Vehicle Performance

1. Introduction

Vehicle performance is the study of the motion of a vehicle. The motion of any vehicle depends upon all the forces and moments that act upon it. These forces and moments, for the most part are caused by interaction of the vehicle with the surrounding medium(s) such as air or water (e.g. fluid static and dynamic forces), gravitational attraction (gravity forces), Earth's surface (support, ground, or landing gear forces), and on-board energy consuming devices such as rocket, turbojet, piston engine and propellers (propulsion forces). Consequently, in order to fully understand the performance problem, it is necessary to study and in some way characterize these interacting forces. Although these four categories of forces are the dominating ones acting on the vehicles of our interest, it should be pointed our that other forces can enter into the performance considerations with varying degrees of participation (e.g. magnetic, electrostatic). These types of forces will be neglected for the present studies.

For performance studies vehicles are usually assumed to behave as rigid bodies, that is structural deflections are generally ignored. In most cases this is a good assumption and it simplifies analysis considerably. Under this assumption, a theorem from the dynamics of rigid bodies states that the motion of a rigid body can be separated into the motion of the center-of-mass (translational motion), and the motion about the center-of-mass (rotational or attitude motion). Further it can be shown that the center-of-mass motion or translational motion is caused only by the forces that act on the vehicle while the rotational motion is caused only by the moments about the center-of-of mass that act on the vehicle.

For detailed studies of vehicle motion, both rotational and translational motions must be considered simultaneously since these motions cause both forces and moments to act on the vehicle (e.g. aerodynamic forces and moments). Hence there is a coupling between the two motions through the forces and moments that they generate (translational motion can cause moments as well as forces and rotational motion can cause forces as well as moments). However, there is a large class of problems that may be considered under the assumption that these two types of motion can be separated. That is we can look at the force - center-of-mass motion, and the moment - rotational motion independent of each other. Studies concerning center-of-mass motion are called trajectory and/or performance analysis while those concerning rotational or attitude motions are called static stability and control analysis. Assumptions allowing such a separation are related to the time scales of the respective motions, and will not be discussed here. The remainder of this study is concerned with trajectory analysis or performance.

Governing Equations

In performance analysis it is assumed that the moments about the center-of-mass (cm) are identically equal to zero and that any desired attitude can be achieved instantaneously. Alternatively, we can say that the vehicle has no moment of inertia and consequently can be treated as a ***point mass***, with all the mass located at the cm. As a result, the equations governing the motion of the vehicle are given by Newton's second law:

$$\vec{F} = m\,\vec{a}_{cm} = m\,\ddot{\vec{r}}_{cm} \tag{1}$$

where $\vec{F}$ = all applied forces - fluid dynamic, gravity, ground, propulsion
m = total mass of vehicle
$\vec{a}_{cm}$ = acceleration of the center-of-mass (cm)

Equation (1) applies to all vehicles, be they automobiles, aircraft, helicopters, rockets, satellites, ships at sea, and submarines. If we could integrate it with respect to time, then we could determine the velocity in time, and if integrated again, the position in time, of the vehicle. These are the principle ingredients that characterize the performance of the vehicle. Typical problems that may be of interest are: how long does it take to reach a given speed, or a given distance, or a given altitude from a specified starting point, or what is the time and distance for take-off? These types of problems are generally called ***unsteady or accelerating performance*** problems and are the most difficult to solve. It turns out however that a large portion of vehicle motion is ***non-accelerating or steady.*** For example such problems include cruising flight. In many other cases the acceleration is not zero, but it is small and can be neglected. Such motion is called ***quasi-steady motion.*** Consequently, a large portion of the vehicle performance can be analyzed using the steady or quasi-steady assumption, that is that the acceleration is negligible. Under this assumption, the governing equations for performance are:

$$\vec{F} = 0 \tag{2}$$

Equation (2) is in the form of an algebraic equation, instead of the form of an ordinary differential equation such as Eq. (1). As a result, quasi-steady performance can be investigated using algebraic concepts. Although deceptively simple in appearance, Eq. (2) is the subject of a major portion of this study.

In order to fully appreciate Eqs. (1) and (2) it is necessary to characterize the forces that are acting on the vehicle. Furthermore, several coordinate systems must be introduced to help in the description of the forces. For the problems of interest to us, we will assume a flat Earth, with a constant gravity field. Aerodynamic, propulsion, and gravity forces are important in so-called up-and-away flight, while aerodynamic, propulsion, gravity, and ground forces are important for takeoff and landing. To help with the description of these forces we will need to introduce several coordinate systems.

Coordinate Systems

We will introduce three coordinate systems: the **inertial system** in which Newton’s laws hold, the **local horizontal** system that is parallel to the inertial system, the **body-fixed** system that is fixed in the vehicle, and the **wind axes** system that has some specified alignment with the wind.

