Lecture 4: Cyclic loading and fatigue

Safe working life:[1]

All structures will be broken or destroyed in the end – just as all people will die in the end. It is the purpose of medicine and engineering to postpone these occurrences for a decent interval.

But what is a decent interval? Every structure must be built so as to be "safe" for what may be considered an appropriate working life. For a rocket case this may be a few minutes, for a car or an aircraft ten or twenty years.

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[1] Adapted from Gordon, J.E., *Structures – or why things don't fall down*, Da Capo Press, Inc., New York, N.Y., 1978, Chapter 15.

Prediction of safe life

It is impossible in practice to plan for a “safe” life of exactly so many hours or years. We can only consider the problem in statistical terms and in the light of accumulated data and experience. We build in whatever margin of safety seems reasonable. All the time we are working on a basis of probability and estimates.

Prediction of safe life (concluded)

If we make the structure too weak we may save weight and money, but then the chance of the thing breaking too soon is unacceptably high. Contrariwise, if we make a structure so strong that, in human terms, it is likely to last “forever” – which is what the public would like – then it will probably be too heavy and expensive.

Because we are working on a statistical basis, when we design a practical structure for a realistic life we have to accept some finite risk, however small, of premature failure.

Cyclic, or fluctuating, loads

Airplane structures are subjected repeated loads, called cyclic loads, and the resulting cyclic stresses can lead to microscopic physical damage to the materials involved. Even at stresses well below the material's ultimate strength, this damage can accumulate with continued cycling until it develops into a crack or other damage that leads to failure of the component.

The process of accumulating damage and finally to failure due to cyclic loading is called fatigue. An insidious cause of loss of strength.

Comet airplane accidents 1953 & 1954

The Comet was one of the earliest airplanes to have a pressurized fuselage for passenger comfort. In a pressurized airplane the fuselage becomes, in effect, a cylindrical pressure vessel, which is pressurized and relaxed every time the aircraft climbs and descends. The Comet was built from aluminum alloys

In each of these accidents the cracks seem to have started from the same small hole in the fuselage and spread, slowly and undetected, until they reached a critical length. Whereupon the skin tore catastrophically and the fuselage exploded like a blown-up balloon.

