Buckling

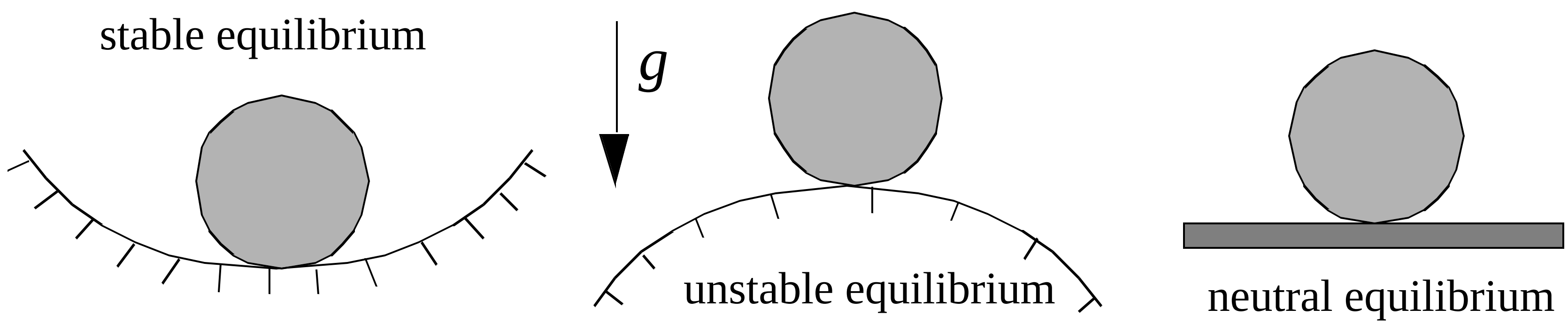
Buckling of a structure means

- failure due to excessive displacements (loss of structural stiffness), and/or
- loss of stability of an equilibrium configuration of the structure

The rule of thumb is that buckling is considered a mode of failure for slender members in compression, or for thin panels in compression or shear. About 50% of an airplane design may be limited by the buckling of thin skins.

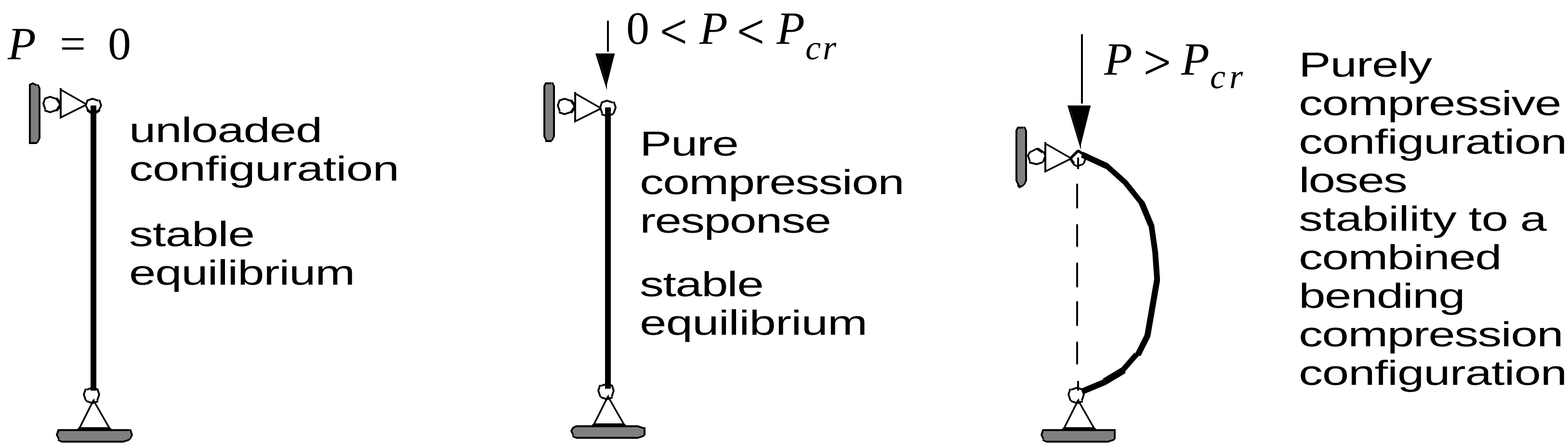
Concept of stability of equilibrium

Stability of equilibrium means that the response of the structure due to a small disturbance from its equilibrium configuration remains small; the smaller the disturbance the smaller the resulting magnitude of the displacement in the response.If a small disturbance causes large displacement, perhaps even theoretically infinite, then the equilibrium state is unstable.



