Introduction to Aerospace Engineering

4. Basic Fluid (Aero) Dynamics

Here, we will try and look at a few basic ideas from the complicated field of fluid dynamics. The general area includes studies of incompressible, and compressible, inviscid (frictionless) and viscous, subsonic and supersonic flow. The exact type of flow fields we study depends upon what assumptions can be made and how accurate we want the results. There are two assumptions that we will make that will limit the application of the results that we obtain.

Assumptions:

1) We will deal only with subsonic flow $M_a < 1$ and that the Mach numbers of interest will be less than $M_a < 0.4$. Under these circumstances the air can be considered ***incompressible***. (That right, low speed air is just like water, ρ = constant!).

2) We will assume the fluid is inviscid. We have discussed the fact the viscosity of air only affects the flow field near the surface of an object immersed in the flow (called the boundary layer). If we move away from that boundary layer, then the flow can be treated as inviscid.

It turns out that for certain calculations, the above assumptions are very good. On the other hand, there are certain calculations that will yield poor results using these assumptions. Experience and experimentation helps discern when our calculations are suitable.

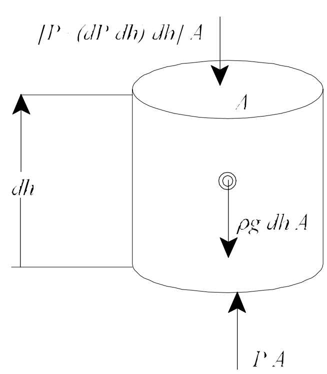
4.1 The hydro static equation

We encountered this equation previously when we dealt with the atmosphere. He we will apply it to an incompressible fluid that could be a short column of air, or a tank full of water. The equation of interest is obtained by summing the vertical forces on a chunk of air:

$$P\,A - (P + dP/dh)\,dh\,A = \rho\,A\,dh\,g$$

or

$$-dP = \rho\,g\,dh \qquad (1)$$



Under the assumption of incompressible fluid, everything in Eq. (1) is constant except *dP* and *dh*. Hence we can integrate to get:

$$\int_{P_1}^{P_2} dP = -\int_{h_1}^{h_2} \rho\,g\,dh = \rho\,g \int_{h_1}^{h_2} dh$$

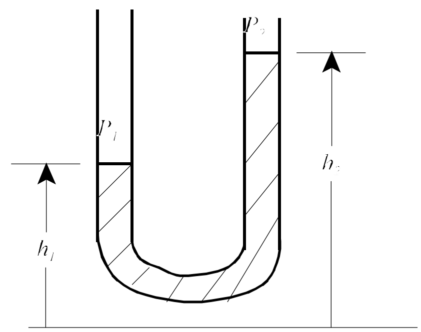
or

Fluid Static Equation

$$P_2 - P_1 = -\rho g (h_2 - h_1) \qquad (2)$$

We can use this equation to make a device to measure pressures in fluid flows. The device is called a manometer and consists of a U tube with the ends open, one attached to a known pressure, and the other to the unknown pressure, or if we are interested in the difference of two pressures, just connected to each of the unknown pressures. The tube is partially filled with a fluid of known density (typically water, alcohol, or mercury).

The figure at the right represents a U tube manometer that is open to two pressures, P_1 and P_2. Typically one of these pressures would be a known atmospheric pressure and the other would be the pressure to be measured, say a static pressure in a wind tunnel. For example P_1 could be atmospheric pressure, and P_2 would be the wind tunnel pressure to be measured.



Here we can apply the hydrostatic to the column of manometer fluid between P_1 and P_2.

Just applying the equation strictly as it is written, we have:

$$P_2 - P_1 = -\rho_w g (h_2 - h_1)$$

where ρ_w is the density of the fluid in the manometer

Hence if we know the properties of the fluid in the manometer, and can measure the height difference, we can determine the pressure difference, and if we know one of the pressures, we can determine the other.

Example:

A mercury barometer works by putting mercury into a closed tube and inverting it and putting the open end in a reservoir of mercury. Hence the pressure on the reservoir surface is atmospheric, and the pressure on the upper surface of the column of mercury is zero since it is in a vacuum. If we designate the surface of the reservoir as point 1, and the upper surface of the mercury column as point 2, we can write:

$$P_2 - P_1 = -\rho_{hg} g (h_2 - h_1)$$

We would like to find, the barometric reading for the standard atmosphere at sea-level. For our problem, $P_2 = 0$ since it is a vacuum, (we will assume it is a vacuum) and the above equation becomes:

$$-P_1 = -P_{atm} = -\rho_{hg} g (h_2 - h_1)$$

The specific gravity of mercury is 13.598. If we use US customary units we have:

$$2116.217 \text{ lbs.ft}^2 = [\, 13.598\,(1.940)\,(32.174)\,] \text{ lbs/ft}^3 \cdot (h_2 - h_1)$$

$$(h_2 - h_1) = 2.4933 \text{ ft} = 29.92 \text{ inches of hg} = 0.0.760 \text{ m}$$