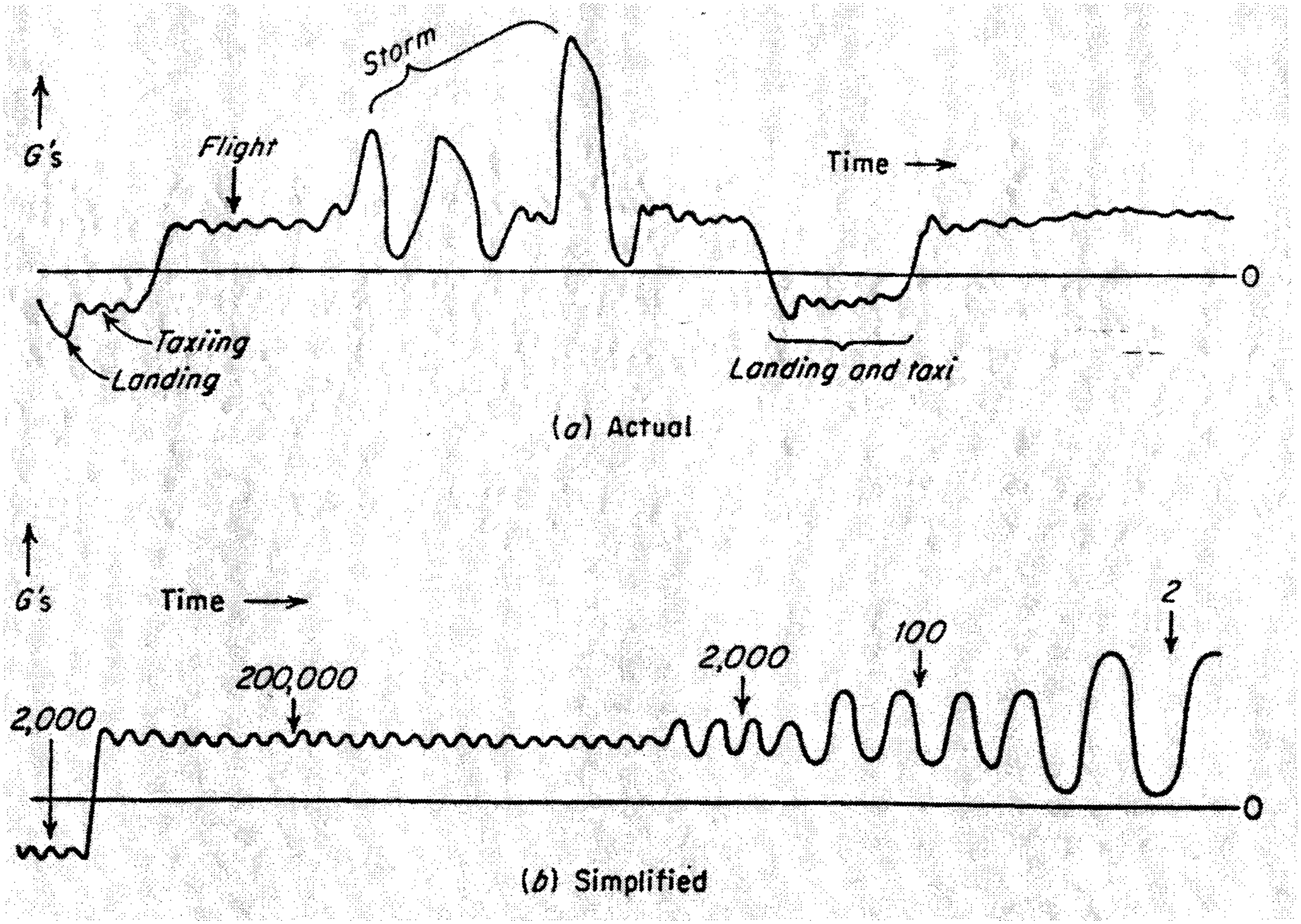
Comet accidents 1953 & 1954 (concluded)

By repeatedly pressurizing a Comet fuselage in a large tank of water at Farnborough, Sir Arnold Hall was able to reproduce the effect so that it could be observed, as it were in slow motion.

The lethal mistake in the design of the Comet lay in not realizing sufficiently the danger of “fatigue” occurring at stress concentrations in the metal fuselage under repeated cycles of pressurization and de-pressurization.

[adapted from J.E. Gordon, 1978, pp. 336 & 337]

