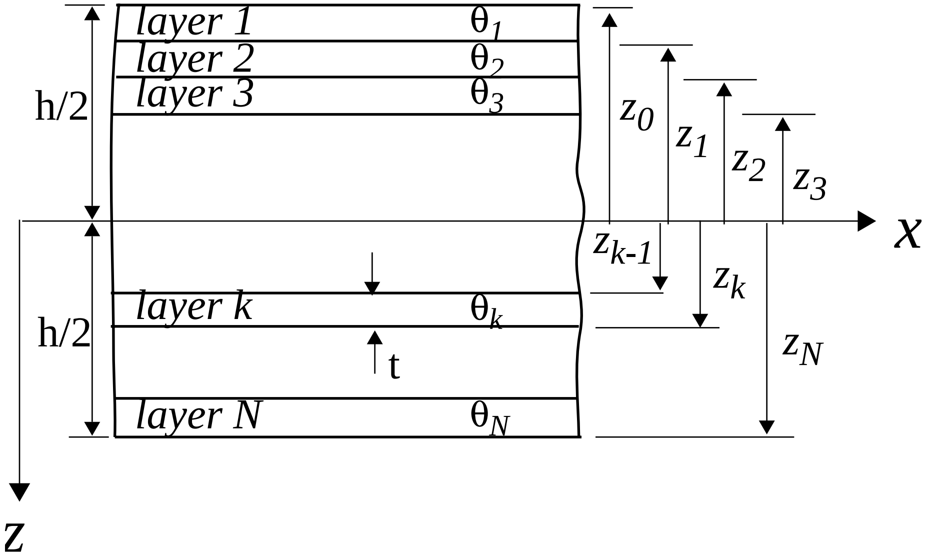
OUTLINE

- Constitutive Relations for a Single Orthotropic Layer
- Response of a Single Off-Axis Layer
- Laminated Composites, Classical Lamination Theory: Strains
- Elastic Coupling
- Engineering In-plane Stiffness Properties

