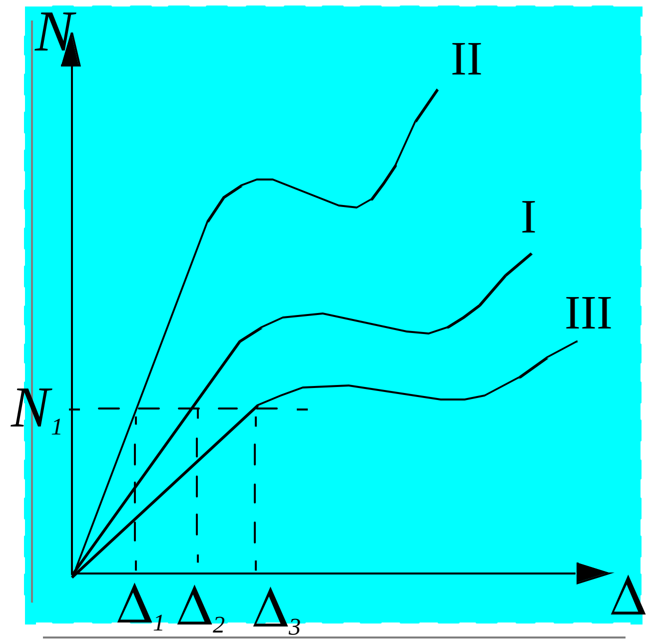
Elongation is measured by an extensometer, and the load is measured by the load cell.





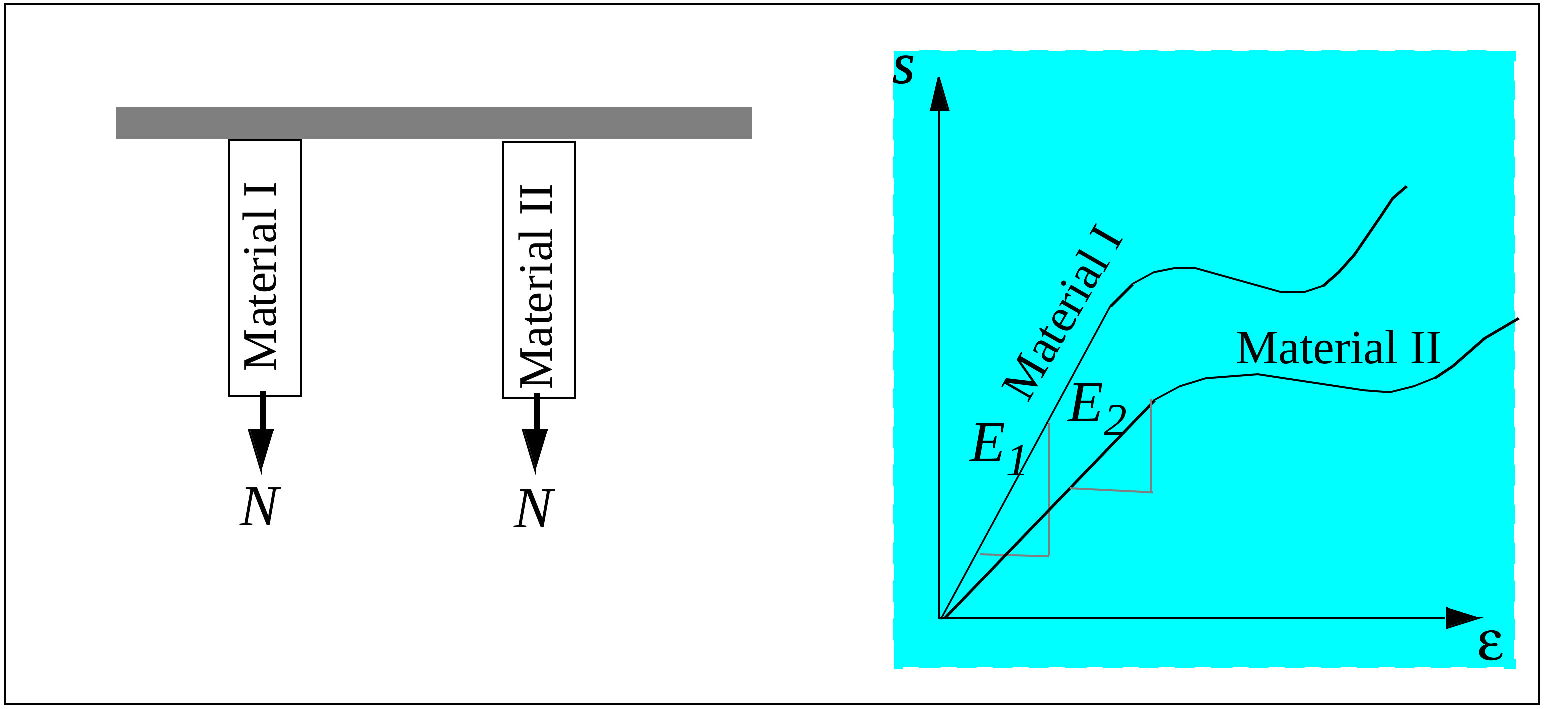
Tensile test with different geometry bars





