AOE 2104 Introduction to Aerospace Engineering Problem Sheet 6 (ans)

The following problems (unless otherwise indicated) are concerned with an aircraft that has the following properties:

weight W = 17,578 lbs   Area S = 260 ft^2 Span b = 27.5 ft   mean aero chord $\bar{c}$ = 10.8 ft

The vehicle is flying at Mach 0.8 at 35,000 ft (density = 0.0007382 slugs/ft^3 , speed of sound - 973.14 ft/sec)

26. a) Find the lift coefficient
b) Assuming that at zero angle of attack, the lift coefficient is zero, find the angle of attack for flight at this condition if $\frac{dC_L}{d\alpha} = 4.0 \ \text{/rad}$ (express answer in degrees)
c) Sketch the lift coefficient vs angle of attack for this aircraft.

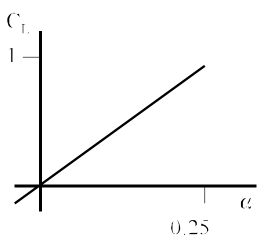
a) Determine the airspeed: $V = M_a a = 0.8\,(973.14) = 778.512 \ \text{ft/sec}$

Then the lift coefficient is given by: $C_L = \frac{W}{1/2\,\rho\,V^2\,S} = \frac{17578}{1/2\,(0.0007382)\,778.512^2\,(260} = 0.3022$

b) Using the lift coefficient math model we have:

$$C_L = \frac{dC_L}{d\alpha}\bar{\alpha} = 0.3022 = 4.0\,\bar{\alpha} \quad \Leftrightarrow \quad \bar{\alpha} = 0.0756 \ \text{rad} = 4.33 \ \text{deg}$$

c)



The key feature to this graph is that the lift curve goes through the origin of the graph, i.e. when the angle of attack is zero, the lift coefficient is zero (definition of $\bar{\alpha}$ and of α for this problem.

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27. Under these flight conditions (altitude and speed), find the value of the pitch-moment coefficient if the lift were zero, that is find $C_{m_{0L}}$. Note that under these flight conditions:

$$\frac{dC_m}{d\alpha} = -0.39 \ \text{/rad}$$

We require that the pitch moment be zero:

$$C_m = 0 = C_{m_{0L}} + \frac{dC_m}{d\alpha}\bar{\alpha} = C_{m_{0L}} + (-0.39)\,(0.0756) \quad \Rightarrow \quad \underline{C_{m_{0L}} = 0.0295}$$

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28. If the elevator is deflected in a negative direction (trailing edge up), in such a manner that the zero lift intercept increased by 0.01 (i.e. $C_{m_{0L}}$ was increased by 0.01):

a) What would be the new angle of attack (in deg)?
b) What would be the new equilibrium flight speed (assuming thrust adjusted so T = D)?

a) We deflect the elevator in the negative direction to get $\Delta C_m = \frac{dC_m}{d\delta_e}\delta_e = 0.01$

then:

$$C_{m_{0L_{new}}} = C_{m_{0L}} + \Delta C_m = 0.0295 + 0.01 = 0.0395$$

and

$$C_m = 0 = C_{m_{0L_{new}}} + \frac{dC_m}{d\alpha}\bar{\alpha} = 0.0395 + (-0.39)\bar{\alpha} \quad \Rightarrow \quad \underline{\bar{\alpha} = 0.1013 \text{ rad} = 5.80 \text{ deg}}$$

b) $C_L = a\bar{\alpha} = 4.0\,(0.1013) = 0.4052$

$$V = \sqrt{\frac{W}{1/2\,\rho\, S\, C_L}} = \sqrt{\frac{17578}{1/2\,(0.0007382)\,260\,(0.4052)}} = 672.3 \text{ ft/sec}$$

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29. We can fly at stall speed, $C_{L_{\max}} = 1.5$, in two different ways:

a) We can adjust $C_{m_{0L}}$ to maintain balance flight ($C_m = 0$), Find the value of $C_{m_{0L}}$ required to fly at stall speed.
b) We can move the center of gravity. Under this scenario, the value of $C_{m_{0L}}$ remains the same as its original value (calculated in problem 27), but the slope of the pitch curve, $\frac{dC_m}{d\alpha}$ is changed. Determine the new slope (longitudinal stability parameter) required so that the aircraft is balanced at stall speed (in units - /rad)
c) Is the aircraft more stable (more negative slope) or less stable then in the cruise condition of problem 26 if we use the CG movement method to balance the aircraft at stall?

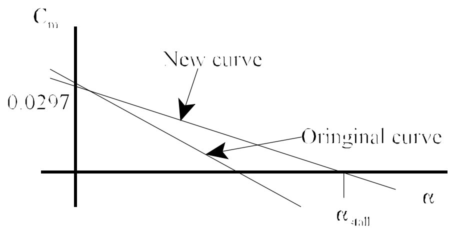
a) The stall angle of attack is given by:

$$C_{L_{\max}} = a\,\alpha_{stall} = 1.5 = 4.0\,\alpha_{stall} \quad \Rightarrow \quad \alpha_{stall} = 0.375 \text{ rad} = 21.486 \text{ deg}$$

$$C_m = 0 = C_{m_{0L_{new}}} + (-0.39)(0.375) \quad \Rightarrow \quad \underline{C_{m_{0L_{new}}} = 0.1462}$$

b) $C_{m_{0L}} = C_{m_{0L}} + C_{m_\alpha}\bar{\alpha} = 0.0295 + C_{m_\alpha}(0.375) \quad \Rightarrow \quad \frac{dC_m}{d\alpha} = C_{m_\alpha} = -0.0787 \text{ /rad}$

c) -0.0787 > -0.39 New value is less negative then original, hence less stable



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30. You have looked at performance and plotted thrust and drag vs airspeed. The solution to the equation T = D occurs where the two lines intersect. We also know that Newton's law apply,

$\vec{F} = m\dfrac{d\vec{V}}{dt}$ in general an that $\vec{F}$= 0 at those points. We will assume that thrust is independent of airspeed (T = const). Describe how drag must vary with airspeed to insure that the aircraft is ***statically stable*** with respect to velocity. Then indicate the stability status of the two reference flight conditions - thrust limited maximum airspeed, and the thrust limited minimum airspeed.

(You folks are on your own on this one - I will only answer questions on clarity of the problem, not how to do it)

We can approach this problem by using the definition of static stability. First we find the reference flight condition. In this case it is:

RFC T = D

Then we look at the case where there is a disturbance (in this case , in airspeed) where the vehicle is no longer in equilibrium:

T - D = m dV/dt

If we increase airspeed, we want the unbalanced force caused by the increase to act in a direction to decrease it (that is we want dV/dt to be negative. Consequently we require:

$$\left.\frac{\Delta(T-D)}{\Delta V}\right|_{ref} \Rightarrow \left.\frac{d(T-D)}{dV}\right|_{ref} < 0$$

This is a general requirement for "speed" stability. However for our problem, the thrust is independent of speed, and hence dT/dV = 0, and the above requirement becomes:

$$\left.\frac{d(T-D)}{dV}\right|_{ref} = \left.\frac{d(-D)}{dV}\right|_{ref} < 0 \quad \text{or} \quad \left.\frac{dD}{dV}\right|_{ref} > 0$$

The two equilibrium points are at 1 and 2 in the figure where T = D. At point 1 we see that the drag curve has a negative slope or that $\left.\dfrac{dD}{dV}\right|_1 < 0$ and hence is ***statically unstable***. On the other hand at point 2, the slope is positive, $\left.\dfrac{dD}{dV}\right|_2 > 0$ and hence it is ***statically stable***.