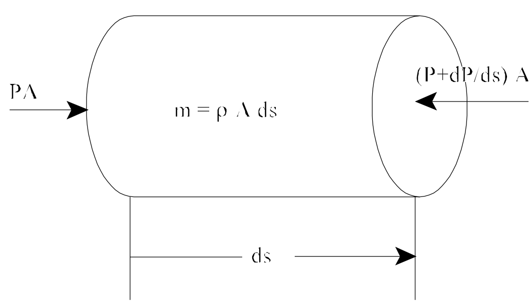
Hence the "pressure" at sea-level in a standard atmosphere is designated as 29.92 inches of mercury. To get the real pressure you need to convert that number to feet, and then multiply by the "weight density" of mercury (or the specific gravity times the density of water times the gravitational constant)

4.2 Euler's Fluid Dynamic Equation (inviscid flow)

We can apply Newton's second law to a chunk of fluid to obtain the fluid dynamic equation. However when we do this we get a result that is a little different then what we expected. We end up with a relation between pressure and velocity!

$$\Sigma F = m a$$

$$P A - [P + (dP/ds)\, ds\,] A = \rho A\, ds \frac{dV}{dt}$$



$$-dP\, A = \rho A\, ds\, V \frac{dV}{ds} = \rho A\, V dV$$

or

$$dP + \rho V dV = 0$$

If we assume that the flow is incompressible (ρ = constant), then we can easily integrate the equation to get:

Bernoulli's equation for incompressible flow

$$P + \rho \frac{V^2}{2} = \text{const} = P_0 \qquad (3)$$

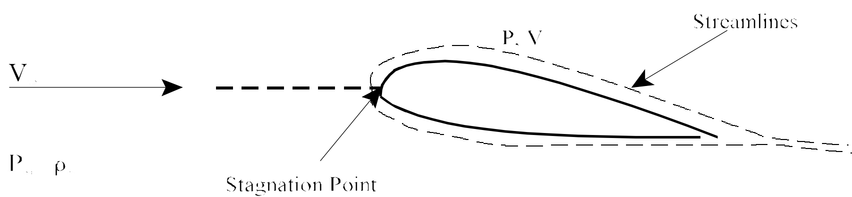
where:

P	=	static pressure
ρ	=	density
V	=	airspeed - Velocity
P_0	=	stagnation pressure

also $\quad \frac{1}{2}\rho V^2 \equiv \bar{q}$ = dynamic pressure

These terms require further explanation as they are often confused. To help in understanding and using Bernoulli's equation we can consider the following idea. When we are discussing flow over a body, be it an aircraft, automobile, or building, we should think of the vehicle (or building) as being fixed, and the air moving over it (as in a wind tunnel). The motion of a vehicle thorough still air is the same as if the vehicle were fixed and the air is moving over it. In order to use Bernoulli's equation correctly, we must take the latter view point, the air moving over a fixed body.

Under these circumstances, we can think of the body being immersed in a stream of air, the properties of which, measured well upstream of the body, are called the free stream properties. These include pressure, density, temperature, and airspeed (velocity). These properties are designated as the free stream properties, sometimes designated with a subscript infinity, such as V_∞ or P_∞ and sometimes designated without the subscript, V, P . This free stream pressure, and in fact, the pressure measured anywhere in the flow field is designated as the ***static pressure***. Now from Bernoulli's equation we see that if the flow comes to rest ($V = 0$) at some point in the flow, that the pressure will be P_0, and is called the ***stagnation (or total) pressure***. The point at which the flow comes to rest is called the ***stagnation point***.



If we know the conditions in the free stream, and the velocity at any other point in the fluid, we can determine the pressure at that point using Bernoulli's equation.

Example: A vehicle is about to land and is flying at 100 kts. If we are at standard sea level conditions, what is the stagnation pressure, and what is the static pressure?

First we convert from knots to basic units:

$$100 \text{ kts} \cdot \frac{0.5144 \text{ m/s}}{1 \text{ kt}} = 51.44 \text{ m/s} \qquad \text{or} \qquad 100 \text{ kts} \cdot \frac{1.6878 \text{ ft/sec}}{1 \text{ kt}} = 168.78 \text{ ft/sec}$$

Sea level conditions (free stream): These are the static pressures in the free stream!

