Homework Set 5 (ans)

23. Using the DATCOM formula for calculating the lift curve slope, and considering a wing with constant aspect ratio of 4.0, make the following two plots:

a) plot how the lift curve slope varies vs Mach for a constant (half chord) sweep angle of 45 degrees. Mach numver range from 0 - 0.95

b) plot how the lift curve slope varies vs sweep angle for a constant Mach number of 0.7 Sweep angle range from 0 to 75 degrees

A MATLAB code for part (a)
Main code:

```
%homework 5 problem 23a - Plot liftcurve slope vs M and Sweep
fid= fopen('C:\matlab\work\3104\liftcurve.txt','wt');
fprintf(fid,'      mach         liftcurve slope\n');
%
% enter contants of problem
a2D= 2*pi; ar= 4; lambdaonehalf=45*pi/180;
for i = 1: 20
    M(i)=0.05*(i-1);
    a(i)= liftcurve(M(i), lambdaonehalf, ar, a2D);
    fprintf(fid,'   %8.4f           %8.4f      \n', M(i), a(i));
end
plot(M,a);
xlabel('Mach Number');
ylabel('Lift Curve Slope');
fclose(fid);
```

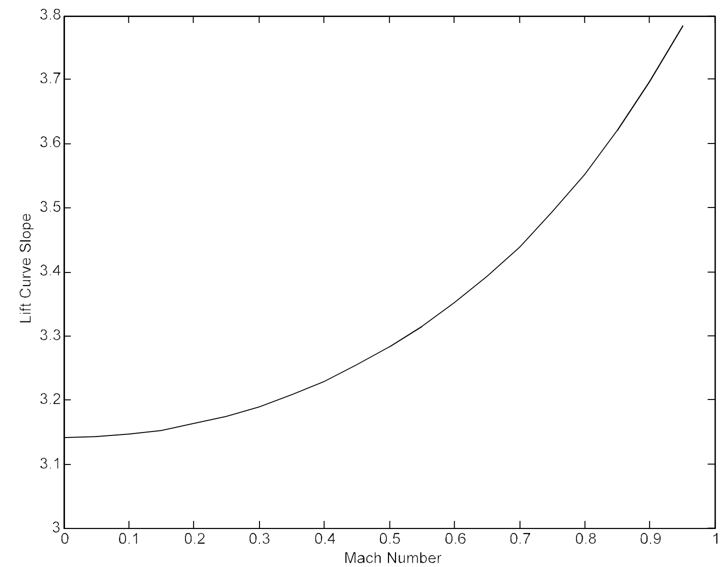
Subroutine “liftcurve”:

```
function [a]=liftcurve(M, halfsweep, ar, a2D)
% liftcurve computes the liftcurve slope for a straight tapered wing given Mach number, M
%(subsonic), halfchord sweep angle, halfsweep, aspect ratio, ar, and the 2D lift curve slope,
%a2D.
%
% compute k
k = a2D/(2*pi);
beta= 1-M^2;
num = 2*pi*ar;
den = 2+sqrt(ar^2*beta/k^2*(1+(tan(halfsweep))^2/beta)+4);
a = num/den;
```

The results of using code:

mach	liftcurve slope
0.0000	3.1416
0.0500	3.1429
0.1000	3.1468
0.1500	3.1534
0.2000	3.1628
0.2500	3.1749

0.3000	3.1899
0.3500	3.2080
0.4000	3.2293
0.4500	3.2540
0.5000	3.2824
0.5500	3.3147
0.6000	3.3514
0.6500	3.3929
0.7000	3.4397
0.7500	3.4925
0.8000	3.5521
0.8500	3.6195
0.9000	3.6960
0.9500	3.7832

