AOE Problem Sheet 4

Due 2 October, 2003

Read Section 5 on the website <www.aoe.vt.edu/~lutze/AOE2104>

16. A Boeing 747 weighs 750,000 lbs. The geometry of the aircraft is as follows: Wing area = 5500 ft^2, Wing span = 196.68 ft. Calculate the lift coefficient for cruise at Mach = 0.75 at

a) Sea level

b) 35,000 ft

a) Determine airspeed: $V = M_a a = 0.75\,(1116.4) = 837.3$ ft/sec

Then: $$C_L = \frac{W}{1/2\,\rho\,V^2\,S} = \frac{750000}{1/2\,(0.002377)\,(837.3^2)\,(5500)} = 0.1636 \text{ at sea level}$$

b) Determine airspeed: $V = M_a a_{35} = 0.75\,(973.14) = 729.85$ ft/sec

Then: $$C_L = \frac{750000}{1/2\ 0.0007382)\,(729.85^2)\,(5500)} = 0.6945 \text{ at 35,000 ft}$$

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17. The 747 aircraft that is described in question 16, has an Oswald efficiency factor of 0.9. The zero lift drag coefficient is $C_{D_{0L}} = 0.018$ at $M_a = 0.75$.

a) Estimate the induced drag parameter K

b) Estimate the induced drag coefficient at sea level at $M_a = 0.75$

c) Estimate the induced drag coefficient at 35,000 ft at the same Mach number.

a) $$K = \frac{1}{\pi\,AR\,e} \qquad AR = \frac{b^2}{S} = \frac{(196.68^2)}{5500} = 7.0333$$

$$K = \frac{1}{\pi\,(7.0333)\,0.9} = 0.0503$$

At sea level:

$$C_{D_i} = K\,C_L^2 = 0.0503\,(0.1636^2) = 0.001346$$

and

$$D_{i_{SL}} = C_{D_i}\,1/2\,\rho\,V^2\,S = 0.001346\,(1/2)\,0.002377\,(837.3^2)\,5500 = 6170 \text{ lbs}$$

At 35,000 ft:

$$C_{D_{i_{35}}} = 0.0503\,(0.6945^2) = 0.02426$$

and

$$D_{i_{35}} = 0.02426\,(1/2)\,0.0007382\,(729.85^2)\,5500 = 26{,}234 \text{ lbs}$$

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18. For the aircraft in problems 16 and 17, and at Mach = 0.75 find the following

a) At sea level

i) Parasite drag (lbs)

ii) Induced drag (lbs)

iii) Total drag (lbs)

b) At 35,000 ft

i) Parasite drag (lbs)

ii) Induced drag (lbs)

iii) Total drag (lbs)

a) Sea level

$$D_{0L} = C_{D_{0L}} \, 1/2 \, \rho \, V^2 S = 0.018\,(1/2)\,0.002377\,(837.3^2)\,5500 = 82{,}489 \text{ lbs}$$

$$D_{i_{SL}} = \frac{K W^2}{1/2 \, \rho \, V^2 S} = \frac{0.0503\,(750000^2)}{1/2\,(0.002377)\,837.3^2\,(5500)} = 6174.0 \text{ lbs}$$

$$D_{total} = D_{0L} + D_i = 82489 + 6174 = 88663 \text{ lbs}$$

b) 35000 ft

$$D_{0L} = 0.018\,(1/2)\,0.0007382\,(729.85^2)\,5500 = 19465 \text{ lbs}$$

$$D_i = \frac{0.0503\,(750000^2)}{1/2\,(0.0007382)\,729.85^2\,(5500)} = 26165 \text{ lbs}$$

$$D_{total} = 19465 + 26165 = 45630 \text{ lbs}$$

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19. At sea level, the thrust of a single engine of a 747 is 40,000 lbs. Assuming that thrust varies with altitude proportional to density,

a) Find the thrust available with four engines operating at 35,000 ft

b) Assuming the values of $C_{D_{0L}}$ and K calculated above are constant with Mach number (that is they don't change with airspeed), determine the maximum and minimum speeds at which the aircraft can fly in cruise at 35,000 ft (if it can fly at all)

c) In cruise configuration, (wheels up, flaps retracted) $C_{L_{max}} = 1.5$, What would be the stall speed at 35,000 ft? Compare this value with the thrust-limited minimum cruise speed determined in (b).

a) for one engine we have: $T = T_{SL} \dfrac{\rho}{\rho_{SL}} = 40{,}000 \left(\dfrac{0.0007382}{0.002377} \right) = 12{,}422.4 \text{ lbs}$

Hence

$$T_{total} = 4\,(12.4224.4) = \underline{49{,}670 \text{ lbs}}$$

b)

$$T = D = C_{D_{0L}} \, 1/2 \, \rho \, S V^2 + \frac{K W^2}{1/2 \, \rho \, S V^2}$$

Substituting in the numbers we have:

$$49670 = 0.018\,(1/2)\,0.0007382\,(5500\,V^2 + \frac{0.0503\,(750000^2)}{1/2\,(0.0007382)\,5500\,V^2}$$

$$= 0.03654\,V^2 + \frac{1.39375\,x\,10^{10}}{V^2}$$

We can solve this using a non linear equation solver, or in this case, we can just rearrange it and solve it as a quadratic in V^2.

$$0.03654\,V^4 - 49670\,V^2 + 1.39375\,x\,10^{10} = 0$$

$$V^2 = 395543.2\ \text{ft}^2/\text{sec}^2 \qquad V^2 = 964322.7\ \text{ft}^2/\text{sec}^2$$

$$V_1 = 628.9\ \text{ft/sec}, \qquad V_2 = 982.0\ \text{ft/sec}$$

$$M_{a_1} = \frac{628.9}{973.14} = 0.65 \qquad M_{a_2} = \frac{982.0}{973.14} = 1.009!$$

Note that the assumption that $C_{D_{0L}}$ and K are constant is not a good one since the Mach number is in the transonic range at the higher speed. In general, the drag would be higher than we estimated with constant parameters, and we would be at or beyond the drag rise, giving higher values of drag then we calculated and thus reducing the maximum speed we can achieve (to less than $M_a = 1$

c) $V_{stall} = \sqrt{\dfrac{W}{1/2\,\rho\,S\,C_{L_{max}}}} = \sqrt{\dfrac{750000}{1/2\,(0.0007382)\,5500\,(1.5)}} = 496.18\ \text{ft/sec}$

Therefore, in this case, the minimum airspeed for level flight at 35,000 ft is 628.9 ft/sec, the thrust limited minimum.

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20. The lift curve slope for the 747 aircraft is $\dfrac{d\,C_L}{d\,\alpha} = a = 5.70\ /\text{rad}$. Determine the angle of attack from the zero lift line for the case of flight at Mach = 0.75 at

   a) sea level
   b) 35,000 ft

a) From problem (16),

$$C_L = 0.1636 = a\,\bar{\alpha} = 5.7\,\bar{\alpha} \quad \Rightarrow \quad \bar{\alpha} = 0.0287\ \text{rad} = 1.644\ \text{deg}$$

b) From problem (16),

$$C_L = 0.6945 = a\,\bar{\alpha} = 5.7\,\bar{\alpha} \quad \Rightarrow \quad \bar{\alpha} = 0.1218\ \text{rad} = 6.98\ \text{deg}$$