AOE 2104 Problem Sheet 3 (ans)

11. An aircraft is flying at Mach = 0.8 at sea level. The compressible Bernoulli's equation (for subsonic flow) is given by:

$$\frac{P_0}{P} = (1 + 0.2M_a^2)^{3.5}$$

where:

P_0	=	stagnation pressure
P	=	static pressure
M_a	=	Mach number

a) Compare the stagnation pressure computed using this equation with that using the incompressible Bernoulli's equation that we derived in class.
b) Assuming the compressible equation is correct, what is the percent error in the stagnation pressure if the incompressible equation is used?

@ sea level:	P = 2116.2 lbs/ft^2	T = 518.69 deg R	a = 1116.4 ft/sec
	P = 101325 N/m^2	T = 288.16 deg K	a = 340.29 m/s

a) From the compressible equation:

$$\frac{P_0}{P} = \left(1 + 0.2M_a^2\right)^{3.5} = \left[1 + 0.2\ (0.8^2)\right]^{3.5} = 1.524$$

Then,

$$P_0 = 1.524\,P = 1.524\,(2116.2) = 3225.8 \ \text{lbs/ft}^2$$

$$= 1.524\,(101325) = 154419.3 \ \text{N/m}^2$$

Need airspeed:

$$V = 0.8\,a = 0.8\,(116.4) = 893.12 \ \text{ft/sec}$$

$$= 0.8\,(340.29) = 272.23 \ \text{m/s}^2$$

Incompressible:

$$P_0 = P + \frac{1}{2}\rho V^2 = 2116.2 + \frac{1}{2}(0.002377)(893.12^2) = 3064.22 \ \text{lbs/ft}^2$$

$$= 101325 + \frac{1}{2}(1.2250)(272.23^2) = 146716.9 \ \text{N/m}^2$$

Error:

$$\%\ \text{error} = \frac{P_{comp} - P_{incomp}}{P_{comp}} \cdot 100 = \frac{3225.8 - 3064.22}{3225.8} \cdot 100 = 5.0\ \%$$

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12. The temperature equation for any Mach number (subsonic or supersonic) is given by:

$$\frac{T_0}{T} = 1 + 0.2 M_a^2$$

a) If an aircraft were to fly at M_a = 3.0 at sea level, what would be the stagnation temperature, T_0, at the leading edge of the wing or any other stagnation point? (deg K and deg R)
b) For incompressible flow it is assumed that the temperature is constant. If that's the case, what would be the percent error in the temperature if we used the incompressible value.

a) $$\frac{T_0}{T} = 1 + 0.2 M_a^2 = 1 + 0.2\,(3)^2 = 2.8$$

Compressible:

$$T_0 = 2.8\,T = 2.8\,(518.69) = 1452.3 \text{ deg } R$$

$$= 2.8288.16) = 806.8 \text{ deg k}$$

Incompressible:

$$T_0 = \text{const} = T = 518.69 \text{ deg } R$$

$$\%\ \text{error} = \frac{T_{0_{comp}} - T_{0_{incomp}}}{T_{0_{comp}}} \cdot 100 = \frac{1452.3 - 518.69}{1452.3} \cdot 100 = 64.3\ \%$$

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13. Attached is a drawing of an F/A 18 aircraft. By measuring needed values off the drawing (generally not a good thing to do!), use the DATCOM lift curve slope equation to estimate the lift curve slope, a.

The DATCOM equation:

$$a = \frac{2\pi AR}{2 + \sqrt{\frac{AR^2(1 - M_a^2)}{k^2}\left(1 + \frac{\tan^2\Lambda_{1/2}}{(1 - M_a^2)}\right) + 2}}$$

From Class: AR = 3.66 Also for low speed we can assume M = 0
From drawing: $\Lambda_{1/2}$ = 12.5 deg

$$a = \frac{2\pi\,(3.66)}{2 + \sqrt{\frac{3.66^2(1 - 0^2)}{1^2}\left(1 + \frac{\tan^2 12.5}{(1 - 0^2)}\right) + 2}} = \underline{3.88\ /\text{rad}}$$

