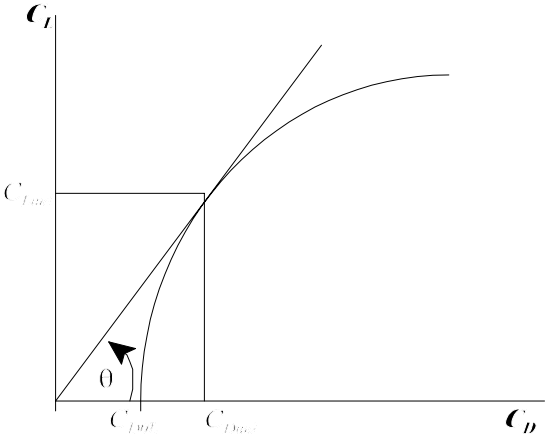
9. Determining Minimum Drag

One of the goals of performance analysis is to do things the best, maximum speed, maximum endurance, minimum take-off distances etc. Consequently, finding the optimum (minimum or maximum) of some function is a constant requirement in performance considerations. Our first encounter with that idea is that of finding the maximum L/D ratio or equivalently (since weight is given, and W = L), minimum drag. Associated with this optimum is the conditions (namely airspeed and possibly angle-of-attack) at which it occurs>

In general, the drag polar is defined as $C_D = f(C_L)$, where the drag coefficient can be any function of the lift coefficient. Although we have considered primarily the parabolic drag polar, $C_D = C_{D_{0L}} + K C_L^2$, compressibility effects can distort the curve from a true parabola. Under the conditions where the drag polar is not purely parabolic, and in fact for any drag polar, we can use graphical methods to determine the maximum L/D ratio. This is accomplished by plotting the drag polar with C_L vs C_D (or vice versa).

If we draw ***any line*** from the origin to any point on the drag polar (it will generally intersect it in two locations), the tangent of the angle the line makes with the horizontal is given by:

$$\tan\theta = \frac{C_L}{C_D} \qquad (1)$$



Therefore, the maximum value of this quantity is determined when the line from the origin is tangent to the curve for the drag polar. That point defines the max L/D ratio and component along the x axis gives the minimum drag coefficient $C_{D_{md}}$, and the y axis gives the lift coefficient for minimum drag, $C_{L_{md}}$.

$$\tan\theta_{max} = \frac{C_{L_{md}}}{C_{D_{md}}} = \left.\frac{L}{D}\right)_{max} \qquad (2)$$

Then we can compute the minimum drag airspeed and equivalent airspeed, from:

$$V_{md} = \sqrt{\frac{W}{1/2\,\rho\, S\, C_{L_{md}}}} \qquad V_{eq_{md}} = \sqrt{\frac{W}{1/2\,\rho_{SL}\, S\, C_{L_{md}}}} \qquad (3)$$

These results are general, and ***hold for any drag polar***.