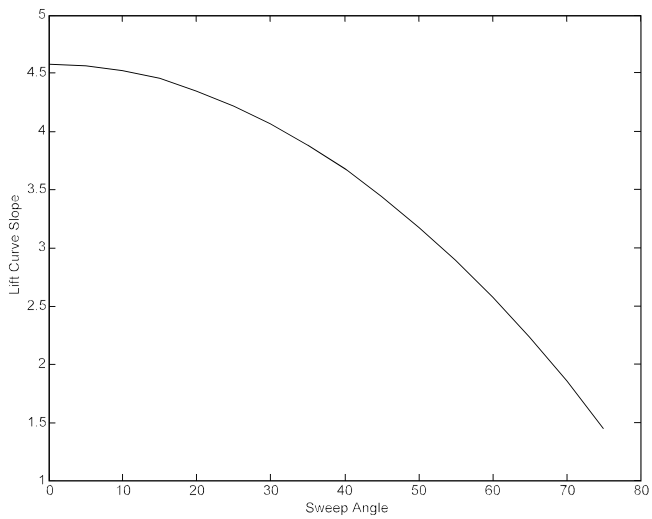


For part (b) the code can be modified as:

```
%homework 5 problem 23b - Plot liftcurve slope vs M and Sweep
fid= fopen('C:\matlab\work\3104\liftcurve.txt','wt');
fprintf(fid,'   sweep angle       liftcurve slope\n');
%
% enter contants of problem
a2D= 2*pi; ar= 4; M=0.7;
for i = 1: 16
   lambdaonehalf(i)=5.0*(i-1)*pi/180;
   a(i)= liftcurve(M, lambdaonehalf(i), ar, a2D);
   fprintf(fid,'   %8.4f           %8.4f      \n', lambdaonehalf(i)*180/pi, a(i));
end
plot(lambdaonehalf*180/pi,a);
xlabel('Sweep Angle');
ylabel('Lift Curve Slope');
fclose(fid);
```

The output of this code is:

sweep angle	liftcurve slope
0.0000	4.5803
5.0000	4.5657
10.0000	4.5221
15.0000	4.4498
20.0000	4.3491
25.0000	4.2207
30.0000	4.0651
35.0000	3.8829
40.0000	3.6744
45.0000	3.4397
50.0000	3.1788
55.0000	2.8912
60.0000	2.5761
65.0000	2.2321
70.0000	1.8576
75.0000	1.4502



<u>Be sure to note how Mach number and sweep angle affect the lift curve slope.</u>

24. Write a computer code that will numerically integrate the equations of motion. Write it so that you can provide any thrust and normal load factor time histories. Use your computer code to solve the problem started in class. That is, find the maximum speed of the aircraft by letting the time grow large for the case of level flight. The parameters of the problem are: Weight = 10,000 lbs., S = 200 ft^2, T = 1,000 lbs., and it is flying level at sea level so that the load factor is 1 and the flight path angle is 0. The drag polar of the aircraft is given by $C_D = C_{D0L} + K\,C_L^2 = 0.02 + 0.05\,C_L^2$. Start the problem with initial conditions of V = 300 ft/s, h = 0, x = 0 (really don't need the x equation unless you want to know how far it travels before it reaches the max speed).

A MATLAB code that will do this integration follows:

```
fid= fopen('C:\matlab\work\3104\trajectory.txt','wt');
fprintf(fid,'      time          airspeed      flight-path angle     altitude        range\n');
%
% enter contants of problem
w = 10000; s=200; T = 1000; rho = 0.002377; cd0l=0.02; k=0.05; g=32.174;
% enter initial conditions
% note that x(1, ) = V, x(2, ) = gamma, x(3, ) = h, x(4, ) = x
x(1,1)=300; x(2,1)=0; x(3,1) = 0; x(4,1) = 0; t = 0;
%fprintf(fid,'    %8.4f          %8.4f          %8.4f          %8.4f        %8.4f\n',...
%    t,x(1,1), x(2,1), x(3,1), x(4,1));
%enter load factor time history;
n = 1; m=7500;
% Euler integration  x(k+1) = x(k) + xdot*deltat
% enter time step
deltat= 0.1;
%
for i = 1:m;
   t(i)=i*0.1; y = x(:,i);
```

```
   x(:,i+1) = y+xdot(y,n,T,w,s,rho,cd0l,k,g)'*deltat;
end
t(m+1)=t(m)+deltat;
for i = 1:100:m
    fprintf(fid,'    %8.4f          %8.4f          %8.4f         %8.4f        %8.4f\n',...
        t(i)-deltat, x(1,i), x(2,i), x(3,i), x(4,i));
end
plot(t,x(1,:));
xlabel('Time');
ylabel('Airspeed');
title('Accelerated Level Flight')
fclose(fid);
```