Stability of equilibrium with respect to load

Practical structures are stable at no load. Now consider increasing the load slowly. We are interested in the value of the load, called the *critical load*, at which buckling occurs. That is, we are interested in when a sequence of equilibrium states as a function of the load, one state for each value of the load, ceases to be stable.



Load-shortening curves for compression

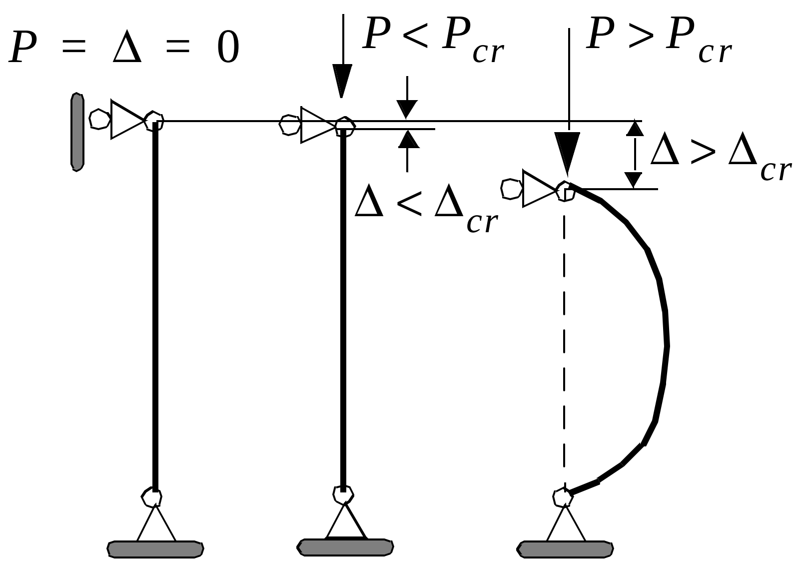
Perfectly straight, elastic column

