AOE/ESM 4084 - ENGINEERING DESIGN OPTIMIZATION
Fall Semester, 2000, Tuesday, October 3
TEST-I

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Show every step of your calculations.

PROBLEM-1 (30%)**:** Minimize the following function along the specified direction, *d*, starting from the specified initial point, using 3-point quadratic polynomial approximation.

$$f(\boldsymbol{x}) = x_1 x_2 - 5x_1 - 20x_2 \qquad d(\boldsymbol{x}) = (1.0, 8.0) \qquad \boldsymbol{x}^o = (12.6, 61.6)$$

a) Construct the one dimensional function, and determine the initial points to start the iterations if the size of the initial interval of uncertainty is α_U-α_L = 0.5.

b) Starting with the following interval perform one iteration of the algorithm, and compute and clearly indicate the information that needs to be used to start the next iteration.

$$\alpha_L = 0.118 \quad \alpha_1 = 0.191 \quad \alpha_U = 0.309$$

PROBLEM-2 (20%)**:** For $x_2 = 1$, determine the range of values of x_1 in which the following function is convex.

$$f(x_1, x_2) = x_1^2 x_2^2 - 2x_1 x_2 + x_2^3$$

PROBLEM-3 (25%)**:** For the following problem, (a) write all the Kuhn-Tucker conditions, (b) find all the points that satisfy Kuhn-Tucker conditions.

$$\begin{aligned} &\text{Minimize} && f(x_1, x_2) = x_1^3 - 16x_1 + 2x_2 - 3x_2^2 \\ &\text{Subject to} && g(x_1, x_2) = x_1 + x_2 \le 3 \end{aligned}$$

PROBLEM-4 (25%)**:** A cabinet is assembled from components C_1, C_2, and C_3. Each cabinet requires eight C_1, five C_2, and fifteen C_3 components. Assembly of C_1 needs either five bolts or five rivets; C_2 six bolts or six rivets; and C_3 three bolts or three rivets. The cost of putting a bolt, including the cost of the bolt, is \$0.70 for C_1, \$1:00 for C_2, and \$0.60 for C_3. Similarly, riveting costs are \$0.60 for C_1, \$0.80 for C_2, and \$1.00 for C_3. A total of 100 cabinets must be assembled daily. Bolting and riveting capacities per day are 6000 and 8000, respectively. Formulate the standard mathematical optimization problem to minimize the cost clearly identifying the design variables and constraint functions.

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AOE/ESM 4084 - ENGINEERING DESIGN OPTIMIZATION
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TEST-I

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$$f(\boldsymbol{x}) = x_1 x_2 - 5x_1 - 20x_2 \qquad d(\boldsymbol{x}) = (1.0, 8.0) \qquad \boldsymbol{x}^o = (12.6, 61.6)$$

a) Construct the one dimensional function, and determine the initial points to start the iterations if the size of the initial interval of uncertainty is α_U-α_L = 0.5.

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