AOE 3104 Aircraft Performance Problem Sheet 2 (ans)

6. The atmosphere of Jupiter is essentially made up of hydrogen, H_2. For Hydrogen, the specific gas constant is 4157 Joules/(kg)(K). The acceleration of gravity of Jupiter is 24.9 m/s^2. Assuming an isothermal (constant temperature) atmosphere with a temperature of 150 K, and assuming that Jupiter has a definable surface, calculate the altitude above that surface where the pressure is one-half the surface pressure.

Find the Pressure ratio in a constant temperature atmosphere:

$$\frac{P}{P_1} = e^{-\frac{g_0}{R T_1}(h - h_1)} = \frac{1}{2}$$

Then

$$\ln\frac{1}{2} = -\frac{g_0}{R T_1}(h - h_1) \qquad \Rightarrow \qquad h - h = -\frac{R T_1}{g_0}\ln\frac{1}{2}$$

$$= -\frac{4157\,(150)}{24.9}(-0.69315)$$

$$= 17358 \text{ m}$$

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7. Consider an airplane flying with a velocity of 60 m/s at a standard altitude of 3km. At a point on the wing, the airflow velocity is 70 m/s. Calculate the pressure at this point. Assume incompressible flow.

@ 3 km $\qquad$ P = 7.010x10^4 N/m^2 $\qquad$ ρ = 0.9090 kg/m^3

Use Bernoulli's equation for incompressible flow:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

To get:

$$70100 + \frac{1}{2}(0.9090)(60)^2 = P_2 + \frac{1}{2}(0.9090)(70)^2$$

$$P_2 = 69509.2 \text{ N/m}^2$$

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8. In early low-speed airplanes, the venturi tube was used to measure airspeed. This simple device is a convergent-divergent duct. (The front section's cross-sectional area decreases in the flow direction, and the back section's cross-sectional area increases in the flow direction. Somewhere in between the inlet and exit of the duct, there is a minimum area, called the throat). Let A_1 and A_2 denote the inlet and throat areas, respectively. Let p_1 and p_2 be the pressures at the inlet and throat, respectively. The venturi tube is mounted at a specific point on the plane where the inlet velocity V_1 is essentially the same as the free stream velocity, i.e. the velocity of the airplane through the air. With a knowledge of the area ratio, A_1/A_2 (a fixed design feature) and a

measurement of the pressure difference p_1 - p_2 , the airplane's velocity can be determined. Find an expression for the velocity in terms of the pressure difference, the area ratio, and the constant density. If the airplane is flying at standard sea level conditions, $A_1/A_2 = 4$, and $p_1 - p_2 = 80$ lb/ft^2, evaluate your expression and determine the speed of the airplane.

Let V_1 = the entrance airspeed (the airspeed of the aircraft), and let V_2 be the airspeed in the smallest area or throat. We need to be able to determine V_1. Use the incompressible Bernoulli's and 1-D continuity equation:

$$P_1 + \frac{1}{2}\rho V^2 = P_2 + \frac{1}{2}\rho V_2^2 \qquad\qquad A_1 V_1 = A_2 V_2$$

$$P_1 - P_2 = \frac{1}{2}\rho (V_2^2 - V_1^2) \qquad\qquad V_2 = \frac{A_1}{A_2} V_1$$

Replace V_2 to get:

$$P_1 - P_2 = \frac{1}{2}\rho V_1^2 \left(\frac{V_2^2}{V_1^2} - 1 \right)$$

$$= \frac{1}{2}\rho V_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$$

$$V_1^2 = \frac{2 (P_1 - P_2)}{\rho \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]}$$

$$\frac{A_1}{A_2} = 4 \qquad P_1 - P_2 = 80 \text{ lbs/ft}^2$$

$$V_1^2 = \frac{2\,(80)}{0.002377\,[4^2 - 1]} = 4481.79$$

$$V_1 = 66.9 \text{ ft/sec}$$

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9. A supersonic transport is flying at a velocity of 1500 mi/hr at a standard altitude of 50,000 ft. Calculate the Mach number of the transport.

@ 50,000 ft T = 392.088 deg R 1500 mi/hr x 88 (ft/sec)/(mi/hr) = 2200 ft/sec

$$a = \sqrt{\gamma R T} = \sqrt{(1.4)(1716.5)(392.088)} = 970.68 \text{ ft/sec}$$

$$M = \frac{V}{a} = \frac{2200}{970.68} = 2.27$$

10. The altimeter of a low-speed Piper Aztec reads 8000ft. A Pitot tube mounted on the wingtip measures a pressure of 1650 lb/ft^2. If the outside air temperature is 500 deg R, What is the true velocity of the airplane?, What is the equivalent airspeed?