Loads for one flight of a fixed wing aircraft

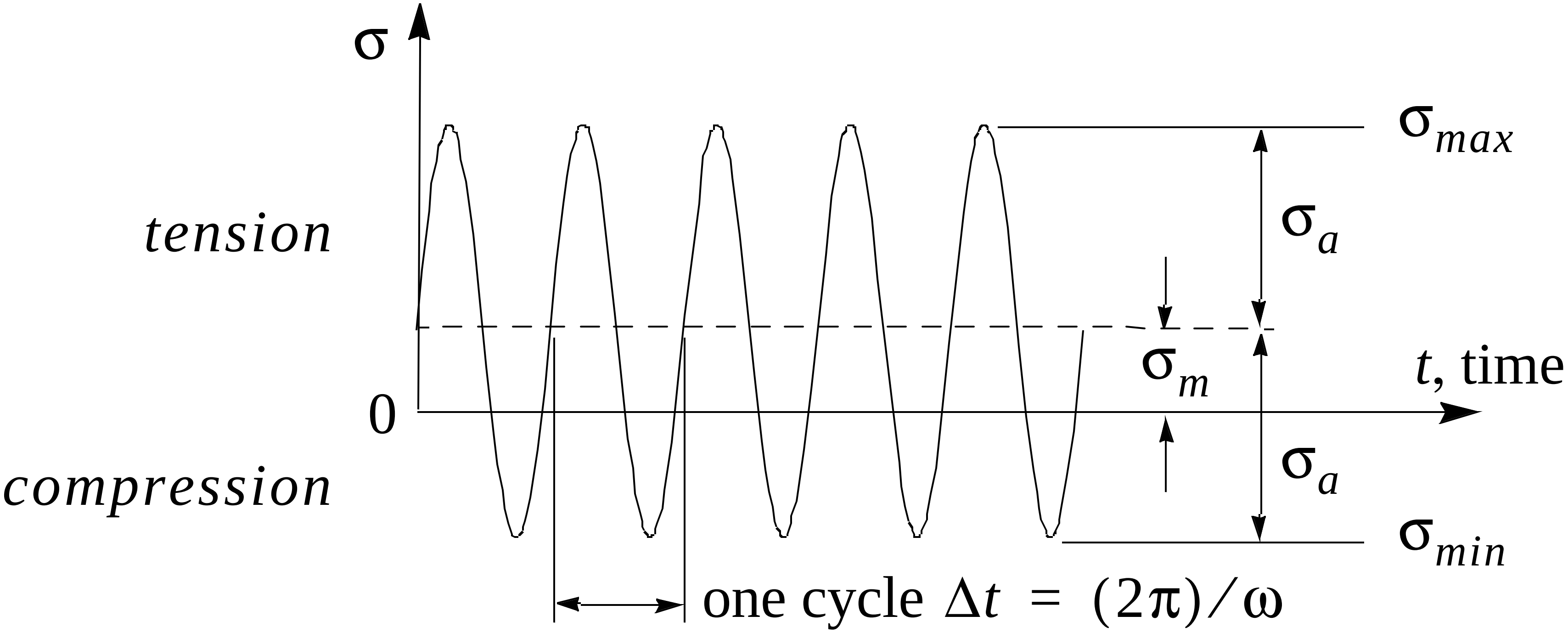


From
N.E. Dowling
“Mechanical
Behavior of
Materials,”
Prentice Hall,
Inc.,
1993, p. 351.

Cyclic stressing of laboratory specimens

$$\sigma(t) = \sigma_m + \sigma_a \sin(\omega t)$$

ω = frequency in radians/second



Cyclic loading of laboratory specimens

Parameters to character fluctuating stress

$$\sigma_m = (\sigma_{max} + \sigma_{min})/2 \text{ mean stress}$$

$$\sigma_a = (\sigma_{max} - \sigma_{min})/2 \text{ stress amplitude}$$

$$R = \sigma_{min}/\sigma_{max} \text{ stress ratio} \qquad A = \frac{\sigma_a}{\sigma_m} \text{ amplitude ratio}$$