P = 101325 N/m^2	P = 2116.22 lbs/ft^2
ρ = 1.2250 kg/m^3	ρ = 0.002377 slugs/ft^3
T = 288.16 deg K	T = 518.69 deg R

The stagnation pressure is obtained from Bernoulli's equation:

$$P_0 = P + \frac{1}{2}\rho V^2 \qquad\qquad P_0 = P + \frac{1}{2}\rho V^2$$

$$= 101325 + \frac{1}{2}(1.2250)(51.44^2) \qquad\qquad = 2116.22 + \frac{1}{2}(0.002377)(168.78^2)$$

$$= 102945.72 \text{ N/m}^2 \qquad\qquad = 2150.08 \text{ lbs/ft}^2$$

This would be the pressure at any point(s) that the air came to rest, typically near the leading edge of the wing and other lifting surfaces.

We could also use Bernoulli's equation to find the pressure at any point in the flow if we knew the airspeed at that point:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 = P_0 \qquad (4)$$

Remember that under the assumptions used to derive this equation, the flow was incompressible, (ρ = const), and inviscid (frictionless). However, it turns out that the pressure distribution in the fluid is not greatly affected in the boundary layer, so the pressures calculated on the surfaces of the airfoil or wing using Bernoulli's equation are good estimates.

4.3 Measuring Airspeed

We can use Bernoulli's equation to help us build a device that can measure airspeed. If we could measure the temperature, the stagnation pressure, and the static pressure, we could measure the airspeed in incompressible flow. The procedure would be as follows: If we know the static pressure and the temperature, we can determine the density from the perfect gas law, $P = \rho R T$.

If we know the stagnation pressure and the static pressure, we can take the difference, and use Bernoulli's equation to obtain the airspeed:

$$P_0 - P = \frac{1}{2}\rho V^2 \qquad \Rightarrow \qquad V = \sqrt{\frac{2(P_0 - P)}{\rho}} \tag{5}$$

Airspeed Indicators

Unfortunately, most airspeed indicators do not have the luxury of knowing the temperature Consequently the local density is unknown. The airspeed indicator is calibrated to just use the pressure difference, and computes the airspeed assuming sea level density. For an incompressibly calibrated airspeed indicator, the result is:

$$V_{cal} = \sqrt{\frac{2(P_0 - P)}{\rho_{SL}}} \tag{6}$$

For this incompressible case, the calibrated airspeed is the same as ***equivalent airspeed*** that is defined as:

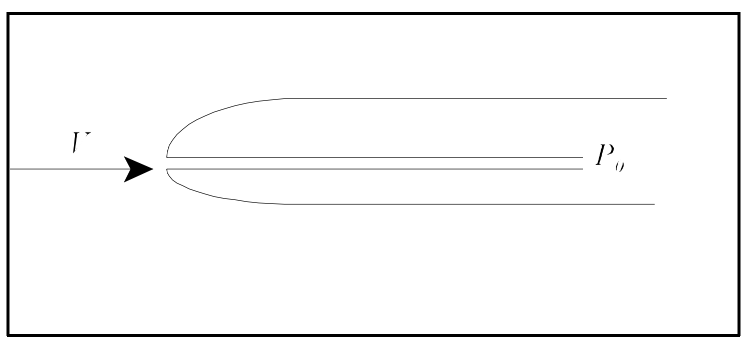
Equivalent airspeed

$$\bar{q} = \frac{1}{2}\rho V^2 \equiv \frac{1}{2}\rho_{SL} V_{eq}^2 \qquad \Rightarrow \qquad V_{eq} = \sqrt{\frac{\rho}{\rho_{SL}}}\,V \tag{7}$$

Note that equivalent airspeed is always less than (or equal to) the true airspeed.

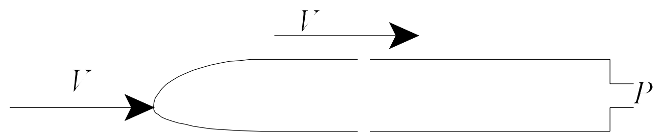
Pitot-Static Tube

The actual hardware used to measure airspeed on aircraft consists of a pitot (total pressure) tube combined with a static tube and hence the name pitot-static tube. A sketch of a pitot tube is shown to the right. The end of the center tube is attached to a pressure sensor and it will read the pressure P_0 since the flow will come to rest at the tip of the tube. These tubes can be observed to be located at various points on different aircraft. In flight- test aircraft it is usually located at the nose on an "instrumentation boom." On typical general aviation aircraft it is located on the outboard of the wing so as not to be in the propeller wash, and on jet propelled aircraft, it can generally be found on the side of the fuselage or on the top of the vertical tail, again, out of the region of jet wash or other jet effects.



Pitot Tube

Static pressure is measured by putting a pressure tap in a surface parallel to the flow. One way to do this is to use a static tube. A static tube is shaped like a pitot tube, but the pressure taps are along the side, rather than at the front. Here the flow is still moving at the free stream speed and the pressure will be the static pressure. In an aircraft, the static pressure taps can be located at points along the fuselage. In fact one of the pre-flight inspection checklist item is to be sure the static pressure ports are not clogged or obstructed. These pressure taps are the source of the static pressure for airspeed measurement, and for the altimeter discussed previously. Generally, as indicated in the drawing, the pressure taps are located on both sides (actually all around) the tube and on both sides of the fuselage. The reason for these locations is to account for any misalignment of the tube (or fuselage) with the wind.