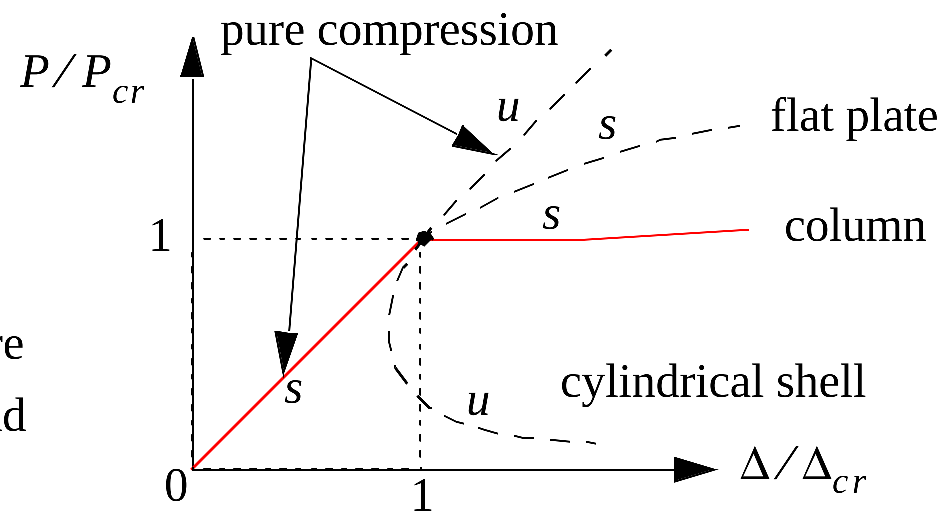
P = compressive axial force

Δ = axial displacement

P_{cr} = critical load of perfect structure

Δ_{cr} = displacement at the critical load





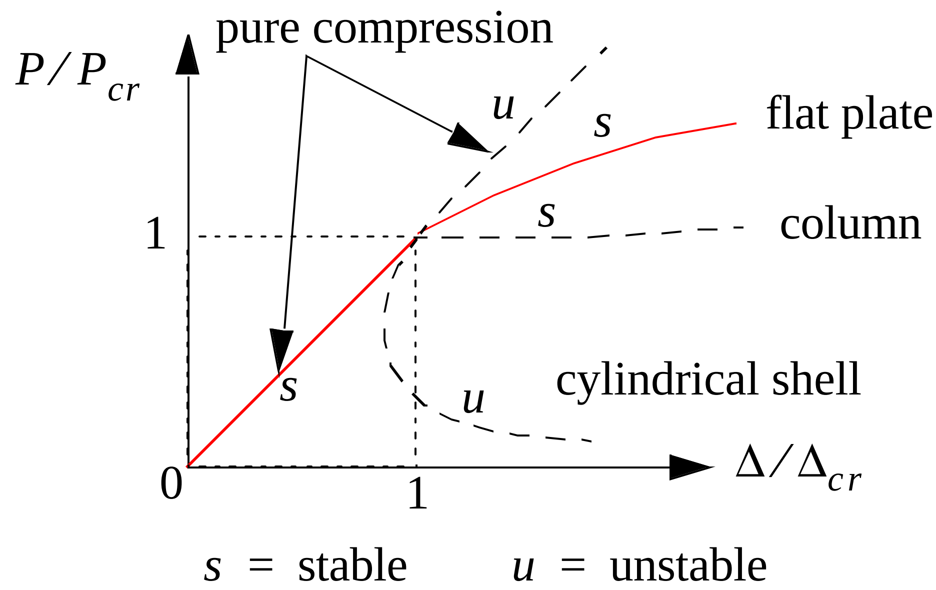
s = stable u = unstable

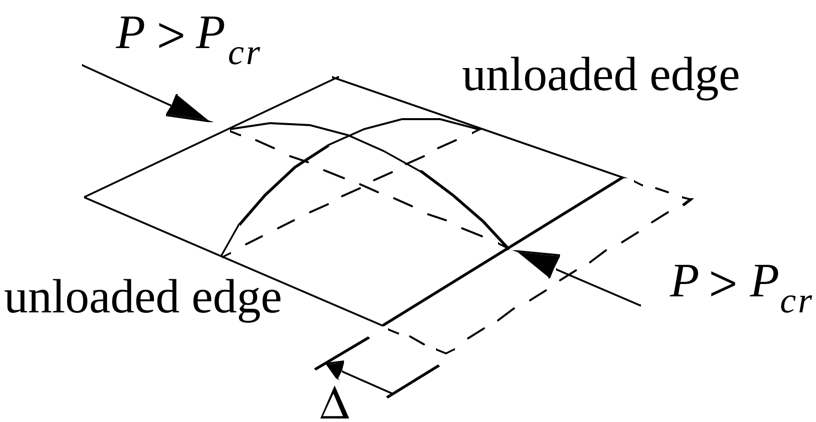
After buckling the column cannot resist much of an increase in load. Its postbuckling stiffness is near zero. It is said the column is neutral in post-buckling

Load-shortening curves (continued)