Inertial Coordinate System (x^I, y^I, z^I)

It is assumed that the Earth's surface serves as an inertial reference point. Consequently the inertial coordinate system is fixed in the Earth with the origin at (or near) sea level. The x^I and the y^I axes form the horizontal plane, and the z^I axis is directed or points ***down***. Generally the x^I axis points in some reference direction (North, down the runway, in the desired direction of motion, or in the initial direction of motion, etc.) Under these assumptions, these axes are also called ***ground*** axes.

Local Horizontal Axes System (x^h, y^h, z^h)

The local horizontal system is parallel to the ground axes system with the origin located at the center-of-mass of the vehicle and moving with it.

Body-Fixed (Body) Axes System (x^b, y^b, z^b)

The body-fixed axes system is fixed in the vehicle as it translates and rotates. The origin is usually located at the center-of-mass, with the x^b axis directed forward, the z^b axis directed down relative to the vehicle, and the y^b axis completing the right hand set, being directed out the right hand side of the vehicle. Generally, the x^b-z^b plane is a plane of symmetry of the vehicle.

Wind Axes System (x^w, y^w, z^w)

The wind axes system is such that its origin is located at the center-of-mass of the vehicle, the x^w axis aligns itself with (or is parallel to) the relative wind, the z^w axis lies in the plane-of-symmetry of the vehicle, and is directed downward relative to the vehicle, and the y^w axis completes the right hand set.

In performance calculations, the problem is usually formulated in the wind axes since aerodynamic forces are most conveniently represented in this axis system. However when dealing with moments, generally the body-fixed axes system is used. In most simulations, the problem is formulated using both body-fixed and wind axes systems.

Forces

In this study we will only consider four types of forces: 1) Fluid static and dynamic forces, 2) forces due to gravity, 3) propulsion forces, and 4) ground or support forces. In this brief introduction we will simply define the nomenclature and the direction of the forces.

Fluid Static and Dynamic Forces

Fluid static forces occur because of gravity. Any fluid in a gravitational field has a vertical pressure distribution, the pressure increasing as you move downward. Consequently any vehicle immersed in such a fluid feels more pressure on the bottom than on the top, causing an upward (opposite gravity) or buoyancy force. It can be shown that this buoyant force is equal to the weight of the fluid displaced and is directed opposite the local gravitational force.

The ***fluid dynamic*** forces are caused by the motion of the vehicle through a fluid medium. These forces are caused the motion of the vehicle through a fluid medium. These forces are caused by lifting surfaces, fuselage shapes, and other parts of the vehicle. It is generally convenient to represent the aerodynamic forces with components in the wind axes system. These components have the following nomenclature and definitions:

Lift - The force perpendicular to the relative wind and directed upward in the vehicle plane of symmetry is called the lift, and is usually designated with an L.

Drag - The force parallel to the relative wind and directed in the same direction as the relative wind is called the drag, and is usually designated with a D.

Side-force - The force perpendicular to the relative wind and directed along the y^w axis is called the side-force and is designated with a Y^w. (Sometimes the negative of this force is used and is designated with a Q or some other letter, hence $Q = -Y^w$)

Hence the aerodynamic force vector can be written as:

$$\vec{F}_A^{\,w} = \begin{Bmatrix} -D \\ Y^w \\ -L \end{Bmatrix}^w = \begin{Bmatrix} -D \\ -Q \\ -L \end{Bmatrix}^w \tag{3}$$

Gravity Forces

The gravity force is equal to the mass times the acceleration due to gravity and is directed downward in the local horizontal coordinate system. Hence we can write:

$$\vec{F}_g = \vec{W} = m\,g\,\hat{k}^h \tag{4}$$

Propulsion Forces

Propulsion forces are generated by on-board energy consuming devices and are usually designated as thrust and represented by the symbol $\vec{T}$. The thrust forces can be directed in virtually any direction but is usually forward. Usually we need to know the angle between the thrust vector and the relative wind. The thrust vector can be represented in either the body or wind axes systems. For the wind axes system (assuming no side thrust) we can write the thrust vector as:

$$\vec{T}^w = \begin{Bmatrix} T_x \\ 0 \\ T_z \end{Bmatrix}^w = \begin{Bmatrix} T\cos\alpha_T \\ 0 \\ -T\sin\alpha_T \end{Bmatrix}^w \tag{5}$$

Ground Forces

Ground forces will be considered to occur when there is physical interaction with the ground such as a tire or landing gear. The force can have three components. The normal force acts perpendicular to the surface, the rolling friction force acts tangent to the surface, opposite the direction of motion, and the cornering friction force acts perpendicular to the direction of motion and parallel to the surface. If we assume the direction of motion is along the x^w axis of the wind coordinate system (the assumption being that the when moving along the ground, the relative wind is parallel to the ground), the ground force vector can be written as:

$$\vec{F}_f^{\,w} = \begin{Bmatrix} -f_r \\ f_c \\ -N \end{Bmatrix}^w \tag{6}$$

Equations of Motion

Before we write down the governing equations for performance considerations, a few assumptions, definitions and observations can be made. Since most vehicles have a plane of symmetry, it is likely that, unless turning, the attitude of the vehicle would be such that the nose is pointed into the relative wind. Otherwise the projection of the fuselage perpendicular to the wind would cause a large drag force. Consequently, vehicle motions can be divided into two groups, ***symmetric motions***, and ***asymmetric motions***.