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14. In the early 60's there was a very good aircraft called the Lockheed Electra. Unfortunately it was the aircraft that inadvertently discovered "whirl instability," a previously unknown phenomena that coupled the rotating engine and propellor (prop jet), with the flexibility in the wing in such a manner that the wings started tearing off. The powers that be figured out that the phenomena was directly related to the dynamic pressure, and could be avoided if the dynamic pressure was kept below a certain value, approximately 60 % of what they were currently flying. The airline wanted to keep the schedules as they were. The airline's aerospace engineers said "no problem"

all you have to do is fly higher and maintain the original airspeed. If the original flights were flown at 20,000 ft, how high would the aircraft have to fly to maintain the same airspeed and reduce the dynamic pressure 60%. Is this a feasible solution? Why or why not?.

Get the properties of the atmosphere at 20,000 ft. From table: $\rho_{20} = 0.0012673 \ \text{slugs/ft}^3$

Define the dynamic pressures at 20,000 ft and at x,000 ft:

$$\bar{q}_{20} = 1/2\, \rho_{20} V_{20}^2 \qquad\qquad \bar{q}_x = 1/2\, \rho_x V_x^2$$

The conditions on the problem are: $\bar{q}_x = 0.6\, \bar{q}_{20}$ and $V_x = V_{20} = V$

Substituting these relations into the dynamic pressure relations we have:

$$\frac{1}{2} \rho_x V^2 = 0.6 \frac{1}{2} \rho_{20} V^2 \qquad \Rightarrow \qquad \rho_x = 0.6\, \rho_{20}$$

$$\rho_x = 0.6\,(0.0012673) = 0.00076038 \ \text{slugs/ft}^3$$

Interpolate to get altitude:

$$h_L = 34{,}000 \ \text{ft} \qquad \rho_L = 0.00076696 \ \text{slugs/ft}^3$$

$$h_u = 35{,}000 \ \text{ft} \qquad \rho_U = 0.00073820 \ \text{slugs/ft}^3$$

$$h = h_L + \frac{\rho - \rho_L}{\rho_u - \rho_L}(h_u - h_L) = 34000 + \frac{0.00076038 - 0.00076696}{0.00073820 - 0.00076696}(1000) = 34{,}224 \ \text{ft}$$

Its feasible because the altitude is within the upper limits of where current aircraft can fly (assuming the engines can generate enough thrust.

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15. A drag polar is given by $C_D = 0.02 + 0.06\, C_L^2$. If we assume that lift = weight, and that we are considering an aircraft with a weight of 10,000 lbs and a wing area of 500 sq ft, find:

a) The lift coefficient at 100, 200, 300, 400, and 500 ft/sec
b) Find the corresponding drag coefficient and drag (I lbs) at these same speeds.
c) Make a table with V, C_L, C_D, and D as column headings.

From the information given, and comparing with $C_D = C_{D_{0L}} + K C_L^2$, it is clear that $C_{D_{0L}} = 0.02$, and $K = 0.06$. Further, since no altitude is given, we will assume sea level conditions (alternatively we can think of the airspeeds given as equivalent airspeeds). WE can use MATLAB to do these calculations or just do them by hand. Picking the latter, we have:

$$C_L = \frac{L}{1/2\, \rho V^2 S} = \frac{W}{1/2\, \rho V^2 S} = \frac{10000}{1/2\,(0.002377)\,(500)\, V^2} = \frac{16827.9}{V^2} \qquad (1)$$

Also we have $C_D = 0.02 + 0.06\, C_L^2$ (2) and $D = C_D\, 1/2\, \rho V^2 S$ (3)

If we perform the calculations, we can from a simple table that we can compute the columns using equations (1), (2), and (3) in that order:

V ft/sec (given)	C_L (1)	C_D (2)	D (lbs) (3)
100	1.683	0.190	1128.3
200	0.421	0.031	727.8
300	0.187	0.022	1181.5
400	0.105	0.021	1964.5
500	0.067	0.020	3011.3

Note that the drag increase as we go slower and as we go faster. Somewhere there is a speed where the drag is a minimum! Can you find that speed without a lot of trial and error?