Static Tube

Finally, in wind tunnel applications, it is convenient to combine the pitot tube with the static tube to provide a pitot-static tube with a hole in the front to measure the total pressure, and holes around the side to measure the static pressure. We can make use of the total and static pressure to estimate the airspeed. How we do this depends on some assumptions that we make, in our case incompressible flow. However, regardless of the assumptions, the airspeed indicator in any aircraft is driven ***only by the pressure difference*** of the stagnation or total pressure and the static pressure, even in supersonic flight!

Example:

An aircraft is flying at 3000 m and has a true airspeed of 120 kts. What is the reading observed on the airspeed indicator? What is the dynamic pressure, the static pressure and the total pressure?

$$V = 120 \text{ kts x } \frac{0.5144 \text{ m/s}}{1 \text{ kt}} = 61.728 \text{ m/s}$$

$$\begin{aligned} V_{cal_{inc}} = V_{eq} &= \sqrt{\sigma}\, V \\ &= \sqrt{\frac{0.9093}{1.2250}}\,(61.728) \text{ m/s} \\ &= (0.8616)\,(61.728) \\ &= 53.182 \text{ m/s} = 103.4 \text{ kts} \end{aligned}$$

The ***dynamic pressure*** is given by:

$$\bar{q} \;=\; \frac{1}{2}\rho\, V^2 \;=\; \frac{1}{2}\rho_{SL}\, V_{eq}^2 \;=\; \frac{1}{2}\, 0.9093\,(61.728)^2 \;=\; 1732.4 \;\; \mathbf{Pa} \;=\; 36.2 \;\; \mathbf{lbs/ft^2}$$

Note that at these low speeds, the dynamic pressure is << than the static pressure at 3000 m (70,121 Pa, or 1,464 lbs/ft^2)

The ***total or stagnation pressure*** is determined from Bernoulli"s equation:

$$P_0 \;=\; P \;+\; \frac{1}{2}\rho\, V^2 \;=\; P \;+\; \bar{q} \;=\; 70{,}121 \;+\; 1732.4 \;=\; 71853.4 \;\; \mathbf{Pa}$$

Since we are interested in pressure differences to measure airspeed, we can attach the two ports on a pitot-static tube and measure the difference with a manometer. The airspeed is then directly related to the difference in heights of the fluid in the two legs of the U tube**.** As a result it is not uncommon to hear wind tunnel operators to discuss airspeeds in "inches of water." For example, what would be the airspeed in a wind tunnel that was measuring 3 inches of water, assuming standard sea level conditions. What would be the airspeed if the same wind tunnel had the same measurements in Blacksburg (altitude = 2000 ft)?

4.4 Airfoil and Wing Aerodynamics

Here we are primarily interested in the forces and moments that are generated by a lifting surface immersed in a flow field. The results we will display can be predicted using aerodynamic theory and/or can be observed from test performed in a wind tunnel. However, before we can make sense out of these aerodynamic properties, we need to introduce the idea of non-dimensional coefficients.

Aerodynamic Coefficients

If we had two wings with exactly the same geometry, but one was twice as big as the other, we would expect the forces on these two wings to be different for the same angle of attack. Consequently it would be impossible to characterize these forces for various sizes and shapes of airfoils and wings since the combinations and permutations of all the parameters would be overwhelming in numbers. As a result we introduce non-dimensional quantities that become independent of size. This same idea allows us to test models in the wind tunnel and use the results for full scale aircraft (at least in theory). The basic idea is to gather all the characteristics associated with an airfoil or wing together and assemble them in a series of non-dimensional groups. Then any one group is some function (to be determined by wind tunnel tests) of the remaining groups, the function being independent of size. There is a systematic way of doing this grouping, but for the purposes of this course we will just display the parameters of interest, and their associated non-dimensional groups.

In general, and from experience, we will assume that the force on a lifting surface is a function of the following variables:

$$F = f(\rho, V, l, \mu, a, \alpha)$$

that are: density, airspeed, length (geometric size), viscosity, speed of sound, and angle of attack, respectively. If we group these parameters in non-dimensional groups, we find the following groups and definitions:

Force coefficient

$$C_f \equiv \frac{F}{\frac{1}{2}\rho V^2 l^2} \qquad (8)$$

Reynolds Number

$$R_e = \frac{\rho V l}{\mu} \qquad (9)$$

Mach Number

$$M_a = \frac{V}{a} \qquad (10)$$

Attitude angle
(Angle of attack)

$$\alpha \qquad (11)$$

More parameters could be included, leading to more non-dimensional groupings. For example camber and thickness. But these would lead to the non-dimensional groupings of t/c and δ/c that are just geometric properties. We will assume that the wind tunnel model and the full scale vehicle are geometrically similar.

