21-25. A jet powered executive jet has the following properties:

b = 54.4 ft — 4 turbojet engines @ 3300 Lbs thrust @ sea level

S = 542.5 ft — $C_{D_{0L}} = 0.028$

W = 42,000 lb — Oswald efficiency factor = 0.9

$C_{L_{max}} = 1.6$

Assume that the thrust level is constant with changes in airspeed, and is proportional to density.

Calculation of the Level Flight Envelope

a) At sea level

i) Calculate, tabulate, and plot, the drag (lbs) (thrust required), vs. airspeed (ft./sec)

ii) Calculate, tabulate, and plot, the thrust available (lbs) vs. airspeed (ft/sec)

iii) Estimate (from the plots) the maximum and minimum thrust- limited airspeeds at this altitude (or calculate them).

iv) Calculate V_{stall} at this altitude

v) Determine the true minimum airspeed at this altitude

a) We can do the sea level calculations by hand and use then to check our computer calculations. Note that doing a hand calculation to check a computer calculation is always good engineering practice. Even if you can't do them all, you should do some, possibly a simple special case.

Calculating the basics:

$$AR = \frac{b^2}{S} = \frac{54.4^2}{542.5} = 5.4550 \qquad K = \frac{1}{\pi\, AR\, e} = \frac{1}{\pi\,(5.45550)\,0.9} = 0.0648$$

sea level thrust: $T_{SL} = 4\ x\ 3300 = 13{,}200\ \text{lbs}$

Calculate the drag: $D = A V^2 + \dfrac{B}{V^2}$ where $A = C_{D_{0L}}\ 1/2\,\rho\, S$ and $B = \dfrac{k W^2}{1/2\,\rho\, S}$

$$A = 0.028\,(1/2)\,0.002377\,(542.5 = 0.01805 \quad B = \frac{0.0648\,(42000)^2}{1/2\,(0.002377)\,542.5} = 1.77286x10^8$$

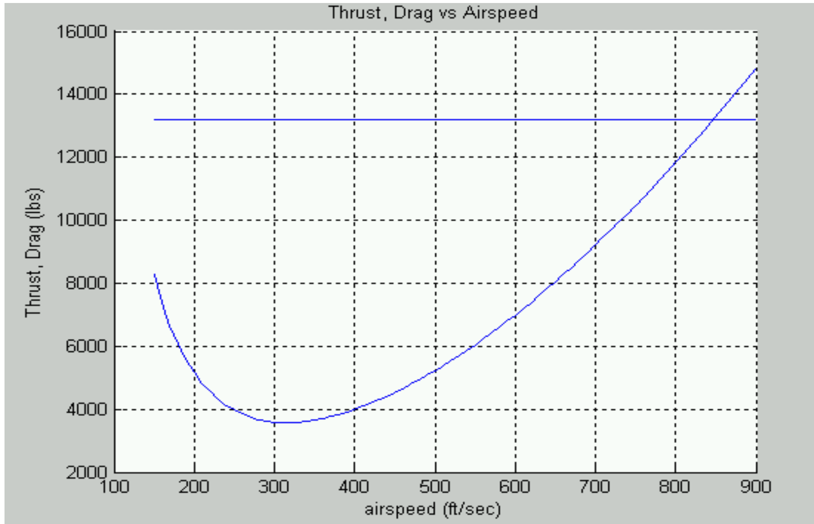
$$D = 0.01805\, V^2 + \frac{1/77289x10^8}{V^2}$$

Evaluate at several airspeeds, e.g. V = 200:

$$D = 0.01805\,(200) + \frac{1.77289x10^8}{200^2} = 5154.2\ \text{lbs}$$

V(ft/sec)	Drag(lbs)
100	17909
200	5154
300	3594
400	3996
500	5222
600	6990
700	9206
800	11839
900	14839

