AOE 2104 Introduction to Aerospace Engineering Problem Sheet 2 (ans)

6. An aircraft is at a pressure altitude of 7000 ft. The temperature is 95 deg F. Determine the density altitude at these conditions. (In ft)

We can find the pressure from the standard atmosphere charts:
@ 7000 ft, P_{7000} = 1633.1 lb/ft^2 T = 95 + 459.688 = 554.688 deg R

From the perfect gas law we have: $\rho = \dfrac{P}{R\,T} = \dfrac{1633.1}{1716.488\,(554.688)} = 0.0017152 \text{ slugs/ft}^3$

We interpolate to get the density altitude:

hu = 11,000 ft ρu = 0.0017011 slugs/ft^3

hL = 10,000 ft ρL = 0.0017556 slugs/ft^3

$$h = h_L + \frac{\rho - \rho_L}{\rho_u - \rho_L}(h_u - h_L)$$

$$= 10{,}000 + \frac{0.0017152 - 0.0017556}{0.0017011 - 0.0017556}(11{,}000 - 10{,}000)$$

$$= 10{,}741 \text{ ft} \qquad \text{density altitude}$$

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7. What is the aspect ratio of

a) a wing with a square planform

b) a 747 that has a wing area of 5,500 sq ft and a wing span of 195.68 ft

c) An F14 Tomcat with wing area of 565 sq ft, and a span of 61 ft 10 in

d) calculate the mean geometric chord of the 747 (for comparison, the mean aerodynamic chord is 27.31 ft)

$$AR = \frac{b^2}{S}$$

a) $S = b^2 \quad \Rightarrow \quad AR = \dfrac{b^2}{S} = \dfrac{b^2}{b^2} = 1$

b) $AR_{747} = \dfrac{195.68^2}{5500} = 6.962$

c) $AR = \dfrac{61.833^2}{565} = 6.767$

d) $c_g = \dfrac{S}{b} = \dfrac{5{,}500}{195.68} = 28.107$ compared to $\bar{c} = 27.31$ ft

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8. A wind tunnel operates at 0.2 m of water

a) What is the airspeed in the wind tunnel assuming standard sea level conditions (m/s)

b) What would be the airspeed under these same conditions, but the wind tunnel was located in the mountains at standard atmospheric conditions at 2000 m?

The phrase "operates at 0.2 m of water" refers to the difference of height of columns of water in a U tube manometer measuring the difference of static and stagnation pressure in a pitot-static tube. Hence we have (from the static or buoyancy equation and from Bernoulli's equation):

$$P_0 - P = -\rho_w g(h_0 - h) \qquad\qquad P_0 - P = \frac{1}{2}\rho V^2$$

From the static equation:

$$\rho_w = 1000 kg/m^3 \qquad\qquad P_0 - P = -1000\,(9.807)\,(0 - 0.2) = 1961.4 \text{ N/m}^2$$

From Bernoulli's equation:

$$P_0 - P = 1961.4 = \frac{1}{2}1.2250\,V^2 \quad \Rightarrow \quad V = 56.59 \text{ m/s} \quad @ \quad \text{sealevel}$$

@ 2000 m We need to interpolate to get the density: $\rho = \rho_L + \left(\frac{h - h_L}{h_u - h_L}\right)(\rho_u - \rho_L)$

$$h_u = 2100 \text{ ft} \qquad \rho_u = 0.99649 \text{ kg/m}^3$$

$$h_L = 1800 \text{ ft} \qquad \rho_L = 1.0269 \text{ kg/m}^3$$

$$\rho = 1.0269 + \left(\frac{2000 - 1800}{2100 - 1800}\right)(9.99649 - 1.0269) = 1.00663 \text{ kg/m}^3$$

$$p_0 - P = \frac{1}{2}\rho V^2 = 1961.4 = \frac{1}{2}(1.00663)V^2 \quad \Rightarrow \quad V = 62.42 \text{ m/s}$$

Note the differences in the speeds with the same dynamic pressure!

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9. The Virginia Tech Stability Wind Tunnel can operate up to 200 miles an hour. Assume standard atmospheric conditions at 2000 ft altitude (Blacksburg's altitude). The vehicle being tested has a maximum lift coefficient of $C_{L_{max}} = 1.7$. The model has a wing area of 2 sq ft. What is the maximum load that we can expect to measure?

Convert to basic units: $200 \text{ mi/hr} \cdot \frac{88 \text{ ft/sec}}{60 \text{ mi/hr}} = 293.33 \text{ ft/sec};$ $\rho_{2000} = 0.0022409 \text{ slugs/ft}^3$

$$L = C_{L_{max}}\frac{1}{2}\rho V^2 S = 1.7\left(\frac{1}{2}\right)0.0022409\,(293.33^2)\,2 = 327.8 \text{ lbs}$$

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10. Wind tunnel tests yield the following data:

α	-2 deg	0	2	4	6
C_L	0.0	0.2	0.4	0.6	0.8

Determine the lift curve slope of this vehicle, $a = \dfrac{dC_L}{d\alpha} = ?$

Plotting this curve, one can see that it is a straight line. We can determine the slope by looking at the extreme values:

$$a = \frac{C_{L_u} - C_{L_L}}{\alpha_u - \alpha_L} = \frac{\Delta C_L}{\Delta \alpha} = \frac{0.8 - 0.0}{6 - (-2)} = \frac{0.8}{8} = 0.10 \text{ /deg} = 5.73 \text{ /rad}$$