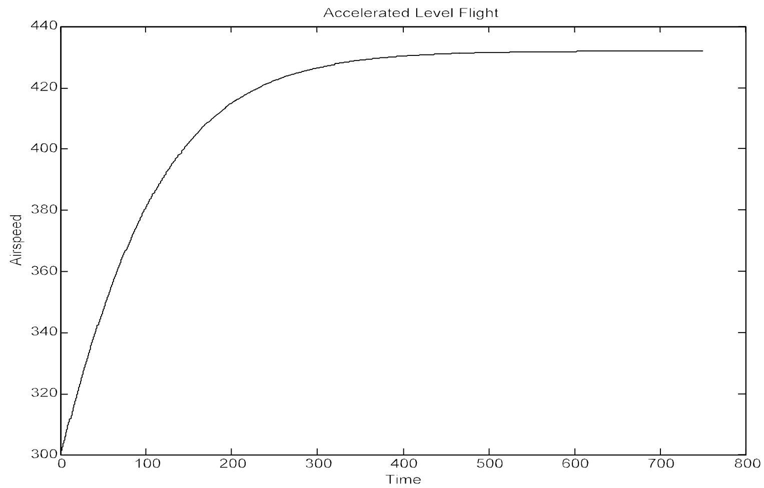
The function xdot is given by:

```
function [z]=xdot(y,n,T,w,s,rho,cd0l,k,g)
%computes right hand side of equations of motion
qbars=0.5*rho*y(1)^2*s;
D=cd0l*qbars+k*w^2*n^2/qbars;
z(1)=(T-D)*g/w-g*sin(y(2));
z(2)=g/y(1)*(n-cos(y(2)));
z(3)= y(1)* sin(y(2));
z(4)= y(1)*cos(y(2));
```

The output is: It takes about 720 seconds to get to a change in airspeed of less than 0.01 ft/sec (About 12 minutes)

time (sec)	airspeed (ft/sec)	flight-path angle	altitude (ft)	range(ft)
0.0000	300.0000	0.0000	0.0000	0.0000
10.0000	310.6521	0.0000	0.0000	3053.1348
20.0000	320.7874	0.0000	0.0000	6210.2779
30.0000	330.3610	0.0000	0.0000	9466.0227
40.0000	339.3463	0.0000	0.0000	12814.6072
50.0000	347.7319	0.0000	0.0000	16250.0802
60.0000	355.5185	0.0000	0.0000	19766.4386
70.0000	362.7166	0.0000	0.0000	23357.7377
80.0000	369.3444	0.0000	0.0000	27018.1774
90.0000	375.4255	0.0000	0.0000	30742.1675
100.0000	380.9875	0.0000	0.0000	34524.3747
110.0000	386.0603	0.0000	0.0000	38359.7544
120.0000	390.6753	0.0000	0.0000	42243.5697
130.0000	394.8645	0.0000	0.0000	46171.4011
140.0000	398.6596	0.0000	0.0000	50139.1473
150.0000	402.0914	0.0000	0.0000	54143.0209
160.0000	405.1899	0.0000	0.0000	58179.5383
170.0000	407.9833	0.0000	0.0000	62245.5071
180.0000	410.4985	0.0000	0.0000	66338.0115
190.0000	412.7607	0.0000	0.0000	70454.3954
200.0000	414.7932	0.0000	0.0000	74592.2455
210.0000	416.6177	0.0000	0.0000	78749.3737
220.0000	418.2542	0.0000	0.0000	82923.8003
230.0000	419.7209	0.0000	0.0000	87113.7369