Stress strain law

Tensile test with different material bars



Hooke's Law $\sigma = E\,\varepsilon$

Data from the tensile test & definitions

Young's modulus of elasticity E

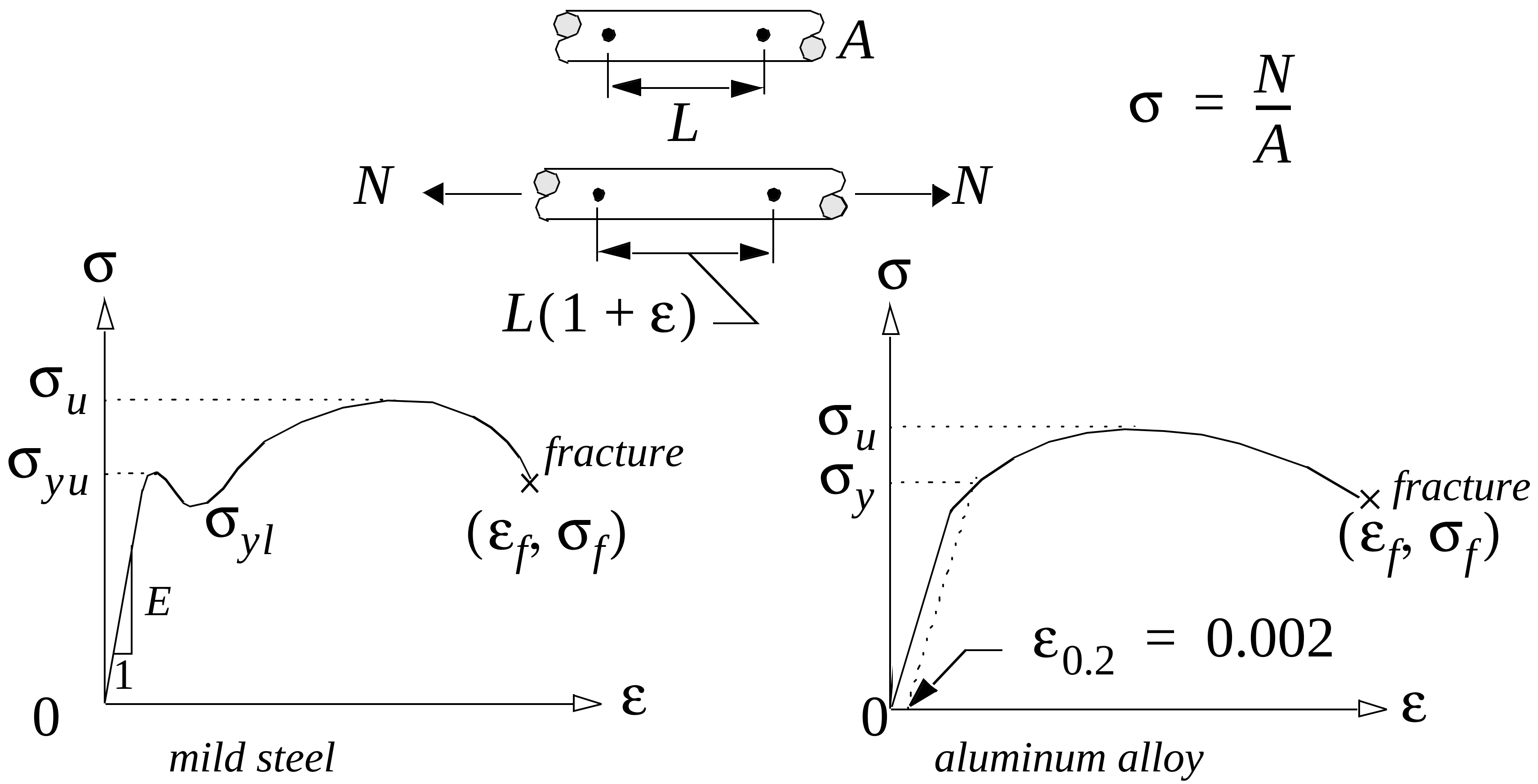
The slope of the linear portion of the stress-strain curve. Units $10^6\text{psi} = 1Msi$, $10^9Pa = 1GPa$.

Hooke's law: $\sigma = E\varepsilon$ in linear region

Elastic deformation

The deformation that disappears on removal of the load. The largest stress for which elastic deformation occurs is called the elastic limit. The elastic limit of a material is difficult to measure precisely since the specimen must be unloaded and reloaded to determine it

Typical stress-strain curves for mild steel and aluminum alloy from tensile tests



$$\sigma = \frac{N}{A}$$

Deformations under axial loading

$$\sigma = E \varepsilon$$

Substituting the definitions of the stress and strain

$$N/A = E\,(\Delta/L)$$

Rearranging

$$\Delta = N\,L\,/\,AE$$

Axial bar with multiple sections with different preperties and internal loads

$$\Delta = \sum_i \frac{N_i L_i}{A_i E_i}$$