The force coefficient has the term l^2 in it that can represent an area. For atmospheric vehicles the reference area used is typically the planform (projected top view) area of the wing for aircraft, and the largest cross sectional area of a missile. Note that the planform area of a wing is that area that includes the area bounded by the leading and trailing edges extended to the centerline of the vehicle.

The force coefficients generally are designated with a subscript associated with the force. For example we have the lift, drag, and side force coefficients:

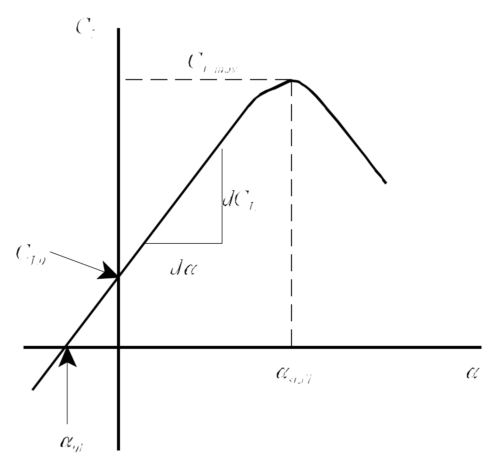
$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \qquad C_D = \frac{D}{\frac{1}{2}\rho V^2 S} \qquad C_Q = \frac{Q}{\frac{1}{2}\rho V^2 S} \qquad (12)$$

Aerodynamic Properties

Lift

The aerodynamic properties of primary interest to us at this time are those associated with the forces of lift and drag. The properties of lift associated with a two dimensional airfoil and a three dimensional wing (with the same airfoil) are similar but not the same. Here we will look at the generic properties that can be applied to both. If we go into a wind tunnel and test a wing or an airfoil, we can measure the lift and see how it changes with changes in angle of attack. If we do so, we can make the following observations: 1) For small angles of attack, the lift varies with angle of attack in a linear (straight line) manner, 2) At higher angles of attack the relation is no longer "linear" and that there exists a maximum value of lift and that the angle of attack at which it occurs is called the ***stall angle of attack.*** Rather than deal with lift, however, we convert all our measurements to the non-dimensional lift coefficient.

From the figure, we can see that there are two intercepts that we can designate, one on the alpha axis at zero lift, designated as α_{0L}, the zero-lift angle of attack, and the other on the C_L axis at zero angle of attack, designated as C_{L_0}, the lift at zero angle of attack. Generally, we are interested in the behavior of the wing in the region of low angle of attack where the lift curve is linear. In this region we are said to be assuming "linear aerodynamics."



With the assumption of linear aerodynamics, we can create a mathematical model of how the lift coefficient varies with angle of attack. To simplify the resulting expressions, we can first define the lift-curve slope:

Lift Curve Slope

$$a = \frac{dC_L}{d\alpha} \qquad (13)$$

Note that the lift curve slope of an airfoil (2-D wing) is usually designated with a subscript 0, e.g. a_0. Also the angle of attack of an airfoil (2-D Wing) is called the 2-D angle of attack, designated as α_0 or by α_{eff}(effective angle of attack). The mathematical model for the lift-curve in the linear region of flight can then be given in general by:

The Lift Curve Model

$$C_L = C_{L_0} + a\alpha = a(\alpha - \alpha_{0L}) \qquad (14)$$

Clearly $C_{L_0} = -a\,\alpha_{0L}$ or $\alpha_{0L} = -\dfrac{C_{L_0}}{a}$. My preference is to use the bottom representation of the lift curve, and to introduce the notation $\bar{\alpha} = \alpha - \alpha_{0L}$, where $\bar{\alpha}$ is the angle of attack measured from the zero lift line. Hence we have

Preferred Lift Curve Model

$$C_L = a\,\bar{\alpha} \qquad (15)$$

Lift Curve Slope

The lift curve slope depends upon many factors, including Reynolds number, Mach number, and the geometry, particularly the aspect ratio. For non-swept or slightly swept wings with aspect ratios around six and above, and operating at low speeds (incompressible) the following equation can be used to estimate the lift curve slope:

3-D Lift Curve Slope Estimation (Prandtl's formula)

$$a = \frac{a_0}{1 + \dfrac{a_0}{\pi\,AR\,e}} \qquad (16)$$

where a_0 = ***two dimensional*** lift curve slope (for thin wing $\approx 2\pi$ /rad)
AR = aspect ratio
e = efficiency factor

For higher subsonic speeds, and all aspect ratios we can approximate the lift-curve by the semi-empirical formula:

Straight tapered wing lift-curve slope (subsonic, all speeds and AR) (DATCOM Formula)

$$a = \frac{2\pi AR}{2 + \sqrt{\frac{AR^2(1-M^2)}{k^2}\left[1 + \frac{\tan^2 \Lambda_{1/2}}{(1-M^2)}\right] + 4}} \tag{17}$$

Where $k = \frac{a_0}{2\pi}$, and a_0 = actual 2-D lift-curve slope of airfoil section. If unknown assume $k = 1$.