Laminates



stacking sequence

$(\theta_1/\theta_2/\theta_3/ \ldots\ldots /\theta_N)_T$; $h = N\,t$

if mid-plane symmetric,

$(\theta_1/\theta_2/\ldots. /\theta_N/\theta_N/\ldots. /\theta_2/\theta_1)_T$

or $(\theta_1/\theta_2/\ldots. /\theta_N)_S$; $h = 2\,N\,t$

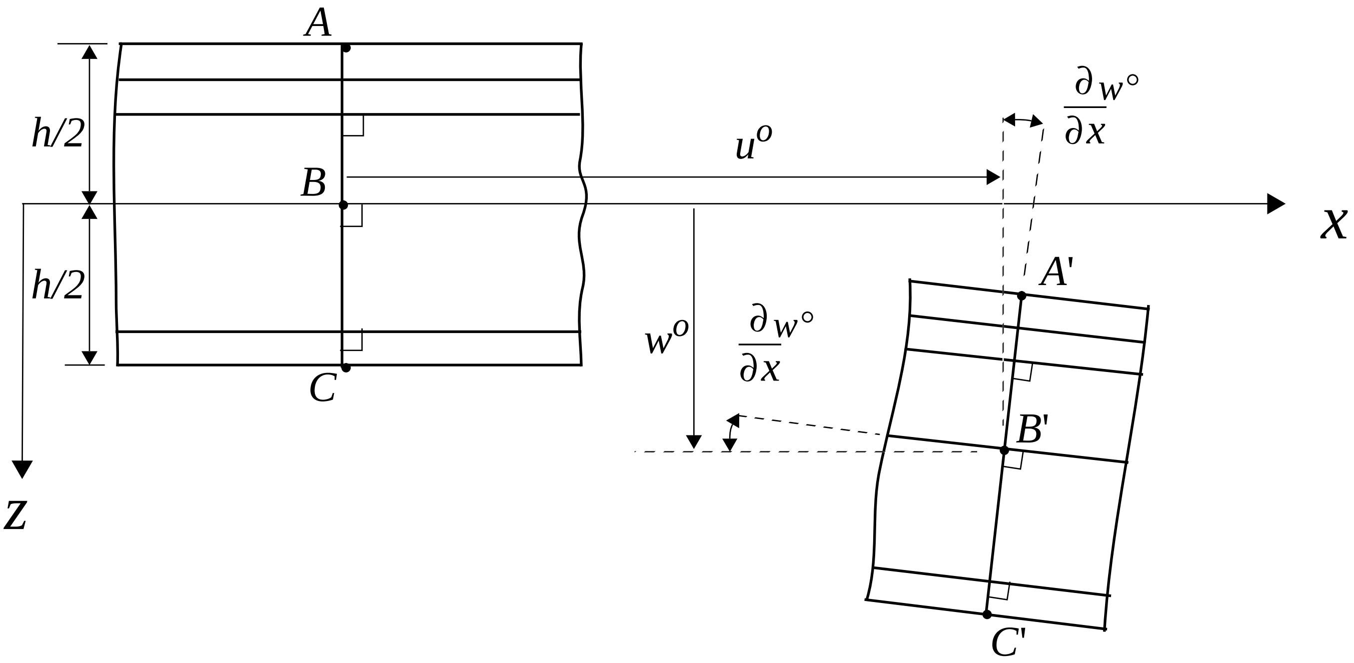
balanced symmetric laminates,

$(\pm\theta_1/\mp\theta_2/\ldots. /\pm\theta_N/)_S$; $h = 4\,N\,t$

Classical Lamination Theory

- Layers are perfectly bonded together
- Each layer is in state of plane stress
- Kirchhoff hypothesis holds for the laminate

Kirchhoff Hypothesis



Kirchhoff Hypothesis (continued)

Displacements of all points are related to the mid-plane displacement

$$u(x, y, z) = u^\circ(x, y) - z\frac{\partial w^\circ(x, y)}{\partial x}$$

$$v(x, y, z) = v^\circ(x, y) - z\frac{\partial w^\circ(x, y)}{\partial y}$$

$$w(x, y, z) = w^\circ(x, y)$$

Laminate Strains

- Linear elastic strain-displacement relations, 3-D

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Classical Lamination Theory

- Substituting the displacements from the previous page

$$\varepsilon_x(x, y, z) = \varepsilon_x^\circ(x, y) + z\kappa_x^\circ(x, y)$$

$$\varepsilon_y(x, y, z) = \varepsilon_y^\circ(x, y) + z\kappa_y^\circ(x, y)$$

$$\gamma_{xy}(x, y, z) = \gamma_{xy}^\circ(x, y) + z\kappa_{xy}^\circ(x, y)$$

$$\varepsilon_z(x, y, z) = 0$$

$$\gamma_{xz}^\circ(x, y, z) = 0$$

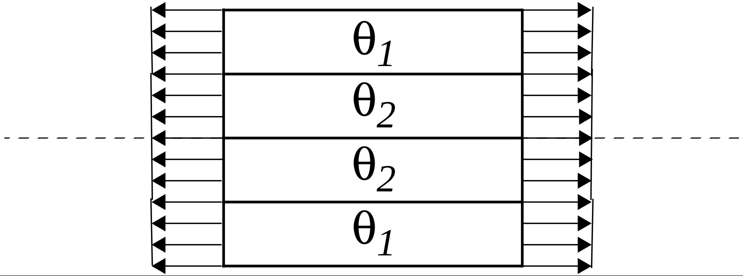
$$\gamma_{yz}^\circ(x, y, z) = 0$$

where

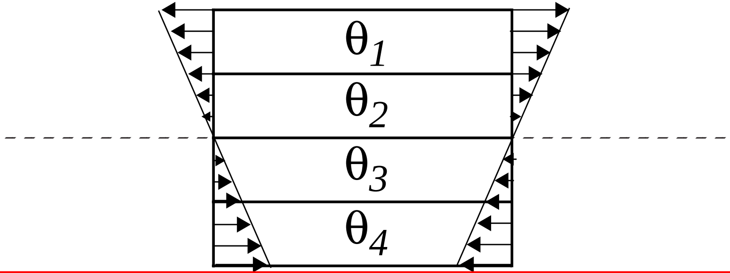
$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial x^2} \\ 2\dfrac{\partial^2 w}{\partial xy} \end{Bmatrix}$$

Laminate Strain Distribution

- If the curvatures are zero, $\kappa_x = \kappa_y = \kappa_{xy} = 0$



- If the mid-plane strains are zero, $\varepsilon_x^o = \varepsilon_y^o = \gamma_{xy}^o = 0$



Laminate Strain Distribution (continued)

- General state of strain