A perfectly flat rectangular plate

All four edges supported; elastic buckling

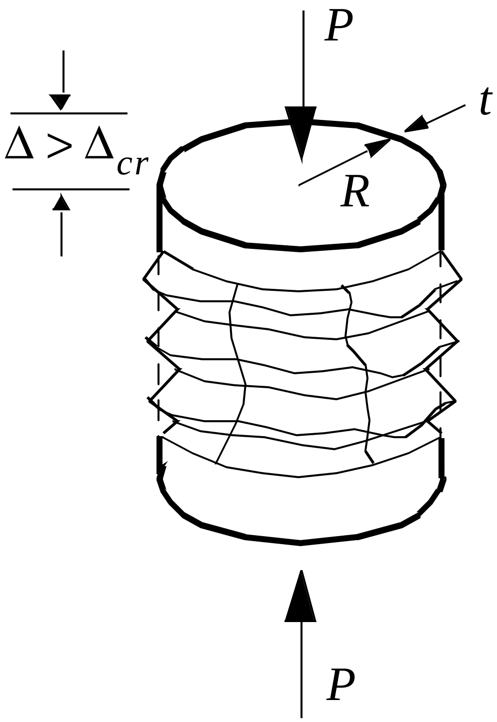


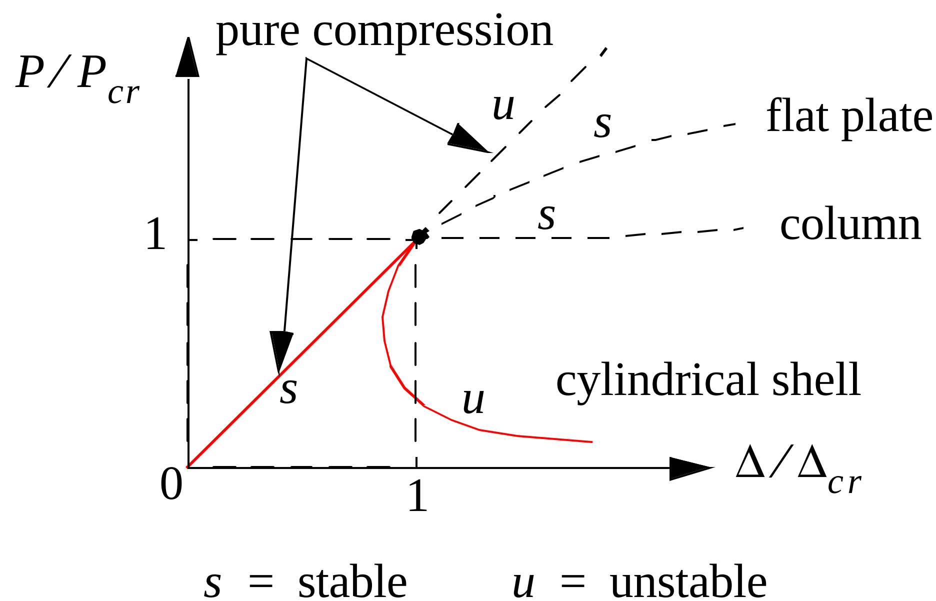


The plate can resist increased load after buckling because the unloaded edges are supported. It's stiffness is reduced in postbuckling. The plate is said to have postbuckling strength.

Load-shortening curves (continued)

A circular cylindrical shell



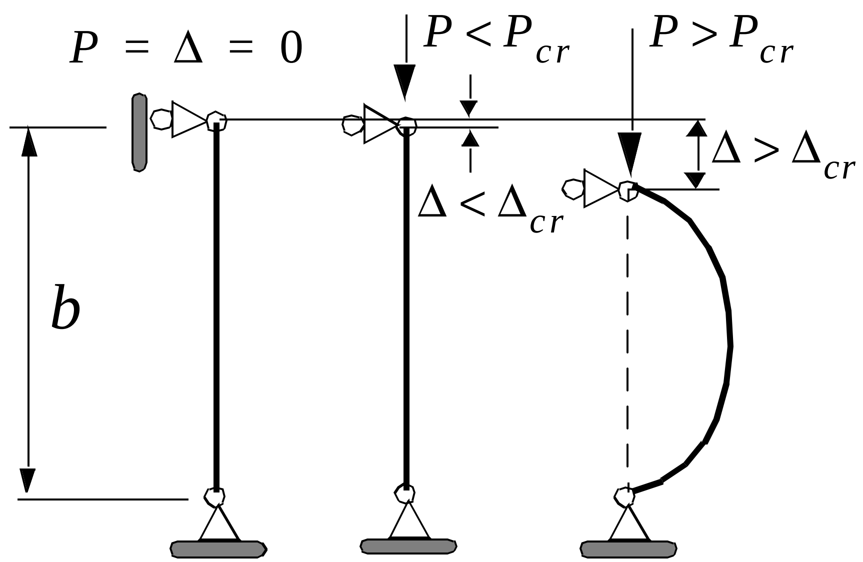


The shell cannot resist increased load after buckling. The load and displacement decrease on the initial, unstable postbuckling equilibrium path. The shell has no postbuckling strength. Designers have to “knockdown” the value of P_{cr} obtained from the theory of the perfect shell by a substantial amount.

Euler load for a pinned-pinned column

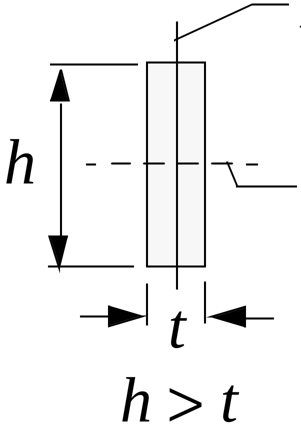
$$P_{cr} = \frac{\pi^2 EI}{b^2}$$

The critical load increases with increased bending stiffness *EI*, and decreases with increasing column length *b*. N.B., Eq. 7.24, p. 263, in the text is incorrect.