@ 8000 ft (Pressure is standard) P = 1571.9 lb/ft^2, T given as 500 deg R

Pitot tube total pressure P_0 = 1650 lb/ft^2

From Perfect Gas Law: $\rho = \dfrac{P}{R\,T} = \dfrac{1571.9}{1716.5\,(500)} = 0.001831 \text{ slug/ft}^3$

(Note we are looking for the density of the atmosphere where the free stream airspeed is V and hence we us static pressure - Recall we consider the aircraft to be sitting still and the atmosphere to be moving past it).

a) Using Bernoulli's incompressible equation:

$$P_0 = P + \frac{1}{2}\rho V^2 = 1650 = 1571.9 + \frac{1}{2} 0.001831\, V^2 \qquad V = 292.16 \text{ ft/sec}$$

b) Equivalent airspeed is defined by: $\frac{1}{2}\rho_{SL} V_{eq}^2 = \frac{1}{2}\rho V^2$

Hence:

$$V_{eq} = \sqrt{\frac{\rho}{\rho_{SL}}}\, V = \sqrt{\sigma}\, V \qquad V_{eq} = \sqrt{\frac{0.001831}{0.002377}}\,(292.16) = 256.19 \text{ ft/sec}$$

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11. The altimeter on a low-speed airplane reads 2 km. The airspeed indicator reads 50 m/s. If the outside air temperature is 280 k, what is the true velocity of the airplane?

@ h = 2 km P = 79480 Pa T (given) = 280 deg K

From the perfect gas law: $\rho = \dfrac{P}{R\,T} = \dfrac{79480}{287.05\,(280)} = 0.9889 \text{ kg/m}^3$

It's a low speed aircraft so assume airspeed indicator is incompressibly calibrated or that the Mach number effects are negligible. Then:

$$V_{cal} = V_{eq} = 50 \text{ m/s} \qquad V = \frac{V_{eq}}{\sqrt{\sigma}} = \frac{50}{\sqrt{\dfrac{0.9889}{1.2250}}} = 55.64 \text{ m/s} \quad \text{Where } \sigma = \sqrt{\frac{\rho}{\rho_{SL}}}$$

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12. A high-speed subsonic Boeing 707 airliner is flying at a pressure altitude of 12 km. A Pitot tube on the vertical tail measures a pressure of 2.96×10^4 N/m^2. At what Mach number is the airplane flying?

@ 12 km the pressure in a standard atmosphere is 19320 N/m^2

The pitot tube read stagnation pressure that is P_0 = 29600 N/m^2

We can use the compressible form of Bernoulli's equation:

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = \frac{29600}{19320} = \left(1 + \frac{1.4 - 1}{2} M^2\right)^{\frac{1.4}{1.4 - 1}}$$

or

$$1.5321 = \left(1 + 0.2\, M^2\right)^{3.5}$$

$$M = 0.805$$

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13. A high-speed aircraft is flying at Mach 0.95 in a standard atmosphere at 30,000ft. Determine: a) True airspeed
b) indicated airspeed on a incompressibly calibrated airspeed indicator
c) indicated airspeed on a compressibly calibrated airspeed indicator
d) equivalent airspeed

@ 30,000 ft, P = 628.50 lbs/ft^2, T = 411.77 deg R, ρ = 0.000890 sllugs/ft^3

a) The true airspeed can be obtained by finding the speed of sound, since the Mach number is given.

$$a = \sqrt{\gamma R T} = \sqrt{(1.4)(1716.5)(411.77)} = 994.75 \text{ ft/sec}$$

The true airspeed is then $V = a M = (994.75)(0.95) = 945.01$ ft/sec

b) The first thing we need to do is to calculate the total pressure since all airspeed indicators only use P_0 - P. Since at high subsonic speed we need to use the compressible form of Bernoulli's equation.

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = \left[1 + 0.2\,(0.95^2)\right]^{3.5} = 1.7874$$

$$P_0 = \left(\frac{P_0}{P}\right) P = (1.7874)(628.50) = 1123.40 \text{ lbs/ft}^2$$

Now, for an incompressibly calibrated airspeed indicator we have:

$$V_{cal_{inc}} = \sqrt{\frac{2\,(P_0 - P)}{\rho_{SL}}} = \sqrt{\frac{2\,(\,1123.40 - 628.50\,)}{0.002377}} = 645.3 \text{ ft/sec}$$

c) For a compressibly calibrated airspeed indicator we have:

$$V_{cal} = \left\{ \frac{2\,a_{SL}^2}{\gamma - 1} \left[\left(\frac{P_0 - P}{P_{SL}} + 1 \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \right\}^{1/2}$$

We need a_{SL}: $a_{SL} = \sqrt{\gamma\, R\, T_{SL}} = \sqrt{1.2\,(1716.5)\,518.69} = 1116.43 \text{ ft/sec}$

$$V_{cal} = \left\{ \frac{2\,(116.43)^2}{0.4} \left[\left(\frac{1123.40 - 628.50}{2116.43} + 1 \right)^{0.2857} - 1 \right] \right\}^{1/2} = 621.31 \text{ ft/sec}$$

d) Equivalent airspeed is defined as $1/2\, \rho_{SL}\, V_{eq}^2 \equiv 1/2\, \rho\, V^2$

or

$$V_{eq} = \sqrt{\frac{\rho}{\rho_{SL}}}\, V = \sqrt{\sigma}\; V = \sqrt{\frac{0.000890}{0.002377}}\,(945.01) = 578.25 \text{ ft/sec}$$