For supersonic flow, low aspect ratio, straight tapered wings we have the following mathematical model for the lift-curve slope:

Straight tapered wing lift-curve slope (supersonic, low aspect ratio, thin wing)

$$a = \frac{4}{\sqrt{M^2 - 1}}\left(1 - \frac{1}{2AR\sqrt{M^2 - 1}}\right) \tag{18}$$

Aircraft Lift Curve

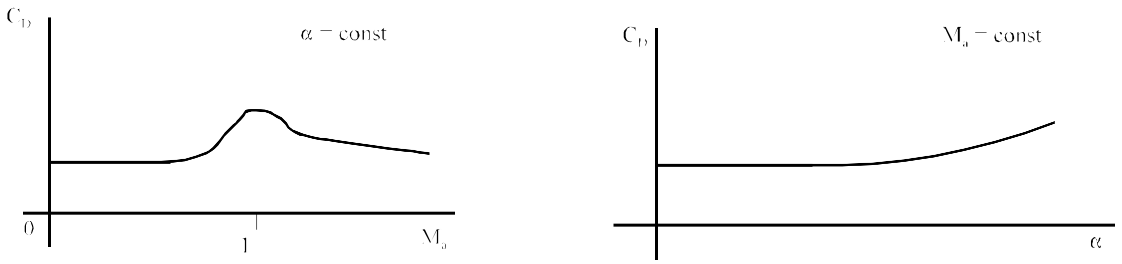
The lift curve model as given by Eqs. (14) and (15) can be used for an entire aircraft or missile. The lift due to all the surfaces as well as the contribution from the fuselage is generally linear with angle of attack, so that when we combine them into the vehicle lift, it to behaves linear with angle of attack and can be modeled with a vehicle lift curve slope, a vehicle zero lift angle of attack, etc. The vehicle lift curve slope is generally slightly greater (say 10%) than the wing lift curve slope for the wing alone as predicted by Eqs. (16 - 17) applied to the wing.

Drag

The drag of an airfoil, wing, or aircraft is more difficult to model, and is strongly affected by three dimensional considerations. If we put an airfoil (2-D) wing in a wind tunnel, the behavior of the drag curve as angle of attack is increased is significantly different then if we put in a 3-D wing (or aircraft). This behavior has to do with the air flowing about the ends of the wings. The fact that a wing is of finite length has considerable effect on its aerodynamic drag properties.

Airfoil (2-D) drag and 3-D profile or parasite drag

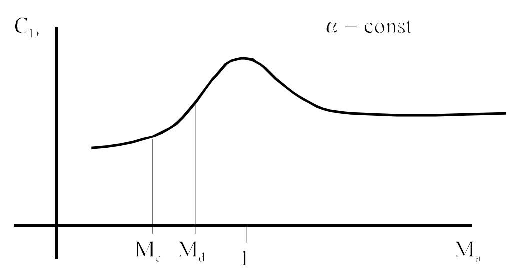
The drag on an airfoil (2_D wing)is primarily due to viscous effects at low speed, and compressibility effects (wave drag) at high speed. In addition, the flow can “separate” from the upper surface and cause additional drag. The drag coefficient depends on (at least) three quantities, Reynolds number (affects separation and drag due to viscous effects), Mach number (compressibility effects), and angle of attack. These are the only drag effects on an airfoil or 2-D wing. It turns out, as might be expected, that the drag on a 3-D wing or an entire aircraft behaves, in part, in a similar manner. Hence the curves shown below are for an airfoil and for a portion of the 3-D and/or complete aircraft drag.



Here we see that the drag coefficient is nearly constant at subsonic speeds and tends to rise just before Mach = 1. The biggest variation is in the neighborhood of Mach 1, called the transonic region. Above that region, say about Mach 1.2, the drag coefficient tends to be constant or it could increase or even decrease slightly. The figure on the right represents a typical change of drag coefficient with angle-of-attack at a given Mach number. It tends to increase slightly with angle-of-attack at low angles, and increases more rapidly at high angles-of-attack. The curve is approximately quadratic in angle-of-attack.

If we look at a close up of the drag coefficient in the transonic region, we can define certain specific Mach numbers.

As the Mach number is increased, the first specific Mach number that we encounter is the ***critical Mach number***, M_c.



Definition: Critical Mach Number

The critical Mach number is defined as the Mach number at which the flow somewhere on the vehicle is sonic, Mach = 1.

The next Mach number encountered is called the ***Drag Divergence Mach Number***.

