

Introduction to Aerospace Engineering

3. Standard Atmosphere

The concept of a standard atmosphere is required so that we can perform comparisons of performance among different aircraft. On any given day it is likely that the atmosphere will not be standard. Consequently the results of flight tests would be different, even for the same aircraft, if the tests were performed on different days, say mid summer and mid winter. However these results can be reduced to conditions in a standard atmosphere and then compared.

The assumptions for a standard atmosphere are:

Assumptions:

1. The atmosphere is static ($V = 0$)

The atmosphere obeys what is called the aero-hydro static equation:

$$dP = -\rho g_0 dh \quad (1)$$

where ρ = density

g = acceleration due to gravity

h = altitude (technically called the geopotential altitude)

(Note that this equation can be derived by summing the forces on a vertical chunk of fluid).

2. The atmosphere also behaves as a perfect gas and hence satisfies the perfect gas equation:

$$P = \rho R T \quad (2)$$

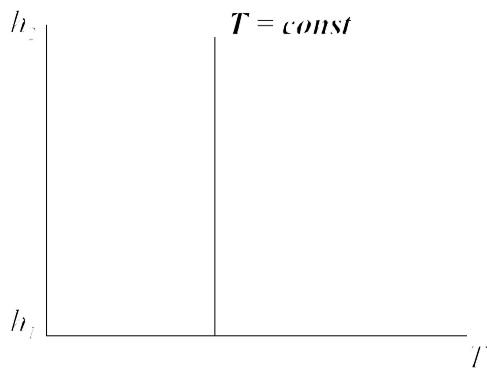
Where ρ = density

R = perfect gas constant

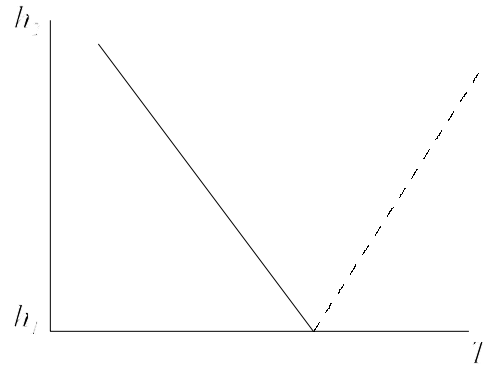
T = Temperature

Here we have two equations to establish three atmospheric properties, P , ρ , T , each as a function of altitude. Therefore, in order to be able to obtain a solution for the atmosphere, we need another equation!

3. It is assumed that the atmosphere is divided into standard layers that either have constant temperature, or a constant temperature gradient (change in temperature with altitude). The temperature profile would look as shown in one of the following diagrams:



Constant Temperature Layer



Constant Temperature Gradient Layer

_____ decreasing with altitude
 ----- increasing with altitude

In general the temperature profile (in a given layer) can be expressed as:

$$T(h) = T_1 + K(h - h_1) \quad (3)$$

Where: T_1 = the temperature at the bottom of the layer
 h_1 = the altitude at the bottom of the layer
 h = altitude measured from the ground

$$K = \frac{dT}{dh} = \text{constant} = \text{temperature gradient in that layer} \quad \left\{ \begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array} \right.$$

Then the three equations that govern the description of the standard atmosphere are:

$$\begin{aligned} dP &= -\rho g dh \\ P &= \rho R T \\ T &= T_1 + K(h - h_1) \end{aligned} \quad (4)$$

We now need to use these equations to find $P(h)$, and $\rho(h)$. The functions $T(h)$ are the means by which we specify the Standard Atmosphere. We have to define the temperature distribution in the standard atmosphere. We will deal with this next.

Temperature Distribution (Standard Atmosphere)

As indicated previously the standard atmosphere is defined by the temperature profiles in different atmospheric layers. The basis for the temperature, pressure and density distribution with altitude are these values at the Earth's surface, and are called sea-level values. Hence we have:

Standard sea-level conditions:

Variable	US	SI
Pressure - P	2116.21695 lbs/ft ²	1.01325x10 ⁵ N/m ² (Pascals)
Density - ρ	0.002376919 slug/ft ³	1.2250 kg/m ³
Temperature	59 deg F = 518.688 deg R	15 deg C = 288.16 deg K
Gravitational Acceleration- g ₀	32.174 ft/sec ²	9.807 m/s ²
Universal Gas Constant	1716.488 ft-lb/slug deg R	287.074 Joules/kg deg K

1st Layer of Standard Atmosphere (Troposphere) The region of air above the Earth’s surface where the temperature drops linearly with altitude.