Symmetric motions are motions that take place in the plane of symmetry of the vehicle. Such motions include motion in a vertical plane. Such motions include (but are not limited to) vehicles in cruise, climbing flight, and take-off and landing. Performance characteristics that can be studied are maximum and minimum speeds, rate of climb, take-off time and distance etc.

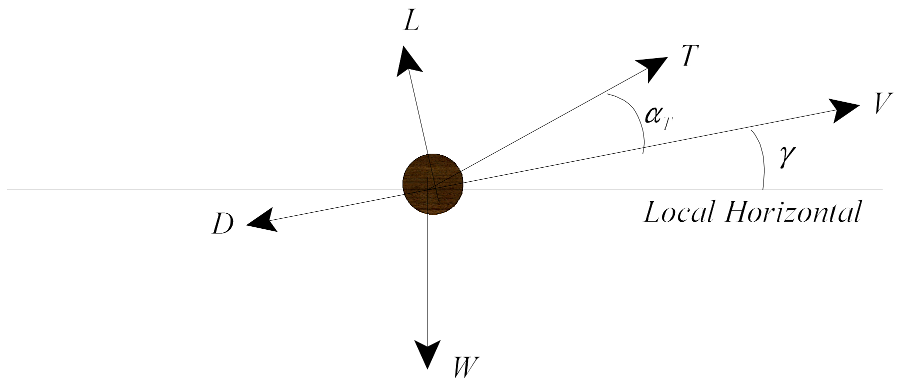
Asymmetric motions are motions that cause the vehicle plane of symmetry to change its angular position, or to cause it to move laterally. Such motions include turning flight, rolling flight, and so-called side-step maneuvers. Performance characteristics that can be studied for this type of flight include radius and rate of turn, and time to roll to a given angle.

For the present, we will consider motion in the vertical plane and take up turning performance at a later time. For in-plane motion, we can define two additional angles:

Definition: Flight Path Angle - (γ) The flight path angle is the angle between the vehicle velocity vector and the local horizontal plane. It is defined as positive for a positive climb rate.

Definition: Thrust angle-of-attack - (α_T) The thrust angle-of-attack is the angle between the thrust vector and the relative wind or velocity vector. In many cases, this angle is small and for convenience (if nothing else) it is assumed zero. (Note 1, this definition is valid only in symmetric flight. Note 2, The assumption that this angle is zero reduces the difficulty of the problems by a tremendous amount, so often the assumption is made even when it is not a good one!)

Remember that we are looking only at the motion in a vertical plane. We also assumed that the vehicle could be represented by a point mass. So that the resulting ***Free-Body-Diagram*** for up-and-away flight appears as:



The equations of motion can be established from Newton's second law applied along the velocity vector and perpendicular to the velocity vector:

$$\begin{aligned} T\cos\alpha_T - D - W\sin\gamma &= m\dot{V} \\ T\sin\alpha_T + L - W\cos\gamma &= m\frac{V^2}{r} = mV\dot{\gamma} \end{aligned} \tag{7}$$

In addition we can present two so-called kinematic equations that resolve the velocity vector into vertical and horizontal components:

$$\begin{aligned} V\sin\gamma &= \dot{h}_G \\ V\cos\gamma &= \dot{x}^g = \dot{R} \end{aligned} \tag{8}$$

Equations (7) are called the ***<u>differential equations of motion</u>*** for a flight vehicle moving in a vertical plane. Equations (8) are called the kinematic equations or the ***<u>trajectory equations</u>***.

The new terms that appear in these equations are:

r = The instantaneous radius of the arc along which the vehicle is flying
h_G = Geometric altitude measured from sea-level
R = Range measured along the (horizontal) ground

Note that the above equations are valid for the case where there is no motion of the air with respect to the ground (***no wind blowing)***!

If we now consider the vehicle to be on a runway with a slope γ, the above equations would be modified by adding the friction and normal forces to give:

$$\begin{aligned} T\cos\alpha_T - D - f - W\sin\gamma &= m\dot{V} \\ T\sin\alpha_T + L + N - W\cos\gamma &= m\frac{V^2}{r} = mV\dot{\gamma} \end{aligned} \qquad (9)$$

where f is the rolling friction, and N is the normal force. Note that the lift and drag act perpendicular and parallel to the runway direction respectively since the direction of the relative wind (velocity) is parallel to the runway (until take-off).

If we now examine these equations of motion, it is clear that in order to do anything with them such as extracting performance measures, we must know something about the thrust, T, the lift, L, and the drag, D. Since all these forces depend on the density, as we will see, and the density depend upon the altitude in the atmosphere, we must consider the idea of standardizing the atmosphere so that we can compare various performance characteristics among various vehicles of interest.