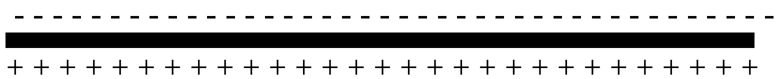
Definition: Drag Divergence Mach Number

The drag divergence Mach number is that Mach number where the drag coefficient increases by 0.002 or by 2 "counts" of drag. Note that this definition is not universal. Other definitions exist, some based on the slope rather than the value itself.

These definitions apply to 2-D, 3-D, and complete aircraft. Of importance to us is the application to a complete aircraft.

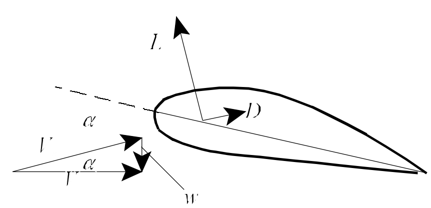
3-D effects on Drag

The primary effect on wing or vehicle drag is to add an additional term to the drag expression. This term comes about when we consider the span-wise lift distribution (it is no longer constant), caused by the flow about the wing tips. In normal operating conditions, the wing will have high pressure on it lower surface and a low pressure on its upper surface. This pressure difference is what generates the lift. However, this same pressure difference causes flow from the under side of the wing to the upper side of the wing around the wing tips.



This type of flow swirls off the tips of the wing and tracks downstream in the form of vortices. In fact there is a vortex distribution across the entire span of the wing with the strongest vortices at the wing tips. These vortices trail downstream behind the wing and rotate in the direction upward on the outboard side, and downward on the inside. Vortices on the right hand side of the wing (looking from the rear) rotate counter clockwise, and those on the left hand side of the wing rotate clockwise. The general result is that the vortices induce a downward flow at the wing interior. This downward flow is called downwash, and it influences the flow in front of, at, and behind the wing. This downward flow causes a change in the local wing angle-of-attack such that each wing section sees a different angle-of-attack then the one that it sees with respect to the free stream.

The velocity induced by the vortices is designated as the downwash, w. This downwash decreases the local angle-of-attack by an amount called the induced angle-of-attack, α_i that is approximately (small angle approximation) given by



$$\alpha_i = \frac{w}{V} \qquad (18)$$

As a result, the local ***airfoil*** only “sees” the “effective” angle-of-attack(or what we called in the previous section the 2-D angle-of-attack). Hence we have:

$$\alpha_{eff} = \alpha - \alpha_i \qquad (19)$$

where:	α	=	actual angle-of-attack measured with respect to the free stream
	α_i	=	induced angle-of-attack
	α_{eff}	=	effective angle-of-attack, the local angle of attack seen by the local

airfoil section, also could be called the 2-D angle-of-attack

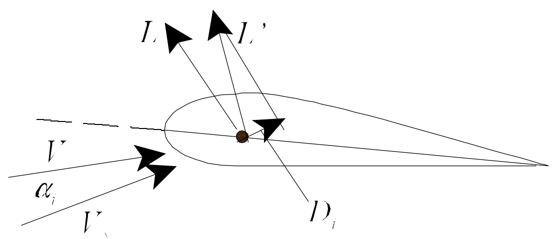
At this point, we can look at the local section and determine the lift from the 2-D section properties that we examined previously. In doing so, we can observe that the effect on the local airfoil section calculations is to:

1) reduce the amount of lift for a given α since

$$c_l = a_0(\alpha_{eff} - \alpha_{0L}) < a_0(\alpha - \alpha_{0L})$$

E.g. the local airfoil sees a smaller angle-of-attack then the free stream angle-of-attack

2) cause the relative wind to come in at a different direction (lower angle-of-attack) and hence the lift perpendicular to it will be in a different direction from the defined lift direction (perpendicular to the free stream) thus producing a component of force in the drag direction called induced drag.



From the figure we have:

$$\begin{aligned} L &= L' \cos\alpha_i \approx L' \\ D_i &= L' \sin\alpha_i \approx L'\alpha_i \end{aligned} \tag{20}$$

Therefore the finite wing causes a downwash that causes a change in the ***local relative wind direction*** so that the lift generated perpendicular to this local relative wind is no longer perpendicular to the free-stream velocity. It is tilted backward a small amount. The component of this lift parallel to the original free stream direction is called the induced drag.

Induced Drag

The induced drag is equal to the lift times the induced angle of attack as seen in the previous figure. It turns out that the induced angle of attack is proportional to the lift coefficient.

