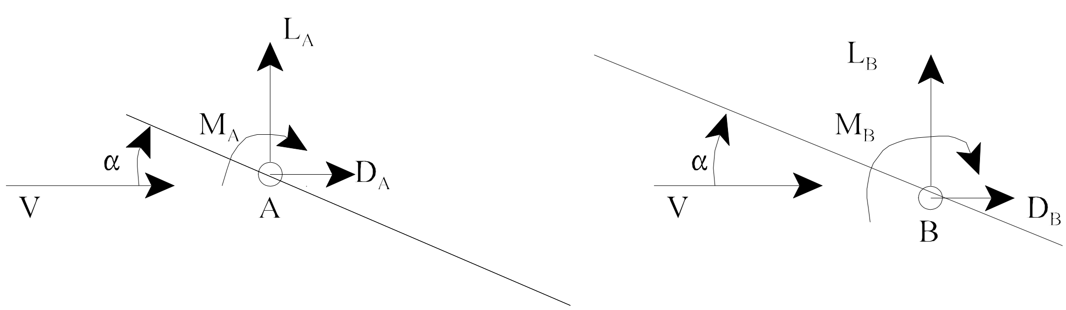
Stability and Control
Some Characteristics of Lifting Surfaces, and Pitch-Moments

The lifting surfaces of a vehicle generally include the wings, the horizontal and vertical tail, and other surfaces such as canards, winglets, etc. What they all have in common is that there purpose is to create an aerodynamic force and moment. Whether the surface is infinite (2-D) or finite (3-D), we can assign to it a force and moment. Usually we designate a 2-D lifting surface as an airfoil, and a 3-D lifting surface as a wing (or tail, or winglet, of whatever). Here all lifting surfaces will be assigned the name - wing.

Generally the information concerning forces and moments on a wing are determined from wind tunnel tests or from computer codes. Unlike a force, a ***moment must be referred to a specific reference point.*** Often time the information given is not referenced to the point of your interest. Consequently we would like to be able to convert results referenced to one point to equivalent information with respect to a different reference point. We will later use these same ideas applied to a complete aircraft.

Consider two identical wings mounted in a wind tunnel. One wing is supported at point A, and the other at point B. At each of these support points there is an electronic “balance” used to measure the forces and moments acting on the wing at those respective points. For simplicity the wings will be represented as flat plates.



Since the wings are identical we can expect that the aerodynamic forces will be the same. The moments will not be the same since the ***reference point is different***. From this observation we can state that:

$$L_A = L_B \equiv L \qquad \text{Lift is the same}$$

and

$$D_A = D_B \equiv D \qquad \text{Drag is the same}$$

For the moment calculation we will use the leading edge as a common reference point in both cases (note that any common point could be used, point A would be a good choice also)

For system A

$$M_{LE} = M_A - L l_A \cos\alpha - D l_A \sin\alpha$$

and similarly for system B

$$M_{LE} = M_B - L l_B \cos\alpha - D l_B \sin\alpha$$

where l_A *and* l_B are the distance from the leading edge to the points A and B respectively. Since the wings are the same, the moments about the same point must be the same so that we can set these two equations equal to each other. If we do that, we can then solve for either the moment about A or the moment about B. Let us select the moment about B.

$$M_B = M_A + (L\cos\alpha + D\sin\alpha)(l_B - l_A)$$

We generally like to deal with equations in non-dimensional or coefficient form. Consequently we will divide Eq. (1) by $\bar{q} S \bar{c}$ and obtain the coefficient form of the above equation.

$$C_{m_B} = C_{m_A} + (C_L \cos\alpha + C_D \sin\alpha)\left(\frac{l_B}{\bar{c}} - \frac{l_A}{\bar{c}}\right) \qquad (1)$$

Eq. (1) is the general equation for the equivalent moment relation. Given a moment and forces at one point on a wing, what is the moment around another point? Eq. (1) answers that question.

If we deal with small angles of attack, then we can make a good simplifying approximation to Eq. (1). If the angle-of-attack is small, then $\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$. In addition we can note that for most (efficient) lifting surfaces, $L >> D$ and hence $C_L >> C_D$. If we compare the magnitudes of the terms in the first set of parentheses, we see we can compare $C_L \cos\alpha \approx C_L$ with $C_D \sin\alpha \approx C_D \alpha$. Since C_D is already significantly smaller than C_L, and it is multiplied by the small α, the second term can be neglected compared with the first. Hence we can make a good approximation of Eq. (1) with

$$C_{m_B} = C_{m_B} + C_L\left(\frac{l_b}{\bar{c}} - \frac{l_A}{\bar{c}}\right)$$

Finally, we can introduce the definition of the non-dimensional distance behind the leading edge of the wing,

$$h_x = \frac{l_x}{\bar{c}} \qquad \text{or} \qquad h_A = \frac{l_A}{\bar{c}} \quad \text{and} \quad h_B = \frac{l_B}{\bar{c}}$$

Then we can write the final form of the moment equation:

$$C_{m_B} = C_{m_A} + C_L(h_B - h_A) \qquad (2)$$

Note that the equation is equally valid if we interchange A and B. As such we can write Eq. (2) in the form

$$C_{m_1} = C_{m_2} + C_L(h_1 - h_2) \qquad (3)$$

where points 1 and 2 can be ***any two*** points on the wing.