Troposphere:

Range	0 - 36,089 ft	0 - 11,000 m
Temperature gradient	-0.00356616 deg R/ft	-0.0065 deg K/m

Definition: Tropopause - The boundary between the troposphere and the next layer, the stratosphere, the top of the troposphere. Hence it occurs at 36,089 ft or 11,000 m.

Stratosphere: The region or layer of the atmosphere above the troposphere where the temperature is constant

Range	36089 - 82021 ft	11,000 - 25,000 m
Temperature gradient	0 deg R/ft	0 deg K / m
Temperature - T	389.988 deg R (-69.7 deg F)	216.66 deg K (-56.5 deg C)

There are regions above this that are defined in various “standard” atmospheres. In the next higher region, the temperature actually increases linearly, then the next higher region the temperature is constant, then decreases, constant, etc. You should be aware that there are different “standard” atmospheres put out by different organizations and also revised as more information is gathered.

Standard Atmosphere Equations

The governing equations for developing the pressure and density distributions with altitude in a standard atmosphere can now be developed. First we will develop the equations for

the constant temperature layers, and then for the constant temperature gradient layers.

General:

The two equations of interest are the aero-static equation, Eq. (1) and the perfect gas law, Eq. (2).

$$\begin{aligned} dP &= -\rho g_0 dh \\ P &= \rho R T \end{aligned} \tag{5}$$

Note that we use the what's called the "geopotential" height in these equations. By combining these equations, we can eliminate the density:

$$\frac{dP}{P} = -\frac{g_0 dh}{R T} \tag{6}$$

Equation (6) is a general relation for the atmosphere. We can now examine how the pressure changes with altitude for the different kinds of temperature layers.

Constant Temperature Layer ($T = \text{const}$)

Here, $T = \text{const} = T_1$, where T_1 is the temperature at the base of the layer. In constant temperature layers, $K = 0$ and $T = T_1$. We can substitute into Eq. (8) to obtain:

$$\frac{dP}{P} = -\frac{g_0 dh}{R T_1}$$

since the left hand side is only a function of P, and the right hand side is constant, other than dh , we can easily integrate this equation to obtain the pressure distribution with altitude.

$$\begin{aligned} \ln P \Big|_{P_1}^P &= -\frac{g_0}{R T_1} h \Big|_{h_1}^h \\ \ln P - \ln P_1 &= -\frac{g_0}{R T_1} (h - h_1) \end{aligned}$$

Or

$\frac{P}{P_1} = e^{-\frac{g_0}{R T_1} (h - h_1)} \tag{7}$
--

From the perfect gas law we have:

$$\frac{P}{P_1} = \frac{\rho R T_1}{\rho_1 R T_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT_1}(h - h_1)} \quad (8)$$

Equations (7) and (8) tell us how pressure and density change with altitude in a constant temperature layer.

Constant Temperature Gradient Layer [$T = T_1 + K(h - h_1)$]

We can start with the general equation, Eq. (6) and replace T with the expression that includes the constant gradient:

$$\begin{aligned} \frac{dP}{P} &= -\frac{g_0}{RT} dh = \frac{g_0}{R} \frac{dh}{[T_1 + K(h - h_1)]} \\ &= -\frac{g_0}{RK} \frac{K dh}{[T_1 + K(h - h_1)]} \\ \ln P \Big|_{P_1}^P &= -\frac{g_0}{RK} \ln [T_1 + K(h - h_1)] \Big|_{h_1}^h \\ \ln \frac{P}{P_1} &= -\frac{g_0}{RK} \ln \frac{T(h)}{T(h_1)} \end{aligned}$$

Finally we have:

$$\frac{P}{P_1} = \left(\frac{T(h)}{T_1} \right)^{-\frac{g_0}{RK}} \quad (9)$$

where $T(h) = T_1 + K(h - h_1)$.

From the perfect gas law we have:

$$\frac{P}{P_1} = \frac{\rho R T}{\rho_1 R T_1} = \left(\frac{\rho}{\rho_1} \right) \left(\frac{T}{T_1} \right) \Rightarrow \frac{\rho}{\rho_1} = \left(\frac{P}{P_1} \right) \left(\frac{T}{T_1} \right)^{-1}$$

Then, using Eq. (9), we can write:

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left(\frac{g_0}{RK} + 1 \right)} \quad (10)$$

Equations (9) and (10) tell us how the pressure and density changes with altitude in a layer of atmosphere with a constant temperature gradient.

We now have two schemes for calculating P(h) and ρ(h), and we have T(h) for constant temperature and constant gradient layers of the atmosphere. We also have the values of P, ρ, and T at the surface of the Earth. We can now start with these values, and the values of the gradients presented in the tables previously, to generate the standard atmosphere. Equations (7) - (10) are used to generate the standard atmosphere. For example using the constant gradient model for the troposphere (Eqs. (9) and (10)), we can calculate the reference values at the tropopause for the constant temperature stratosphere where Eqs. (7) and (8) apply. That is how we generate the standard atmosphere.