We can graph these values of drag and T = 13,200 lbs to determine the thrust-limited maximum or minimum airspeeds. Alternatively, since the thrust model and the drag model are



simple (constant thrust and constant drag parameters $C_{D_{0L}}$ and K, we can solve for the max and min speed directly:

$$A V^4 - T V^2 + B = 0 = 0.01895 V^4 - 13200 V^2 + 1.77286x10^8$$

$$V^2 = 13687;\ 717615 \quad \Rightarrow \quad V_{\text{min}} = 117.0 \text{ ft/sec}, \qquad V_{\text{max}} = 847 \text{ ft/sec}$$

These numbers agree with the graph

An alternative formulation is obtained from the non-dimensional form of the T = D equation:

$$T = D = (C_{D_{0L}} + K C_L^2)\, 1/2\, \rho V^2 S \qquad \Rightarrow \qquad \frac{T}{1/2\, \rho V^2 S} = C_{D_{0L}} + K C_L^2$$

But

$$V^2 = \frac{W}{1/2\, \rho S C_L}$$. Substitute and rearrange to get $K C_L^2 - \frac{T}{W} + C_{D_{0L}} = 0$

Putting in the numbers, we have:

$$0.0648\, C_K^2 - \frac{13200}{42000} C_L + 0.028 = 0 \qquad \Leftrightarrow \qquad \underline{C_L = 0.09079, \qquad C_L = 4.7593}$$

and

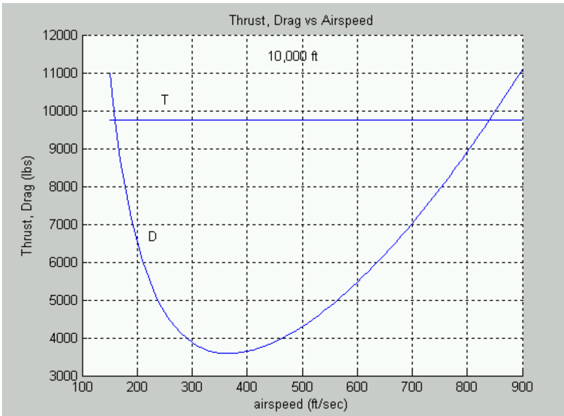
$$V = \sqrt{\frac{W}{1/2\, \rho S C_L}} = \sqrt{\frac{4200}{1/2\,(0.002377)\, 542.5\, (0.9079)}} = 847.04 \text{ ft/sec}$$, and 117.0 ft/sec

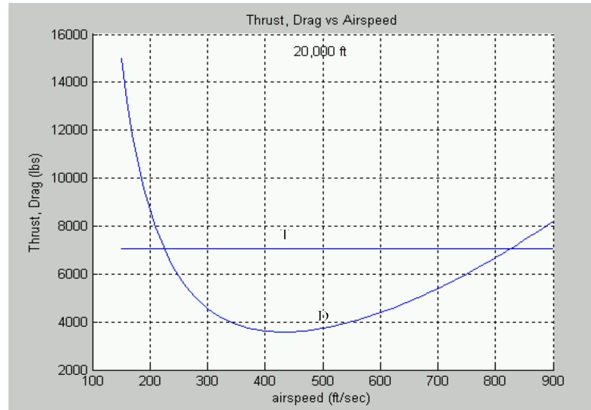
b) Repeat part (a) for 10,000, 20,000, 30,000, and 40,000 ft.

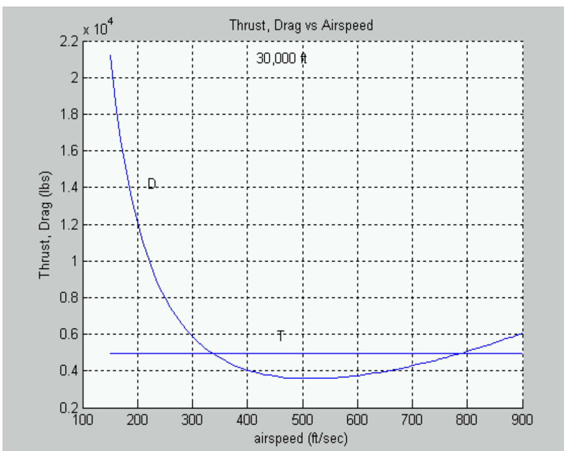
For this part is convenient to develop and MATLAB code. We have some idea of the range of speeds to include. A code that will provide the plots and the tables follows:

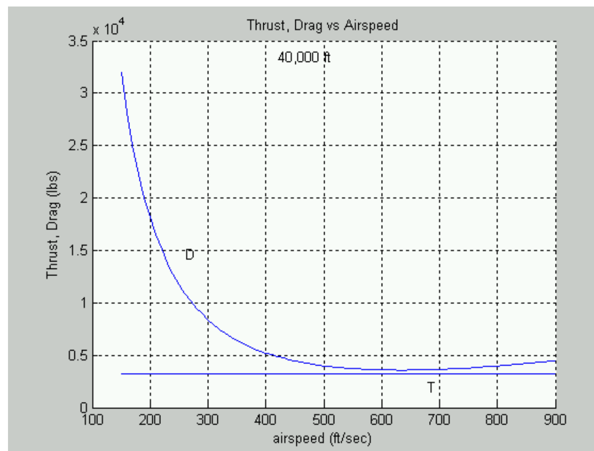
```
%This file computes the thrust and drag curves for problem sheeet 5%
clear all
h = 0:10000:40000;
v=150:10:900;
%
%Enter aircraft properties
w=42000; s=542.5; b=54.4; e0=0.9; cd0l=0.028; tsl=4*3300; clmax=1.6
%
%Enter densities of interest
rho=[0.0023769, 0.0017556, 0.0012673, 0.00089068, 0.00058727];
%
%Preliminary calculations
AR=b^2/s; k=1/(pi*AR*e0);
%
%Calculate thrust and drag - Use indices ii for altitude, and jj for
%airspeed
for ii = 1:length(h);
  for jj = 1:length(v);
      d0(ii,jj)=cd0l*0.5*rho(ii)*v(jj)^2*s;
      di(ii,jj)=k*w^2/(0.5*rho(ii)*v(jj)^2*s);
      dtot(ii,jj)=d0(ii,jj)+di(ii,jj);
      t(ii,jj)=tsl*rho(ii)/rho(1);
  end
  vstall(ii)=sqrt(w/(0.5*rho(ii)*s*clmax));
end
for k=1:length(h);
   figure(k);hold;
   plot(v,dtot(k,:));
   plot(v,t(k,:));
   xlabel('airspeed (ft/sec)');
   ylabel('Thrust, Drag (lbs)')
   title('Thrust, Drag vs Airspeed');
   grid on; hold;
end
fid=fopen('C:\Matlab6p5/work/2104/p503.txt','wt');
for m=1:5
   fprintf(fid,'\n');
   fprintf(fid,'altitude=%8.00f thousand ft,\n', h(m)/1000);
   fprintf(fid,' Airspeed     Thrust      Drag     \n');
   for n= 1:5:length(v)
       fprintf(fid,'%8.0f     %8.4f     %8.4f\n',v(n),t(m,n),dtot(m,n));
   end
end
fclose(fid);
```

The output of this code is the graph above for sea level plus an additional graph at 10, 20, 30, and 40K ft:









