6. Stability Considerations

Before we can discuss the idea of stability, we must first introduce the concept of the ***reference flight condition***. The reference flight condition (RFC) is an equilibrium flight condition. The most common one used is that of straight and level flight. Another could be straight climbing (or diving) flight. If we select a different speed, we have a different RFC. Once we select are reference flight condition, we can consider the static stability of ***<u>that particular reference flight condition</u>***. If we select a different reference flight condition, we must reconsider the stability of the new RFC. When someone says that an aircraft is statically stable, it generally means that it is statically stable over a range of reference flight conditions. To go further, we must introduce the idea of static stability.

When a vehicle is in the reference flight condition, the forces and moments all sum to zero and the vehicle is in equilibrium. If the vehicle is disturbed by some means or other (typically a gust of air), then the vehicle is disturbed away from the equilibrium reference flight condition, and the vehicle is not longer in equilibrium, that is the forces and moments no longer sum to zero. The question asked is do the now unbalanced forces and moments tend to restore the vehicle to equilibrium or do they tend to push it further away from equilibrium? We can now define the concept of statically stability.

Static Stability

If a vehicle is disturbed away from the equilibrium reference flight condition, and the unbalanced forces and moments, ***caused by the disturbance***, tend to move the vehicle in a manner which tends to restore the equilibrium reference flight condition, then the vehicle is said to be statically stable.

Static Stability in Pitch

We can now discuss the concept of statically stability in pitch. Consider an aircraft flying straight and level at some specified reference flight condition. At some instant, a disturbance hits the vehicle causing a nose up displacement in angle-of-attack so that the vehicle is no longer in equilibrium. For static stability, this displacement must cause a moment which tends to lower the nose and reduce the angle-of-attack back to its original value. Hence for static stability in pitch, the change in the pitch-moment due to the change in angle-of-attack must be negative. We can write this mathematically in a very concise way: For ***static stability in pitch***

$$\left.\frac{\Delta M}{\Delta \alpha}\right|_{ref} < 0 \qquad (1)$$

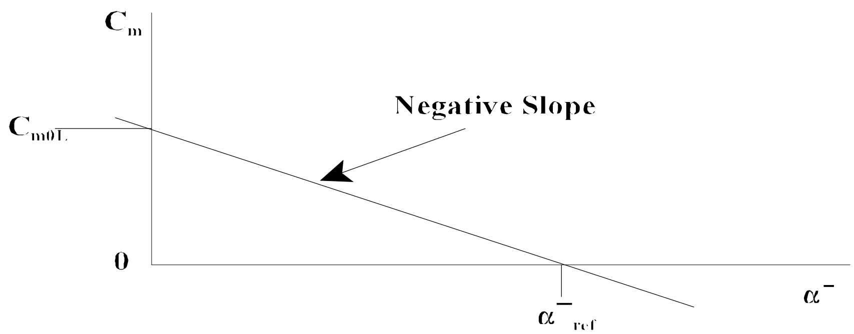
Alternative ways of writing this expression are:

$$\left.\frac{\Delta C_m}{\Delta \alpha}\right|_{ref} = \left.\frac{dC_m}{d\alpha}\right|_{ref} = C_{m_\alpha} < 0 \tag{2}$$

This term C_{m_α} is called the ***longitudinal stability parameter***. It must be negative for a longitudinally statically stable aircraft. If we now assume that the pitch-moment (coefficient) can be represented by a straight line in a manner similar to the lift coefficient, we can write the mathematical model of the pitch-moment coefficient as:

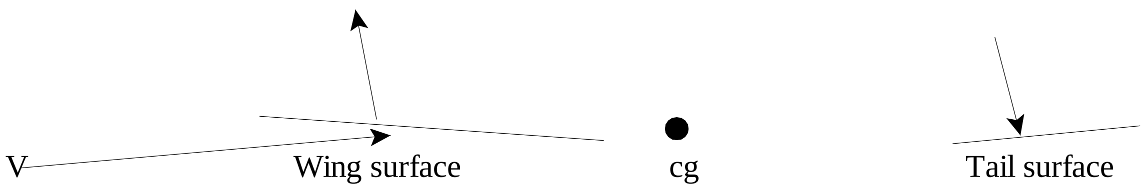
$$C_m = C_{m_{0L}} + \left.\frac{dC_m}{d\alpha}\right|_{ref} \bar{\alpha} \tag{3}$$

For the equilibrium flight condition, $\boldsymbol{C_m = 0}$. Hence we can make the following observation: For a statically stable aircraft, flying at some reference flight condition, the angle-of-attack must be positive in order to generate the required lift, C_{m_α} must be negative for stability, C_m must equal zero, and therefore $C_{m_{0L}}$ must be positive. Otherwise the previous equation could not be satisfied. We can draw a graph to illustrate these points.