However, ***we do not use these equations*** in our daily activities in this class. The standard atmosphere is tabulated in almost every aerospace book that relates to atmospheric vehicles. These tables were generated using the above relations. For the purpose of this class, you will be expected to ***use the tables***, and not the above equations.

Example

Calculate the standard atmospheric properties at 10,000 m altitude. By going into the standard atmospheric tables we see that 10,000 m is not one of the points given in the tables. Hence we must interpolate. If we want to determine the pressure at that altitude we can use the following approach:

@ 9,900 m	P = 26906 Pascals (N/m ²)	(Lower)
@ 10200 m	P = 25701 Pascals	(Upper)

Then: (using linear interpolation!)

$$\frac{h - h_L}{h_u - h_L} = \frac{P - P_L}{P_u - P_L}$$

or

$$P = P_L + \frac{(h - h_L)}{(h_u - h_L)} \cdot (P_u - P_L)$$

Putting in the numbers, we have:

$$P = 26906 + \frac{(10000 - 9900)}{(10200 - 9900)} \cdot (25701 - 26906)$$
$$= 26504 \text{ Pascals}$$

A similar calculation can be done for density leading to

$$\rho = 0.41864 + \frac{(10000 - 9900)}{(10200 - 9900)} \cdot (0.40339 - 0.41864)$$
$$= 0.41356 \text{ kg/m}^3$$

We could interpolate to find the temperature, or we could use the perfect gas equation to solve for the temperature:

$$T = \frac{P}{\rho R} = \frac{26504}{(0.41356)(287.074)} = 223.24 \text{ }^\circ K \quad \text{vs} \quad 223.21 \text{ from interpolation.}$$

Definition: Pressure Altitude - The pressure altitude is the altitude associated with a given pressure assuming a standard atmosphere.

Consequently, if we know the outside pressure, we can determine from standard atmosphere tables, the altitude associated with that pressure. We then say that we are at that pressure altitude.

Example

Given the pressure is $P = 31,000 \text{ N/m}^2$, find the pressure altitude.

From the standard atmosphere tables:

$$\begin{array}{lll} @ h = 8700 \text{ m} & P = 32196 \text{ N/m}^2 & \text{(lower)} \\ @ h = 9000 \text{ m} & P = 30800 \text{ N/m}^2 & \text{(upper)} \end{array}$$

Interpolate:

$$\frac{h - h_L}{h_u - h_L} = \frac{P - P_L}{P_u - P_L}$$

or

$$h = h_L + \frac{P - P_L}{P_u - P_L} (h_u - h_L)$$

$$h = 8700 + \frac{31,000 - 32,196}{30,800 - 32,196} (9000 - 8700)$$

$$= 8957 \text{ m}$$

Hence we are at a pressure altitude of 8957 meters.

Virtually all flight testing is done at a selected pressure altitude. Then temperature corrections are made to the data (beyond the scope of this work), and the results are thus reduced to standard atmosphere results. Differences between the pressure altitude and the actual altitude can be quite large on any given day!

Definition: Density Altitude - The density altitude is the altitude associated with a given density assuming a standard atmosphere. To establish the density, generally the pressure and temperature are measured, and the perfect gas law used to calculate the density.

Altimeters

The variation of pressure with altitude can be used to estimate the altitude of a vehicle. The instrument consists of a pressure source (usually a small hole in the fuselage of the aircraft positioned so that it reads static pressure (not effected by the motion of the aircraft), a corrugated box containing a vacuum, and linkage to a dial. As the pressure increases or decreases, the box contracts or expands, and these motions amplified by mechanical or electronic means to move the dial on the face of the altimeter. Hence for a given pressure, the dial will record a given altitude according to the standard atmosphere.

Unfortunately, on any given day, the atmosphere is not standard. In order to account for differences, altimeters are equipped with an adjustment knob. This knob can be used to set the sea-level atmospheric pressure in a window on the face of the altimeter called the Kollsman window. All weather reports deliver the barometric pressure corrected to sea-level. This information is given in terms of inches of mercury or millimeters of mercury. The value that is used for a standard atmosphere it is 29.92 inches of mercury or 1013 mm mercury (hg). For all flights above 10,000 ft, the altimeter is set at the standard value of 29.92 and the altitude is designated as a “flight level.” For altitudes below that, the altimeter is set to the local (sea-level) barometric pressure. If that is not available, the altimeter can be set before takeoff by turning the knob until the altitude reads the altitude of the airport. Setting the altimeter is an item on the check list! Altimeters set in this manner estimate the altitude within 50 ft or better.