Here we see that the thrust at 40,000 ft is not sufficient to equal the drag an any speed and hence there is no solution. Consequently, the “ceiling” of the aircraft is less than 40,000 ft. The tables that accompany these graphs and the one at sea level follow:

altitude= 0 thousand ft,

Airspeed	Thrust	Drag
150	13200.0000	8290.1526
200	13200.0000	5156.8354
250	13200.0000	3966.5139
300	13200.0000	3595.7225
350	13200.0000	3659.5142
400	13200.0000	3997.0922
450	13200.0000	4531.6392
500	13200.0000	5222.6962
550	13200.0000	6047.3090
600	13200.0000	6991.6681
650	13200.0000	8047.0611
700	13200.0000	9207.7712
750	13200.0000	10469.9213
800	13200.0000	11830.8063
850	13200.0000	13288.4929
900	13200.0000	14841.5691

altitude= 10 thousand ft,

Airspeed	Thrust	Drag
150	9749.6403	10974.0864
200	9749.6403	6537.5192
250	9749.6403	4676.0289
300	9749.6403	3868.5595
350	9749.6403	3593.9329
400	9749.6403	3634.4471
450	9749.6403	3886.0993
500	9749.6403	4294.1124
550	9749.6403	4827.4086
600	9749.6403	5467.2913
650	9749.6403	6201.9648
700	9749.6403	7023.6893
750	9749.6403	7927.2154
800	9749.6403	8908.8810
850	9749.6403	9966.0682
900	9749.6403	11096.8655

altitude= 20 thousand ft,

Airspeed	Thrust	Drag
150	7037.8897	15003.4421
200	7037.8897	8702.6237
250	7037.8897	5924.8470
300	7037.8897	4562.9820
350	7037.8897	3895.0370
400	7037.8897	3619.4275
450	7037.8897	3592.0778
500	7037.8897	3737.1047
550	7037.8897	4011.4562
600	7037.8897	4389.2314
650	7037.8897	4854.0899
700	7037.8897	5395.3095
750	7037.8897	6005.6183
800	7037.8897	6679.9430
850	7037.8897	7414.6571
900	7037.8897	8207.1128

altitude= 30 thousand ft,

Airspeed	Thrust	Drag
150	4946.3486	21191.6461
200	4946.3486	12105.2736
250	4946.3486	7996.9931
300	4946.3486	5868.6843
350	4946.3486	4693.0645
400	4946.3486	4041.0256
450	4946.3486	3707.5703
500	4946.3486	3584.7283
550	4946.3486	3611.2432
600	4946.3486	3750.2623
650	4946.3486	3978.5355
700	4946.3486	4280.8069
750	4946.3486	4646.7296
800	4946.3486	5069.0852
850	4946.3486	5542.7138
900	4946.3486	6063.8477

altitude= 40 thousand ft,

Airspeed	Thrust	Drag
150	3261.3757	32009.7147
200	3261.3757	18127.4263
250	3261.3757	11766.1385
300	3261.3757	8378.7678
350	3261.3757	6407.2911
400	3261.3757	5200.9039
450	3261.3757	4448.6981
500	3261.3757	3986.9211
550	3261.3757	3722.6688
600	3261.3757	3600.0485
650	3261.3757	3583.7983
700	3261.3757	3650.7803
750	3261.3757	3785.3019
800	3261.3757	3976.4154
850	3261.3757	4216.2951
900	3261.3757	4499.2267

c) Plot on a graph with airspeed as the “x” axis and altitude as the “y” axis

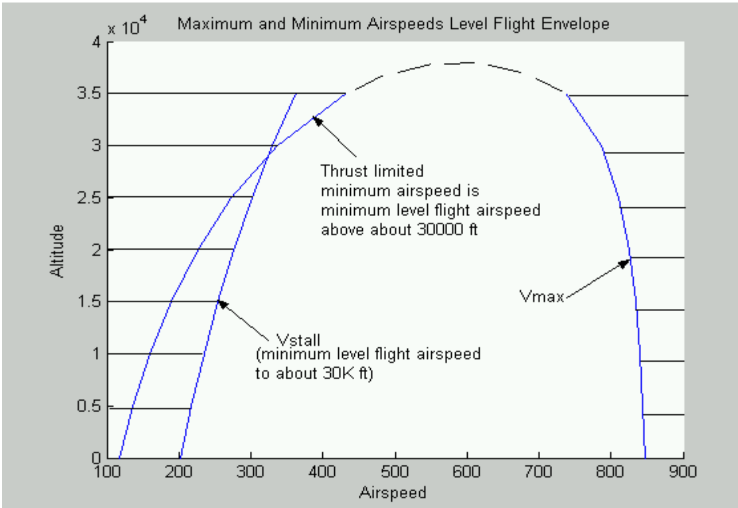
i) altitude vs maximum airspeed

ii) altitude vs ***the greater*** of (stall speed, thrust-limited minimum speed)