Here we see that if the pitch-moment is zero at the reference flight condition, and the slope of the curve is negative, then at zero lift ($\bar{\alpha} = 0$), the pitch-moment coefficient must be positive. Hence all vehicles must have a pitch-moment curve with a negative slope, and have a positive (nose-up) pitch-moment when the angle-of-attack is set so that there is zero lift.

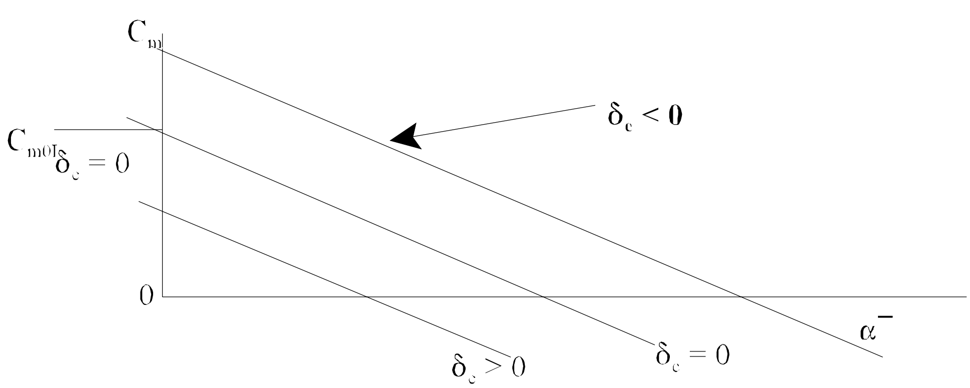
The negative slope of the pitch-moment curve can always be obtained by moving the center-of-gravity far enough forward. (Accept this bit of wisdom on faith). The positive pitch-moment at zero lift is obtained by having the rear lifting surface (horizontal tail for conventional aircraft) with the leading edge down, relative to the wing. In this manner, when the relative wind is at the correct angle-of-attack, the forward lifting surface (the wing in a conventional aircraft) will have a slight up force, and the aft lifting surface will have a slight down force. At the proper angle-of-attack, these two forces will be equal and opposite creating zero lift, but creating a nose up or positive moment. We have the following situation:



The resulting “lift” is zero, and the pitch-moment about the cg is nose up.

Pitch Control

We can observe from the above developments, that if the lift-curve-slope, $\frac{dC_L}{d\alpha}$, the pitch-curve-slope, $\frac{dC_m}{d\alpha}$, and the zero-lift pitch moment coefficient, $C_{m_{0L}}$, are known, then the lift = weight and pitch-moment = 0 give us two equations with only one unknown in them, $\bar{\alpha}$. Consequently there is only one value of $\bar{\alpha}$ that will satisfy both equations, and thus only one speed at which one could fly. If the geometry of the vehicle is fixed (as in our classroom glider), then this is indeed the case. In order fly at different speeds and therefor at different angles-of-attack, we must be able to adjust one of those three parameters. It turns out we can adjust $C_{m_{0L}}$ by deflecting the elevator control. The elevator is usually a flap attached to the trailing edge of the horizontal tail. A ***trailing edge down deflection is a positive elevator deflection***. Such a deflection will create a nose down pitch moment and hence shift $C_{m_{0L}}$ (apparent) downward. In a similar manner, a negative elevator control deflection will shift $C_{m_{0L}}$ (apparent) upward. This shift will cause the intercept with the $C_m = 0$ line (the $\bar{\alpha}$ axis) to move left to a smaller $\bar{\alpha}$ (positive elevator deflection) or to the right to a bigger $\bar{\alpha}$ (negative elevator deflection), allowing us to fly at different angles-of-attack and hence, at different speeds.



Force and Moment Balance

The above ideas can be written formally in terms of two equations for the lift and the pitch-moment. We define an additional term in each equation that models the contribution from the elevator. The two terms are called the elevator effectiveness in the lift coefficient equation, and the elevator power in the pitch-moment coefficient equation. In order to fly in straight and level flight, there must be a force and balance equilibrium. Hence lift must equal weight, thrust equal drag, and the pitching moment must be zero. Here, we will assume that we have the thrust necessary to balance the drag, and concentrate our discussion on satisfying the lift-weight equilibrium and zero pitching moment at the same time If we deal with coefficients we have the following two equations:

$$C_L = \left.\frac{\partial C_L}{\partial \alpha}\right|_{ref} \bar{\alpha} + \left.\frac{\partial C_L}{\partial \delta_e}\right|_{ref} \delta_e = C_{L_\alpha} \bar{\alpha} + C_{L_{\delta_e}} \delta_e \tag{4}$$

$$C_m = 0 = C_{m_{0L}} + \left.\frac{\partial C_L}{\partial \alpha}\right|_{ref} \bar{\alpha} + \left.\frac{\partial C_L}{\partial \delta_e}\right|_{ref} \delta_e = C_{m_{0L}} + C_{m_\alpha} \bar{\alpha} + C_{m_{\delta_e}} \delta_e \tag{5}$$

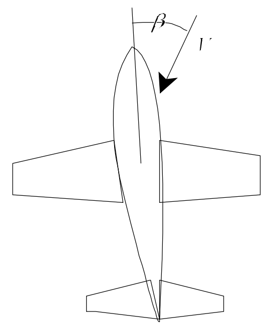
where $C_{L_{\delta_e}}$ is the elevator effectiveness, and $C_{m_{\delta_e}}$ is the elevator power.