240.0000	421.0347	0.0000	0.0000	91317.5705
250.0000	422.2108	0.0000	0.0000	95533.8481
260.0000	423.2631	0.0000	0.0000	99761.2629
270.0000	424.2042	0.0000	0.0000	103998.6402
280.0000	425.0455	0.0000	0.0000	108244.9255
290.0000	425.7973	0.0000	0.0000	112499.1728
300.0000	426.4690	0.0000	0.0000	116760.5340
310.0000	427.0688	0.0000	0.0000	121028.2496
320.0000	427.6044	0.0000	0.0000	125301.6394
330.0000	428.0825	0.0000	0.0000	129580.0950
340.0000	428.5091	0.0000	0.0000	133863.0722
350.0000	428.8899	0.0000	0.0000	138150.0843
360.0000	429.2295	0.0000	0.0000	142440.6965
370.0000	429.5325	0.0000	0.0000	146734.5204
380.0000	429.8028	0.0000	0.0000	151031.2090
390.0000	430.0438	0.0000	0.0000	155330.4527
400.0000	430.2587	0.0000	0.0000	159631.9747
410.0000	430.4503	0.0000	0.0000	163935.5284
420.0000	430.6212	0.0000	0.0000	168240.8935
430.0000	430.7735	0.0000	0.0000	172547.8736
440.0000	430.9092	0.0000	0.0000	176856.2934
450.0000	431.0303	0.0000	0.0000	181165.9966
460.0000	431.1382	0.0000	0.0000	185476.8437
470.0000	431.2343	0.0000	0.0000	189788.7105
480.0000	431.3200	0.0000	0.0000	194101.4861
490.0000	431.3964	0.0000	0.0000	198415.0717
500.0000	431.4645	0.0000	0.0000	202729.3791
510.0000	431.5251	0.0000	0.0000	207044.3298
520.0000	431.5792	0.0000	0.0000	211359.8539
530.0000	431.6274	0.0000	0.0000	215675.8889
540.0000	431.6703	0.0000	0.0000	219992.3792
550.0000	431.7086	0.0000	0.0000	224309.2752
560.0000	431.7426	0.0000	0.0000	228626.5327
570.0000	431.7730	0.0000	0.0000	232944.1124
580.0000	431.8001	0.0000	0.0000	237261.9792
590.0000	431.8242	0.0000	0.0000	241580.1018
600.0000	431.8457	0.0000	0.0000	245898.4522
610.0000	431.8648	0.0000	0.0000	250217.0058
620.0000	431.8819	0.0000	0.0000	254535.7404
630.0000	431.8971	0.0000	0.0000	258854.6362
640.0000	431.9107	0.0000	0.0000	263173.6756
650.0000	431.9227	0.0000	0.0000	267492.8431
660.0000	431.9335	0.0000	0.0000	271812.1246
670.0000	431.9431	0.0000	0.0000	276131.5078
680.0000	431.9516	0.0000	0.0000	280450.9815
690.0000	431.9592	0.0000	0.0000	284770.5359
700.0000	431.9660	0.0000	0.0000	289090.1622
710.0000	431.9720	0.0000	0.0000	293409.8525
720.0000	431.9774	0.0000	0.0000	297729.5999
730.0000	431.9822	0.0000	0.0000	302049.3982
740.0000	431.9865	0.0000	0.0000	306369.2418



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25. Use your code to determine the trajectory starting with the above initial conditions with the thrust at maximum, (T_{max} = 3,000 lbs. but with the load factor equal to 2. Feel free to experiment with initial conditions and load factors to see if it is possible for this aircraft to do a loop! Note that velocity cannot go negative, if that happens, stop your program!