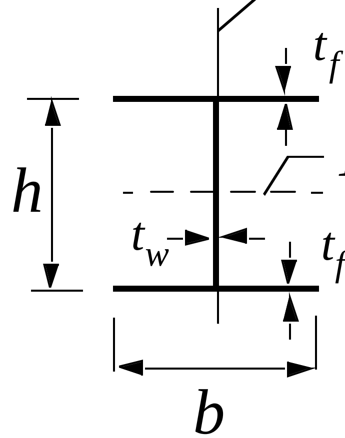
For design of elastic columns the critical load, or Euler load, is used to determine failure by buckling. Also use the minimum I for the cross-sectional area.

For column design use minimum I



$$I_{min} = \frac{t^3 h}{12}$$

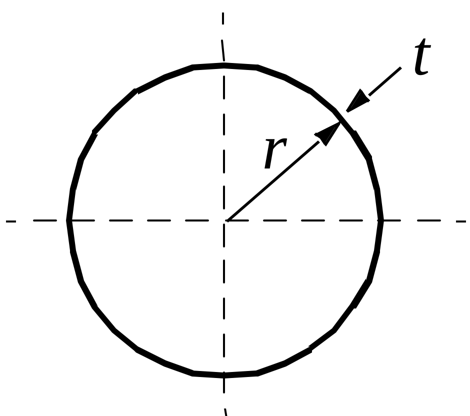
$$I_{max} = \frac{h^3 t}{12}$$



$$I_{min} = \frac{b^3 t_f}{6}$$

$$I_{max} = \frac{h^2 b t_f}{2} + \frac{h^3 t_w}{12}$$

for most I-sections



thin-walled tube

$$I_{min} = I_{max} = \pi r^3 t$$

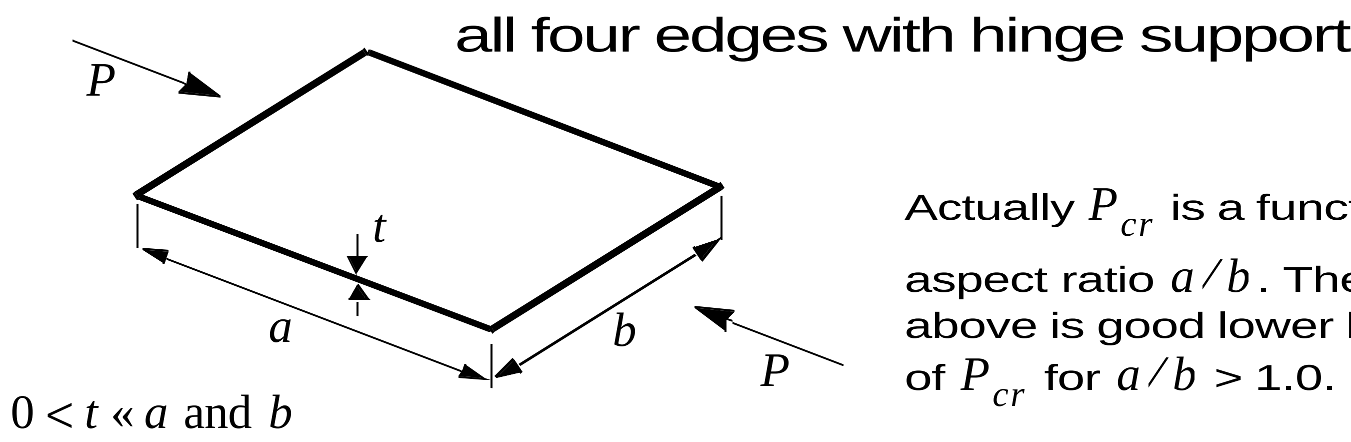
Design buckling load for an elastic plate

For purposes of design, the compressive buckling load of a rectangular, thin plate with all four edges supported by hinges is

$$P_{cr} = \frac{4\pi^2}{b}\frac{Et^3}{12(1-\nu^2)}$$

where ν is Poisson's ratio, a dimensionless material property.

