AOE 2104 Introduction to Aerospace Engineering Problem Sheet 7 (ans)

31-32 The X-Prize is an incentive award for the first privately supported organization to put an astronaut in space, return him/her to Earth safely and then repeat the mission within 14 days. We have discussed in class the mission profile for Scaled Composites' entry for this competition. You are to pick any one of the other entries and to describe its mission profile and any other facts that you can find out about the organization and its design. Your report should indicate the name of the organization, an over view of vehicle of vehicles involved, and a detailed mission profile.

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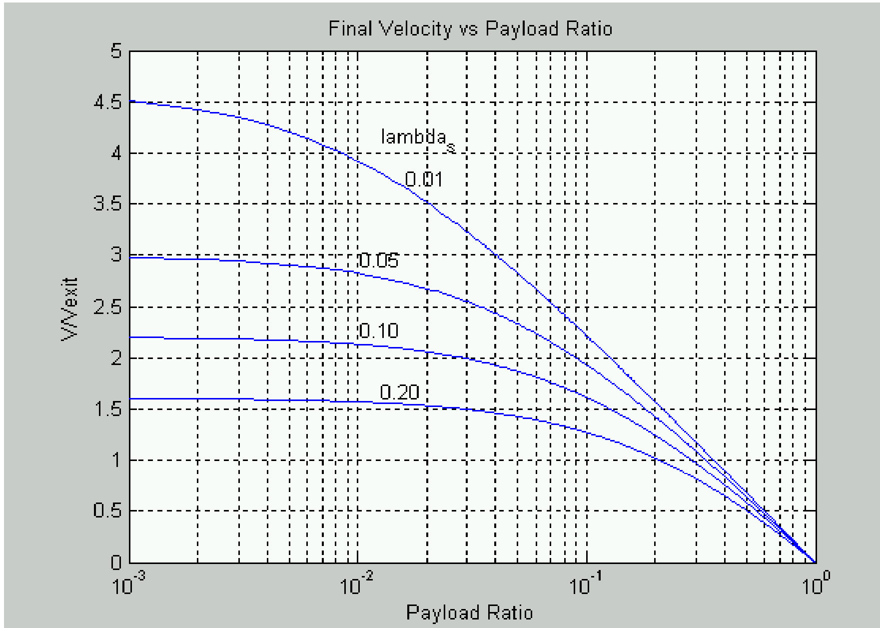
33. Make a plot of final velocity over exhaust velocity vs payload fraction for various structural ratios. The range of structural ratios should include 0.2, 0.1, 0.05, 0.01, and the payload ratios should include the range from 0.001 to 1. It would help to plot the payload ratios on a log scale.

$$\frac{V}{V_e} = \ln\frac{M_1}{m_2} = \ln\frac{1}{\lambda_s + (1 - \lambda_s)\lambda_p}$$

	$\lambda_s = 0.2$	$\lambda_s = 0.1$	$\lambda_s = 0.05$	$\lambda_s = 0.01$
λ_p	V/V_e			
0.001	1.605	2.294	2.977	4.511
0.01	1.570	2.216	2.822	3.917
0.10	1.273	1.660	1.931	2.216
0.20	0	0	0	0

A Matlab code to make the required plots:

```
% determine the non-dimensional final velocity in terms of the payload
% ratio, with structural ratio as a parameter
clear all
lambdas = [0.2, 0.11, 0.05, 0.01];
hold
for ii = 1:4
   for jj= 1:1000
      lambdap(jj)=jj*0.001;
      vr(jj)=log(1/(lambdas(ii)+(1-lambdas(ii))*lambdap(jj)));
   end
   semilogx(lambdap,vr)
end
xlabel('Payload Ratio')
ylabel('V/Vexit')
title('Final Velocity vs Payload Ratio')
grid on
```



34. If we neglect the rotation of the Earth, and if we neglect the drag due to the atmosphere, we can make the following calculations:

A. Calculate the specific energy (Energy per unit mass) at the surface of the Earth
i) kinetic
ii) potential

B. Calculate the specific energy in a circular orbit of 400 km above the Earth' surface. At that altitude the velocity in a circular orbit is 7.6685 km/s
i) kinetic
ii) potential

C. Determine the change in total energy - the energy required to be supplied by the engine in Newton meters per unit mass or Joules per unit mass.