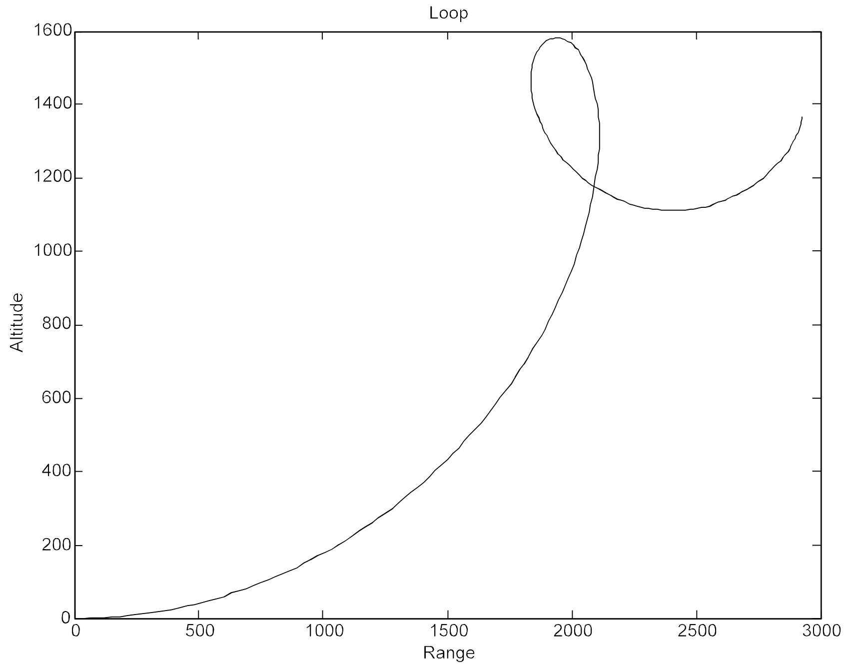
A MATLAB code that will do this follows:

```
%homework 5 problem 25 - Use Euler integration to integrate equaitons of motion
clear
fid= fopen('C:\matlab\work\3104\trajectory.txt','wt');
fprintf(fid,'      time          airspeed      flight-path angle     altitude        range\n');
%
% enter contants of problem
w = 10000; s=200; T = 3000; rho = 0.002377; cd0l=0.02; k=0.05; g=32.174;
% enter initial conditions
% note that x(1, ) = V, x(2, ) = gamma, x(3, ) = h, x(4, ) = x
x(1,1)=300; x(2,1)=0; x(3,1) = 0; x(4,1) = 0; t = 0;
%enter load factor time history;
n = 2; m=240;
% Euler integration  x(k+1) = x(k) + xdot*deltat
% enter time step
deltat= 0.1;
%
for i = 1:m;
        t(i)=i*0.1; y = x(:,i);
        x(:,i+1) = y+xdot(y,n,T,w,s,rho,cd0l,k,g)'*deltat;
end
t(m+1)=t(m)+deltat;
for i = 1:5:m
    fprintf(fid,'    %8.4f          %8.4f          %8.4f         %8.4f        %8.4f\n',...
        t(i)-deltat, x(1,i), x(2,i), x(3,i), x(4,i));
end
plot(x(4,:),x(3,:));
xlabel('Range');
ylabel('Altitude');
title('Loop')
fclose(fid);
```

The output file for this loop is:

time	airspeed	flight-path angle	altitude	range
0.0000	300.0000	0.0000	0.0000	0.0000
0.5000	302.2947	0.0535	3.2273	150.4410
1.0000	303.7424	0.1067	14.5448	301.4711
1.5000	304.3434	0.1601	33.9567	452.2345
2.0000	304.0958	0.2138	61.4217	601.8736
2.5000	302.9957	0.2682	96.8514	749.5265
3.0000	301.0370	0.3237	140.1089	894.3252
3.5000	298.2112	0.3805	191.0055	1035.3934
4.0000	294.5072	0.4391	249.2967	1171.8440
4.5000	289.9116	0.4998	314.6765	1302.7772
5.0000	284.4085	0.5633	386.7708	1427.2790
5.5000	277.9791	0.6300	465.1272	1544.4191
6.0000	270.6028	0.7007	549.2043	1653.2506
6.5000	262.2567	0.7762	638.3556	1752.8099
7.0000	252.9168	0.8577	731.8106	1842.1189
7.5000	242.5597	0.9465	828.6494	1920.1895
8.0000	231.1649	1.0443	927.7706	1986.0344
8.5000	218.7198	1.1535	1027.8492	2038.6845
9.0000	205.2282	1.2772	1127.2819	2077.2219
9.5000	190.7250	1.4195	1224.1184	2100.8369
10.0000	175.3034	1.5864	1315.9757	2108.9263
10.5000	159.1619	1.7860	1399.9442	2101.2659
11.0000	142.6863	2.0293	1472.5138	2078.3044
11.5000	126.5804	2.3300	1529.6110	2041.6450
12.0000	112.0274	2.7017	1566.9689	1994.7350
12.5000	100.7295	3.1462	1581.2145	1943.5036
13.0000	94.4489	3.6352	1571.7876	1896.0139
13.5000	93.9306	4.1095	1542.3324	1860.0775
14.0000	98.2935	4.5176	1499.3322	1840.2629
14.5000	105.7726	4.8453	1449.0831	1837.4642
15.0000	114.7334	5.1050	1396.1956	1850.5900
15.5000	124.0375	5.3143	1343.7827	1877.9468
16.0000	132.9800	5.4880	1294.0009	1917.7730
16.5000	141.1409	5.6366	1248.4175	1968.3640
17.0000	148.2694	5.7674	1208.2110	2028.0808
17.5000	154.2105	5.8857	1174.2796	2095.3359
18.0000	158.8609	5.9951	1147.3037	2168.5768
18.5000	162.1426	6.0985	1127.7828	2246.2703
19.0000	163.9868	6.1983	1116.0570	2326.8885
19.5000	164.3220	6.2964	1112.3174	2408.8940
20.0000	163.0642	6.3947	1116.6070	2490.7251
20.5000	160.1073	6.4951	1128.8112	2570.7789
21.0000	155.3098	6.6000	1148.6369	2647.3921
21.5000	148.4740	6.7122	1175.5731	2718.8182
22.0000	139.3104	6.8357	1208.8239	2783.2006
22.5000	127.3618	6.9766	1247.1887	2838.5405
23.0000	111.8191	7.1458	1288.8364	2882.6632
23.5000	90.9207	7.3658	1330.8109	2913.2085