Completely reversed stressing

$\sigma_m = 0$, hence $R = -1$

Zero-to-tension stressing

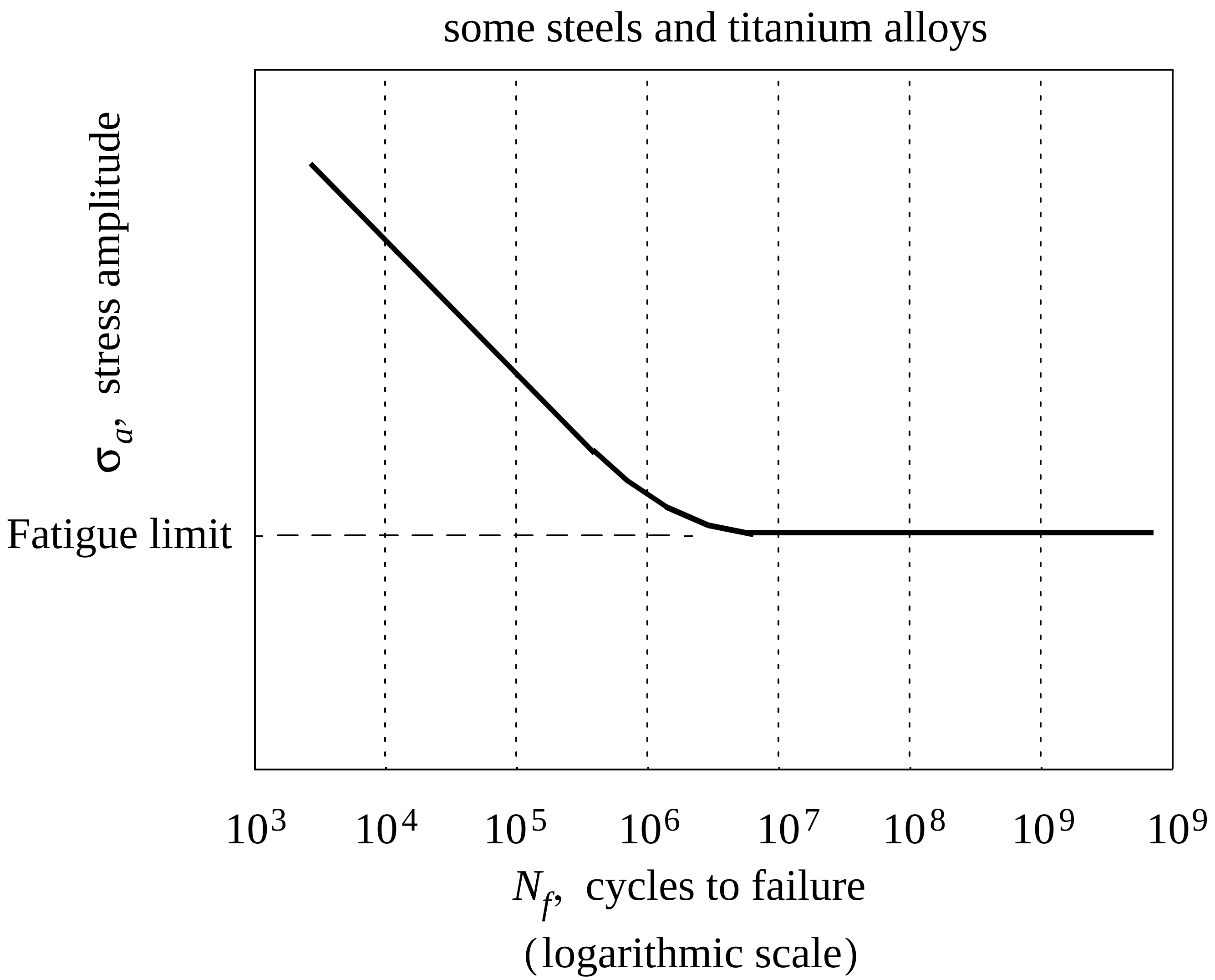
$\sigma_{min} = 0$, hence $R = 0$

ASTM Standard E466

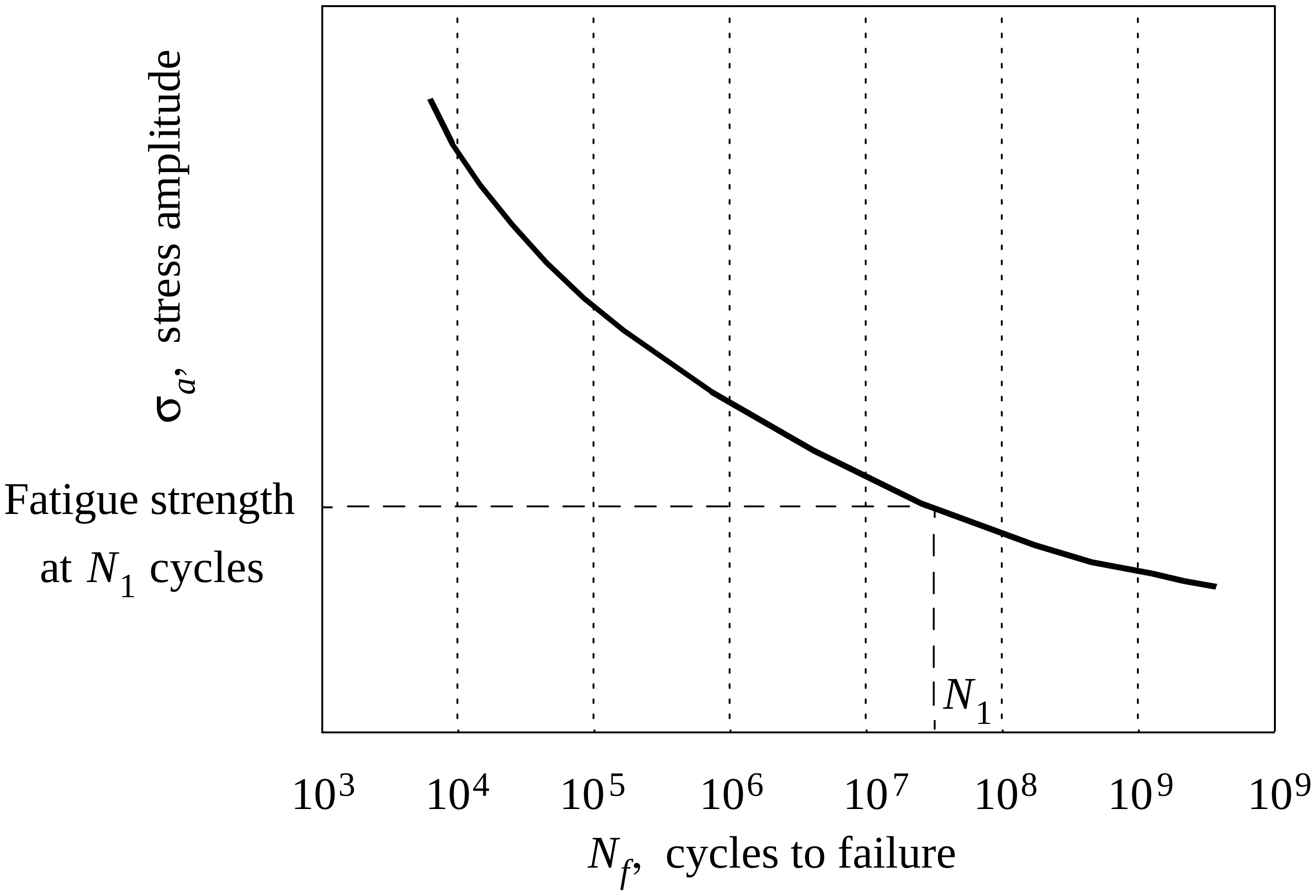
E466: "Standard Practice for Conducting Constant Amplitude Axial Fatigue Tests of Metallic Materials". The frequency is usually 2 to 10 Hz. (1Hz = 1cps = $\omega \frac{\text{rads}}{\text{sec}} \times \frac{1\text{cycle}}{2\pi\text{rads}}$)

Fatigue testing begins by subjecting a specimen to stress cycling at a relatively large maximum stress (σ_{max}); $\sigma_{max} \sim (2/3)\sigma_u$. The number of cycles to failure (N_f) is counted. This procedure is continued on a new specimen of the same material at a lower value of σ_{max}, and N_f is recorded. We plot the results.

Stress-life curves (S-N curves) for R = -1



S-N curve for aluminum alloys R = -1



Power law formula for the S-N curve

$$\sigma_a = a(N_f)^b$$

For 2024-T4 aluminum alloy with $\sigma_m = 0$

$$a = 122 \text{ ksi} \qquad b = -0.102$$ (N.E. Dowling, p.347)

So for $N_f = 10^6$, $\sigma_a = 29.8 \text{ ksi}$. If the tie bar from the previous lecture is to have a fatigue strength of 29.8 ksi, then we design the cross-sectional area at limit load (10000 lb) to be

$$A = \frac{10000 \text{ lb}}{29800 \text{ lb/in}^2} = 0.3356 \text{ in}^2$$

which is a larger area than obtained from static design considerations.