A. i) $U = -\frac{\mu}{r} = -\frac{\mu}{R_e} = -\frac{3.9860x10^5}{6378.1363} = -62.4947 \ \frac{\text{km}^2}{\text{s}^2}$

ii) $T = \frac{1}{2}V^2 = 0.0 \ \frac{\text{km}^2}{\text{s}^2}$

B. i) $U = -\frac{\mu}{r} = -\frac{3.9860x10^5}{6378.1363+400} = -58.8067 \ \frac{\text{km}^2}{\text{s}^2}$

ii) $T = \frac{1}{2}V^2 = \frac{1}{2}(7.6685^2) = 29.4029 \ \frac{\text{km}^2}{\text{s}^2}$

C.

$$\Delta En = En_2 - En_1 = T_2 + U_2 - (T_1 + U_1)$$
$$= 29.4029 - 58.8067 - (0 - 62.49470$$
$$= 33.0909 \ \frac{\text{km}^2}{\text{s}^2} = 33.0909x10^6 \ \frac{\text{m}^2}{\text{s}^2} = 33.0909x10^6 \ \frac{\text{Joules}}{\text{kg}}$$

Note that: $\Delta T = 29.4029 \ \frac{\text{km}^2}{\text{s}^2}$ and $\Delta U = 3.6880 \ \frac{\text{km}^2}{\text{s}^2}$

and that the change in kinetic energy is almost eight times the change in potential energy.

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35. Assuming we used a single stage liquid rocket with an I_{sp} of 400 s, and that we had a structural mass ratio of 0.10:

A. What would be the maximum velocity achieved (if the payload fraction were 0)?

B. In light of your answer in A, what would be the maximum payload mass ratio that we could put in the orbit of problem (34) (both with and without gravity effects). If necessary, assume we want to put a 2000 kg mass in orbit and determine the gross lift-off weight (GLOW).

A. $V_{\text{max}} = V_e \ln \frac{1}{\lambda_s} = g_0 I_{sp} \ln \frac{1}{\lambda_s} = 9.807\,(400) \ln \frac{1}{0.01} = 3922.8 \ln 10 = 9032.6 \ \text{m/s}$

or 9.0326 km/s

Assume no gravity or atmosphere:

B_1.

$$V = V_e \ln \frac{1}{\lambda_s + (1 - \lambda_s)\lambda_p}$$
$$7.6685 = 3.9228 \ln \frac{1}{0.1 + (1 - 0.1)\lambda_p}$$

Or

$$1.9548 = \ln \frac{1}{0.1 + (1 - 0.1)\lambda_p}$$
$$7.0629 = \frac{1}{0.1 + 0.9\lambda_p}$$
$$0.1 + 0.9\lambda_p = 0.1416$$
$$\lambda_p = \underline{0.0462}$$

For a 2000 kg payload, we have $\textbf{GLOW} = \frac{2000}{0.0462} = 43{,}285 \ \text{kg}$

or

19,614 N = 4409 lbs payload requires a gross liftoff weight of 424,492 N = 95,430 lbs

B_2 Assume gravity, but no atmosphere:

From the previous problem, we found that the energy necessary to overcome gravity was 3.6880 km^2/s^2. We can account for this by adding that energy to the kinetic energy and coming up with an equivalent kinetic energy and hence equivalent velocity that we would have achieved if the atmosphere weren't there and treating the problem as we did before in B_1.

$$T_{effective} = T + \Delta U = \frac{1}{2}(7.6685^2) + 3.6880 = 33.0909 \ \frac{\mathrm{km}^2}{\mathrm{s}^2}$$

$$T_{effective} = \frac{1}{2} V^2_{effective} = 33.0909 \qquad \Rightarrow \qquad V_{effective} = 8.1352 \ \mathrm{km/s}$$

Then, following our previous procedure, we have

$$V_{effective} = V_e \ln \frac{1}{\lambda_s + (1 - \lambda_s)\lambda_p}$$

$$8.1352 = 3.9228 \ln \frac{1}{0.1 + 0.9\lambda_p}$$

$$0.1 + 0.9\lambda_p = 0.125733$$

$$\lambda_p = 0.0286$$

For a payload of 2000 kg we have the following numbers:

GLOW~=~70,030 kg $\Rightarrow$ 686,785 N = 154,396 lbs (Over 60 % more weight to take care of gravity. How much additional fuel would be required to take care of atmospheric drag?)