Hence for a given cruise condition, altitude and airspeed, we can solve these two equations for the two unknowns, angle of attack, $\bar{\alpha}$ and elevator angle, δ_e. Without the elevator, we would have to satisfy two equations with only one free variable, the angle of attack. It might be possible to do that, but it would be at only one angle of attack and therefore only for one speed. Consequently, it can be seen that the purpose of the elevator is to allow the aircraft to fly at different speeds and allow the pitch-moment to be zero.

Directional Stability

The concept of static stability applies to directional stability. Here we are interested in the effect of a disturbance in yaw. In the reference flight condition, we fly with the relative wind acting in the plane of symmetry. If the flight is disturbed by a small gust of wind or some other disturbance, then the wind is no longer in the plane of symmetry and makes an angle with the plane of symmetry, called the sideslip angle which is designated by β. Further, β is positive if the relative wind is coming in from the right hand side.

This (asymmetric!) aircraft illustrates the relative wind after a disturbance in yaw. The requirement for static stability is that the disturbance develop a yaw-moment that tends to move the aircraft back to the reference flight condition, in this case with β = 0. Using the right-hand rule and recalling that the z^b axis is down, we can see that a positive yaw moment would push the nose of the vehicle back into the wind. Therefore, for static stability in yaw, the requirement is that:



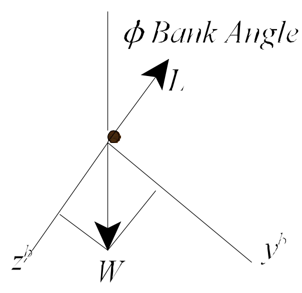
$$\left.\frac{\Delta C_n}{\Delta \beta}\right|_{ref} \Rightarrow \left.\frac{dC_n}{d\beta}\right|_{ref} = C_{n_\beta} > 0 \tag{6}$$

This requirement is called the directional stability parameter or alternatively, the ***weathercock stability*** parameter. An aircraft that is statically stable in yaw is said to be weathercock stable.

In most cases we desire the sideslip angle to be zero. In certain situations, however, it is desirable to have the sideslip angle non-zero. One important case is when the aircraft is landing. At that point we would like the vehicle to be lined up with the runway and not necessarily the wind. To allow that to happen, we need a yaw control, the rudder. The rudder is formed by the trailing edge portion of the vertical tail. A ***positive rudder deflection is trailing edge left*** and it causes a negative yaw-moment.

Lateral or Roll Stability

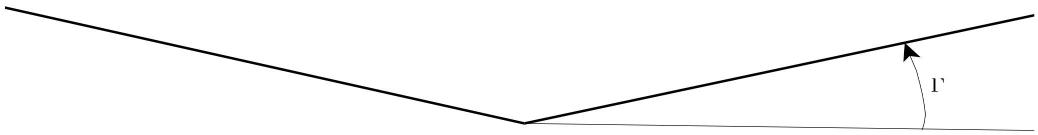
For an aircraft to be statically stable in roll, a roll disturbance (a small bank angle) would generate a restoring roll-moment to bring the wings back to level. Such a force does not exist. ***There is no roll-moment generated by a small bank angle to oppose the bank!*** However, there are also forces that are unbalanced due to the roll angle. In particular there is an unbalanced force in the y direction due to gravity. If we draw the free body diagram we see the component of the weight along the y^b axis is not balanced by any other force. Consequently the vehicle will accelerate along the y^b axis and therefor cause a relative wind from the right (a positive β). If this disturbance in the sideslip angle causes a negative roll-moment, we say the vehicle is (indirectly) statically stable in roll. This stability parameter is called the ***dihedral effect***. Although there is no direct static stability in roll, there is this secondary effect which contributes to stability. The roll stability parameter is given by:



Looking forward from an aft position

$$\left.\frac{dC_l}{d\beta}\right|_{ref} = C_{l_\beta} < 0 \qquad (7)$$

This static stability parameter is called the dihedral effect since one way of achieving this result is to add dihedral to the wings (or horizontal tail). Dihedral is positive if the wing tip chords are higher than the root chords.



Γ = Dihedral angle (positive as shown)