Here we can write another MATLAB code where we can either 1) input the values that we read off the graphs for min and max speeds and plot with altitude (the general case), or 2) calculate the min and max airspeeds as we did for part (a) (for this simple model). Here we did the latter. However reading the values from the graphs and entering them is the usual procedure.

```
%This file computes the "exact thrust limited vmax and vmin using a
%quadratic formula in the lift coefficient.
clear all
h = 0:5000:40000;
%
%Enter aircraft properties
w=42000; s=542.5; b=54.4; e0=0.9; cd0l=0.028; tsl=4*3300; clmax=1.6;
%
%Enter densities of interest
rho=[0.0023769, 0.0020482, 0.0017556, 0.0014962, 0.0012673, ...
    0.0010663, 0.00089068, 0.0007382, 0.00058727];
%
%Preliminary calculations
AR=b^2/s; k=1/(pi*AR*e0);
%
% calculate the lift coefficients for high and low speed T=D flight from
% the equation: KCL^2 - T/W CL + CD0L = 0
for ii=1:length(h);
  t(ii)=tsl*rho(ii)/rho(1);
  cl1=(t(ii)/w+sqrt((t(ii)/w)^2-4*k*cd0l))/(2*k);
  cl2=(t(ii)/w-sqrt((t(ii)/w)^2-4*k*cd0l))/(2*k);
  v1(ii)= sqrt(w/(0.5*rho(ii)*s*cl1));
  v2(ii)= sqrt(w/(0.5*rho(ii)*s*cl2));
  vstall(ii)=sqrt(w/(0.5*rho(ii)*s*clmax));
end
fid=fopen('C:\Matlab6p5/work/2104/p503b.txt','wt');
fprintf(fid,' Altitude      Vstall      Vmin        Vmax  \n');
%We found that the altitude of 40K ft has no solution, so wewill eliminate
%it.
  for m= 1:length(h)-1
    hr(m) = h(m); v1r(m) = v1(m); v2r(m)=v2(m); vstallr(m)=vstall(m)
    fprintf(fid,'%8.0f     %8.4f     %8.4f     %8.4f \n',...
      hr(m),vstallr(m),v1r(m),v2r(m));
  end
fclose(fid);
figure(6); hold;
plot(v1r,hr); plot(v2r,hr); plot(vstallr, hr);
xlabel('Airspeed');
ylabel('Altitude');
Title('Maximum and Minimum Airspeeds Level Flight Envelope');
axis([100, 900, 0, 40000]);
hold
```

The resulting graph is:



Here the hatched area indicates the area outside of the flight envelope of this aircraft.

d) Estimate the ceiling of this aircraft (You do this by "eyeballing" and smoothly joining the max and min airspeed curves, or by looking at your tables and estimating at what altitude the single solution (not two as for lower altitudes) T = D.

From this graph, we could estimate the ceiling at 37000 ft. This is not a good estimate, just an estimate. Can you think of a way to obtain the ceiling exactly (most likely by iteration)?

The table that goes with the above graph is:

Altitude	Vstall	Vmin	Vmax
0	201.7782	117.0260	847.0557
5000	217.3670	136.2703	844.1735
10000	234.7832	159.7635	840.0441
15000	254.3226	188.8223	833.9931
20000	276.3377	225.3955	824.8608
25000	301.2590	272.6386	810.4731
30000	329.6241	336.5154	786.1017
35000	362.0702	432.7870	737.4916