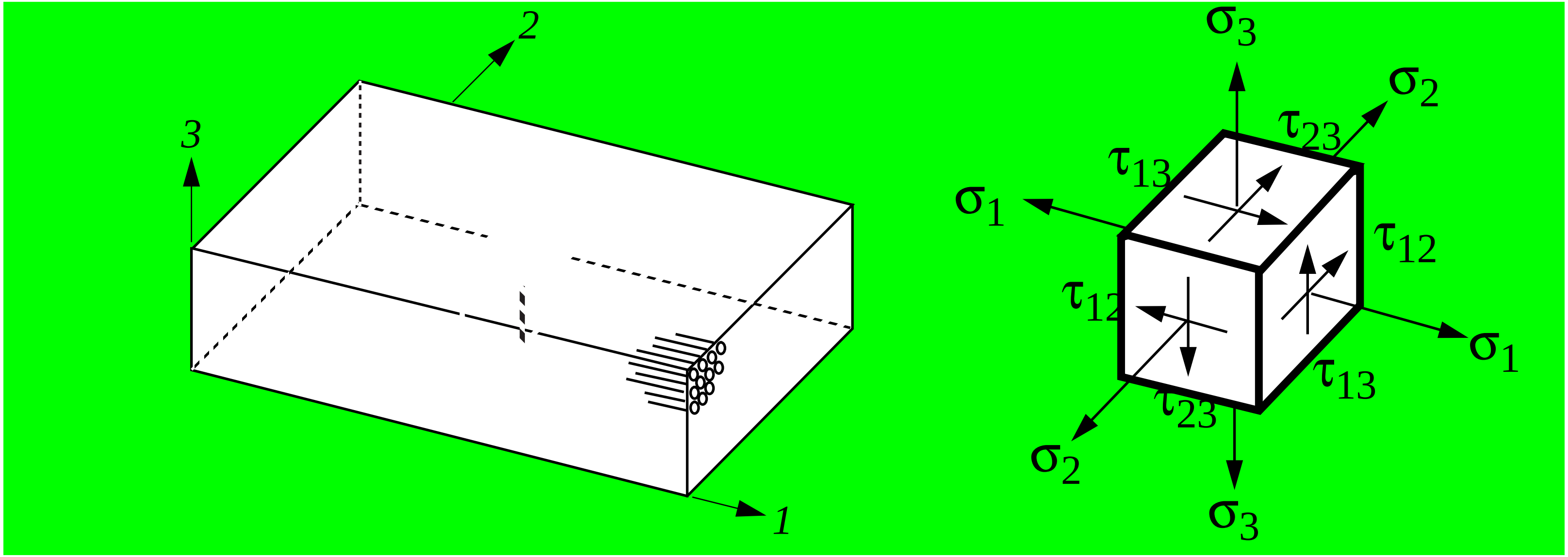
OUTLINE

- Constitutive Relations for a Single Orthotropic Layer
- Response of a Single Off-Axis Layer
- Laminated Composites, Classical Lamination Theory
- Elastic Coupling
- Engineering In-plane Stiffness Properties

Classical Lamination Theory

- Strains Due to Stresses In the Principal Material Directions



Classical Lamination Theory

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$$\varepsilon_1 = \frac{1}{E_1}\sigma_1 - \frac{\nu_{21}}{E_2}\sigma_2 - \frac{\nu_{31}}{E_3}\sigma_3 \qquad \gamma_{23} = \frac{1}{G_{23}}\tau_{23}$$

$$\varepsilon_2 = \frac{-\nu_{12}}{E_1}\sigma_1 + \frac{1}{E_2}\sigma_2 - \frac{\nu_{32}}{E_3}\sigma_3 \qquad \gamma_{13} = \frac{1}{G_{13}}\tau_{13}$$

$$\varepsilon_3 = \frac{-\nu_{13}}{E_1}\sigma_1 - \frac{\nu_{23}}{E_2}\sigma_2 + \frac{1}{E_3}\sigma_3 \qquad \gamma_{12} = \frac{1}{G_{12}}\tau_{12}$$

Classical Lamination Theory

- Compliance Matrix (9 independent material constants)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$\frac{\nu_{13}}{E_1} = \frac{\nu_{31}}{E_3}$$

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}$$

Reciprocity relations

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & \frac{-\nu_{13}}{E_1} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{23}}{E_2} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

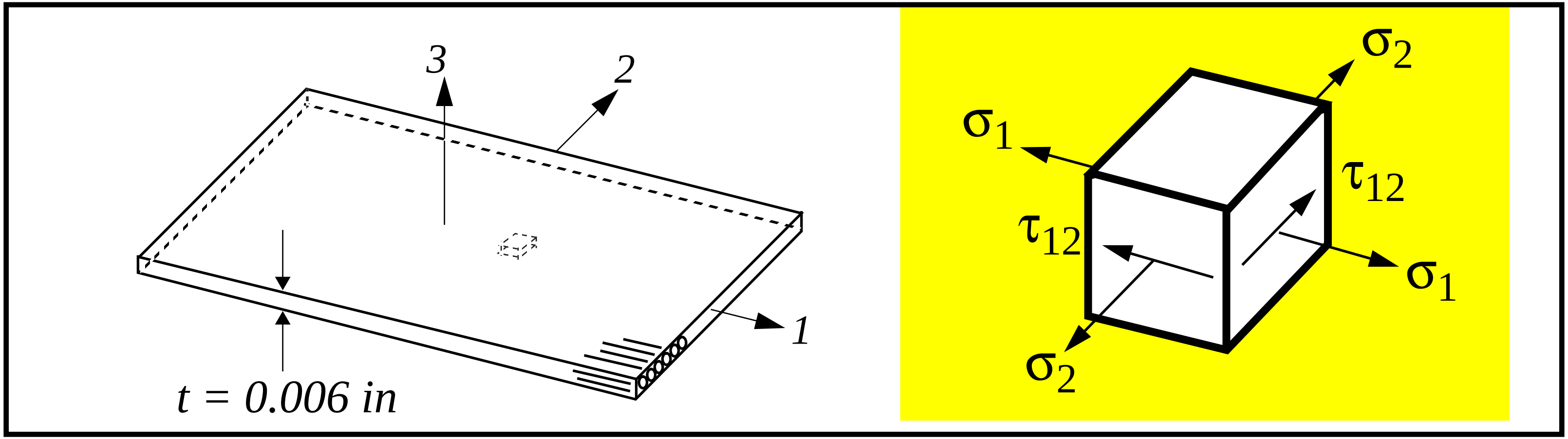
- 3-D Stress Strain Relations

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

$C_{11}, C_{12}, etc = \ldots lengthy \ldots$

$C_{66} = G_{12}$

Plane Stress Assumptions



Places where plane stress does not hold

- Skin-stiffener attachments
- Bonded joints
- Free edges
- Ply termination (dropped plies) regions

Plane Stress Assumption

note that

$$\cancel{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}}$$

Reduced Stiffness Matrix

$$\boxed{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}}$$

$$Q_{ij} = C_{ij} - \frac{C_{i3}C_{j3}}{C_{33}}$$

There is strain in
through-the-thickness direction

$$\boxed{\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2}$$

where the elements of the Reduced Stiffness Matrix are

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \qquad \nu_{21} = \frac{E_2}{E_1}\nu_{12}$$