Further, one can suggest that the aspect ratio would have some affect on this induced angle of attack. In particular, if the wing is infinite (2-D) then there is no flow around the wing tips and hence there is no downwash and hence no induced drag. But as the aspect ratio gets smaller, the effect of the wing tips would be expected have more effect on the downwash since they are closer. As a result, it can be shown that under certain circumstances that the induced angle of attack is given by:

$$\alpha_i = \frac{C_L}{\pi AR}$$

and hence the induced drag is given (see Eq. 20) by:

$$D_i = L\,\alpha_i = L\frac{C_L}{\pi AR} = \bar{q}S\frac{C_L^2}{\pi AR} \qquad \Rightarrow \qquad C_{D_i} = \frac{C_L^2}{\pi AR}$$

This result is for a somewhat restricted case. To extend to the more general case we can use the standard "engineering" method of introducing an engineering "factor." For the general case, we introduce the ***span efficiency factor***, e_w.

$$C_{D_i} = \frac{C_L^2}{\pi AR\, e_w} \tag{21}$$

Again, we can generalize this expression to apply to a complete aircraft in the following way:

$$C_{D_i} = \frac{C_L^2}{\pi AR\, e} = K C_L^2 \tag{22}$$

where e = Oswald (aircraft) efficiency factor

K = induced drag parameter = $\dfrac{1}{\pi AR\, e}$

Finally, we can put together the drag coefficient for the complete aircraft:

Aircraft Drag Coefficient (drag polar)

$$C_D = C_{D_{0L}} + K C_L^2 \tag{23}$$

where: $C_{D_{0L}}$ = zero-lift drag coefficient, parasite drag coefficient

K = induced drag parameter.

Drag Polars

The drag coefficient can be written as functions of many different variables. The strongest dependence of drag is on the lift coefficient because of the induced drag effect. If the mathematical model for the drag coefficient contains the lift coefficient, that expression is called a ***drag polar***. Hence we can make the following definition:

Definition: Drag Polar A drag polar is a any mathematical expression the relates the drag to some function of the lift coefficient, $\boldsymbol{C_D = f(C_L)}$.

There are several different drag polars that are typically used to represent the drag of an aircraft. The following two are the most frequently used when Eq. (23) is not used:

$$C_D = C_{D_{0L}} + K_1 C_L + K_2 C_L^2$$
$$C_D = C_{D_{\min}} + K\left(C_L^2 - C_{L_0}^2\right) \quad (24)$$

where C_{L_0} is the lift coefficient when the drag is a minimum. The last equation in Eq. 24, and Eq. (23) are examples of a particular type of a drag polar, a ***parabolic drag polar***. For the majority of this course we will use the ***parabolic drag polar given in Eq. (23)***

In general the drag coefficient is a function of Reynolds number and Mach number. In general, for the normal flight regime, the dependence on these numbers is the similar to the dependence as observed previously. Typically, in the flight conditions of interest, the Reynolds number has only a small effect while that of the Mach number is prominent in high subsonic, transonic, and supersonic flight. We can indicate this dependence on Mach number in our parabolic drag polar as follows:

$$C_D = C_{D_{0L}}(M) + K(M)\, C_L^2 \quad (25)$$

where $C_{D_{0L}}(M)$ = Mach number dependent zero-lift drag

$K(M)$ = Mach number dependent induced drag parameter

We could also write: $K(M) = \dfrac{1}{\pi\, AR\, e(M)}$, where the Oswald efficiency factor is a function of the Mach number.

Example

An aircraft weighs 40,000 lbs, wing area of 350 ft^2 and a wing span of 50 ft. At sea-level the aircraft flies at 200 and 600 ft/sec. What are the values of the induced drag and the associated drag coefficients for this case. Noting that Lift = weight in level flight:

$$C_{L_1} = \frac{W}{1/2\,\rho\,V_1^2\,S} = \frac{40{,}000}{1/2\,(0.002377)\,(200)^2\,(350)} = 2.400$$

$$C_{L_2} = \frac{W}{1/2\,\rho\,V_2^2\,S} = \frac{W}{1/2\,\rho\,V_1^2\,S} \cdot \frac{V_1^2}{V_2^2} = 2.400\left(\frac{200}{600}\right)^2 = 0.267$$

$$AR = \frac{b^2}{S} = \frac{50^2}{350} = 7.143$$ Also assume e = 0.85

$$C_{D_{i_1}} = \frac{C_L^2}{\pi\,AR\,e} = \frac{2.400^2}{\pi\,(7.143)\,(0.85)} = 0.301$$

$$C_{D_{i_2}} = \frac{0.267^2}{\pi\,(7.143)\,(0.85)} = 0.004$$

$$D_1 = C_{D_i}\,1/2\,\rho\,V^2\,S = 0.301\,(1/2)\,(0.002377)\,(200^2)\,(350) = 5014.7 \text{ lbs}$$ Induced drag only!

$$D_2 = C_{D_i}\,1/2\,\rho\,V_2^2\,S = 0.004\,(1/2)\,(0.002377)\,(200^2)\,(350) = 599.8 \text{lbs}$$ Induced drag only!