For most aluminum alloys $\nu \sim 0.3$.



Actually P_{cr} is a function of the plate aspect ratio a/b. The formula given above is good lower bound estimate of P_{cr} for $a/b > 1.0$.

Buckling load for a circular cylindrical shell

Theory gives the formula for the critical compressive axial normal stress, σ_{cr}, as

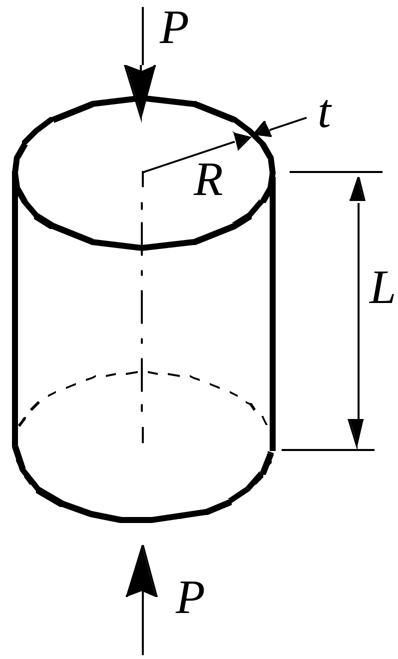
$$\sigma_{cr} = \frac{1}{\sqrt{3(1-\nu^2)}}\left(\frac{Et}{R}\right)$$

The corresponding compressive axial normal force, P_{cr}, at the buckling is obtained from

$$P_{cr} = \sigma_{cr}(2\pi R t)$$

thin shell

$\frac{R}{t} \gg 1$



Design buckling load for the shell

The formula for σ_{cr} above is valid for elastic buckling of a thin, circular cylindrical shell if the shell is moderately long. Moderately long is characterize by parameter $Z > 2.85$, where this parameter (called the Batdorf parameter) is defined by

$$Z = \frac{L^2}{Rt}\sqrt{1-\nu^2}$$

If the shell is too long it will buckle as a column rather than shell, and we use Euler's formula to estimate that critical load.

For design we use a "knockdown" factor γ, which accounts for the fact that experimental values of the buckling load of axially compressed circular cylindrical shells are substantially less than the theoretical prediction. That is, the design buckling load is related to the theoretical value by

$$\sigma_{cr}\Big|_{design} = \gamma\sigma_{cr}\Big|_{theory}$$

Design buckling load for the shell (concluded)

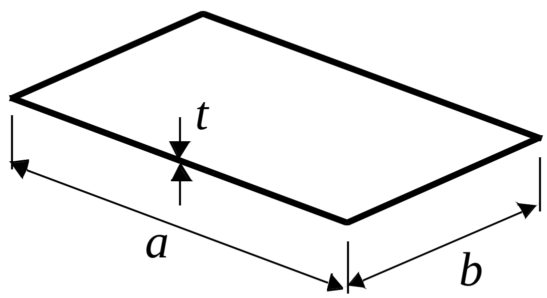
The knockdown factor is a function of the radius to thickness ratio, R/t. Factor γ decreases for thinner shells, i.e., as R/t increases. For example, a design recommendation is

R/t	γ
10	0.84
100	0.58
500	0.32
1000	0.23
4000	0.11

If R/t is small, then the shell will not buckle in the elastic material range.

Structural Performance Indices

For aerospace applications weight is an extremely important measure of the performance of a structure. In comparing different materials one needs to take into account their relative efficiency in terms of strength to weight ratios and stiffness to weight ratios. However, these ratios depend on the type of loading. We will compare these performance indices for a panel of length a, width b, and thickness t.



Tension panel

The weight of the panel is

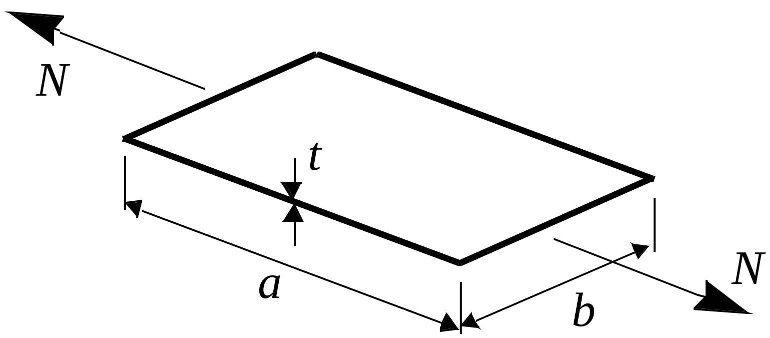
$$w = (\rho g)abt$$

where

$$\rho = \text{mass density}$$

$$g = \text{acceleration due to gravity}$$

Assume the length a and width b are given.