The loop (obtained by plotting h vs x) is



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26. Verify the terminal speed obtained in prob 24. analytically by solving the thrust = drag equation for speed with T = 1000, and find the maximum aircraft speed for T = T_{max} = 3,000 lbs.

The equation that we must solve is T = D. The assumption is that the thrust is independent of speed and is constant. The equation can be written as:

$$T = D = A V^2 + \frac{B}{V^2}$$

where $A = C_{D_{0L}} \frac{1}{2} \rho S$ and $B = \frac{K W^2}{1/2 \rho S}$

Rearranging we have:

$$A V^4 - T V^2 + B = 0$$

The solution of which is:

$$V^2 = \frac{1}{2}\left[\frac{T}{A} \pm \sqrt{\left(\frac{T}{A}\right)^2 - 4\frac{B}{A}} \right]$$

For our problem we have:

$$A = C_{D_{0L}}\ 1/2\ \rho S = 0.02\ (1/2)\ (0.002377)\ (200) = 0.004754\ \text{lb sec}^2/\text{ft}^2$$

$$B = \frac{K W^2}{1/2\ \rho S} = \frac{0.05\ (10000)^2}{1/2\ (0.002377)\ (200)} = 21034917.96\ \text{lbs ft}^2/\text{sec}^2$$

Then $\dfrac{T}{A} = \dfrac{1000}{0.004754} = 210349.1796\ \text{ft}^2/\text{sec}^2$

and $\dfrac{B}{A} = \dfrac{21034917.96}{0.004754} = 4.4247x10^9\ \dfrac{\text{ft}^4}{\text{sec}^4}$

$$V^2 = \frac{1}{2}\left[210349.18 \pm \sqrt{210349.18^2 - 4\ (4.4247\ x\ 10^9)}\ \right] = 186642.34, \quad 23706.84\ \text{ft}^2/\text{sec}^2$$

and hence

$V = \underline{432.02}, \quad 153.97\ \text{ft/sec}$

The higher speed is the maximum level flight airspeed, and the lower speed is the ***thrust-limited minimum*** air speed.

For Thrust equal to 3000 lbs we have $\dfrac{T}{A} = 631047.54\ \text{ft}^2/\text{sec}^2$ and the remaining numbers stay the same. The result is:

V = <u>789.90</u>, 84.21 ft/sec

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27. Determine the speed for minimum drag, and the drag at that speed. Set the initial speed to 50 ft/sec less than the minimum drag speed, and the thrust level to 1000 lbs. Set the load factor to n = 1, use your code to determine what happens to the aircraft. Summarize the results, I don’t want to see printout

The minimum drag lift coefficient is given by $C_L = \sqrt{\dfrac{C_{D_{0L}}}{K}} = \sqrt{\dfrac{0.02}{0.05}} = 0.6324$

$$V_{md} = \sqrt{\frac{W}{1/2\ \rho S C_{L_{md}}}} = \sqrt{\frac{10000}{1/2\ (0.002377\ (200)\ (0.6324)}} = 257.9\ \text{ft/sec}$$

Then $\left.\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{C_{D_{0L}} K}} = \frac{1}{2\sqrt{0.02\,(0.05)}} = 15.81$

and

$$D_{\min} = \frac{W}{(L/D)_{\max}} = \frac{10000}{15.81} = 632.5 \text{ lbs}$$

If you start at 50 ft/sec below min drag speed, approximately 200 ft/sec. the aircraft accelerates until it reaches the same equilibrium speed that we had in problem 24. It takes just about the same amount of time to do it!