For the panel subjected to tension, the axial force N is specified. Let σ_f denote the failure strength of the material. The value of σ_f could be the tensile strength, or the yield strength, or an allowable stress for the material.

Tension panel (continued)

The axial normal stress in the panel is $N/(bt)$, and we set this equal to the failure stress σ_f of the material and solve for the thickness to get $t = N/(b\sigma_f)$. Now eliminate the thickness in the weight equation to find

$$w = N \times a \times \left(\frac{1}{\sigma_f/(\rho g)} \right)$$

Here the axial force *N* is a specified functional requirement, length *a* is specified geometry, and $\sigma_f/(\rho g)$ is a material property.

A material with a large value of $\sigma_f/(\rho g)$ will result in a lower weight panel of specified dimensions *a* by *b* that has to carry the specified tensile force *N*.

Tension panel (continued)

From table 7.3 in the text, we list values of strength to specific weight ratios, $\sigma_f/(\rho g)$, for selected materials.

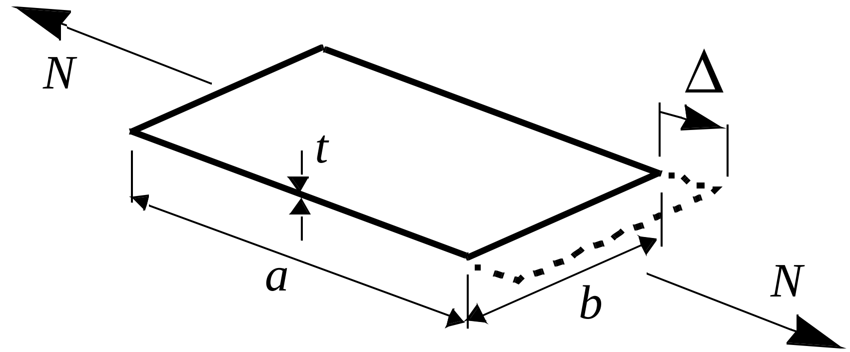
Material	ρ**, slugs/in^3**	$\sigma_u/(\rho g)$**, in 10^3 in**
4340 Steel	0.00879	636
2024-T4 Aluminum	0.00311	570
7075-T6 Aluminum	0.00314	772
Titanium	0.00497	981
Graphite/Epoxy	0.00174	3034[a]
Spruce wood	0.00048	608[a]

a. corrected from the value listed in Table 7.3

Tension panel — specified stiffness

The panel stiffness is specified, where stiffness *K* is defined as

$$K = N/\Delta$$



and Δ is the elongation under tensile load *N*.

From Hooke's law $\sigma = E\varepsilon$, but $\sigma = N/(bt)$ and $\varepsilon = \Delta/a$. So,

$$N = btE(\Delta/a)$$

Note that $K = N/\Delta$ and we see that

$$K = \frac{bt}{a}E.$$

Tension panel — specified stiffness (cont.)

Now solve for t to get

$$t = \frac{Ka}{Eb}$$

Eliminate thickness t in the weight equation and write is as

$$w = Ka^2\left(\frac{1}{E/(\rho g)}\right)$$

where

K is the functional requirement (specified stiffness)

a^2 is the specified geometry

and $E/(\rho g)$ is a material property

Tension panel — specified stiffness (concl.)

A material with a large value of specific modulus results in a lower weight panel. The specific modulus is Young's modulus divided by the specific weight of the material.

Material	ρ, slugs/in^3	$E/(\rho g)$, in 10^6 in	Comments
4340 Steel	0.00879	102[a]	The metal panels all have about the same specific stiffness.
2024-T4 Aluminum	0.00311	107	
7075-T6 Aluminum	0.00314	102[a]	
Titanium	0.00497	100	
Graphite/Epoxy	0.00174	393	impressive
Spruce wood	0.00048	84[a]	

a. Corrected from the value listed in Table 7.3 of the text