It should be pointed out, that under the small angle assumption, the assumption the two reference points are on the same reference line, and the assumption of a reasonable aerodynamic efficiency, Eq. (3) can be applied to any body with aerodynamic forces and moments (including the whole aircraft!)

Definition: ***Aerodynamic Center -*** That point on a lifting surface where the pitch-moment is independent of lift (or angle-of-attack).

The aerodynamic center is usually located at approximately the quarter chord point (h = 0.25) for subsonic flight and at the half-chord point (h = 0.5) for supersonic flight. Although the location of the aerodynamic center usually depends on the angle-of-attack (for any given Mach number), we will assume that it is fixed. (If the aerodynamic model is linear, then the aerodynamic center is fixed as will be shown next.

For our wing problem, we can estimate the location of the aerodynamic center from the moment equation (3) if we know the aerodynamic properties about some other point (say B). Let the location of the aerodynamic center be located a distance $h_{n_w}\bar{c}$ behind the leading edge of the wing. The pitch-moment about the aerodynamic center is given (In terms of a pitch moment about the point B) as:

$$C_{m_{ac_w}} = C_{m_B} + C_L(h_{n_w} - h_B)$$

From the definition of the aerodynamic center, we can just take the derivative of the above equation and set it equal to zero:

$$\frac{dC_{m_{ac}}}{d\alpha} = 0 = \frac{dC_{m_B}}{d\alpha} + \frac{dC_L}{d\alpha}(h_{n_w} - h_B)$$

Further we can define the lift-curve slope as $a_w = \frac{dC_L}{d\alpha}$, where the subscript, w, indicates it's the lift-curve-slope of the wing.

Then, by rearranging the previous equation, and solving for h_{n_w} we have:

$$h_{n_w} = h_b - \frac{1}{a_w}\frac{dC_{m_B}}{d\alpha} \tag{4}$$

We can make a few observations: If the lift-curve is linear with angle-of-attack, that is, it is a straight line, then the lift-curve-slope a_w, is a constant (independent of α). Likewise with the pitch-moment-curve-slope. Consequently, if the aerodynamic lift and pitch moment are linear (straight lines vs alpha), then the aerodynamic center location is constant.

Pitch-moment at zero lift

We can see from Eq. (3) (and for a special case of Eq. (1), where zero lift is at α =0), that the pitch moment at any point on the wing is the same.

$$C_{m_1} = C_{m_2} + C_L(h_1 - h_2)\big|_{C_L=0} = C_{m_2}$$

Therefore, since the points 1 and 2 can be anywhere on the wing chord line, the moment at zero lift is the same at any point. Furthermore, since at the aerodynamic center the change in the pitch-moment with angle of attack is zero, the pitch moment about the aerodynamic center must be constant with changes in angle-of-attack. So that

$$C_{m_1} = C_{m_2} = C_{m_{ac}} = C_{m_{0L}}$$

or

$$\boxed{C_{m_{ac}} = C_{m_{0L}}}$$

Consequently it is convenient to select the aerodynamic center as a reference point. If we do so, then we can write the moment equation for an arbitrary location in the general form:

$$C_m = C_{m_{0L_w}} + C_L(h - h_{n_w})$$

where C_m is the pitch-moment coefficient about the arbitrary point located at the distance $h\bar{c}$ behind the leading edge of the mean aerodynamic chord.

Definition: ***Center of Pressure*** - Point along the chord line, extended if necessary, about which the pitch-moment is zero

$$C_{m_{cp_w}} = C_{m_{0L_w}} + C_{L_w}(h_{cp_w} - h_{n_w}) \equiv 0$$

We can solve for the center of pressure location:

$$\boxed{h_{cp} = h_{n_w} - \frac{C_{m_{0L}}}{C_L}}$$

Note that the center of pressure location changes with the lift coefficient.

<u>Some Nomenclature and Definitions</u>

In the above developments we defined some nomenclature that we will review here. At the same time we will introduce some new nomenclature.

Lift-curve slope

The lift-curve slope is the derivative of the lift coefficient with respect to the angle-of-attack. We generally use a partial derivative since the lift coefficient depends upon other variables (such as Mach number and pitch rate). Hence we have:

$$\frac{\partial C_L}{\partial \alpha} = C_{L_\alpha} = a \qquad (a_w,\ a_{ht},\ a_{wb},\ a_{vt},\ a, ...etc)$$

Pitch-moment curve slope

$$\frac{\partial C_m}{\partial \alpha} = C_{m_\alpha} \qquad (c_{m_{\alpha_w}},\ C_{m_\alpha}, ...)$$

$$\left.\frac{\dfrac{\partial C_m}{\partial \alpha}}{\dfrac{\partial C_L}{\partial \alpha}}\right|_{M_a} = \frac{1}{a}\left.\frac{\partial C_m}{\partial \alpha}\right|_{M_a} = \frac{\partial C_m}{\partial C_L} = C_{m_{C_L}}$$

Lift coefficient

The lift coefficient can be represented in two ways:

$$C_L = C_{L_\alpha}(\alpha - \alpha_{0L}) = C_{L_\alpha}\bar{\alpha} = a\bar{\alpha}$$
$$= C_{L_0} + C_{L_\alpha}\alpha$$

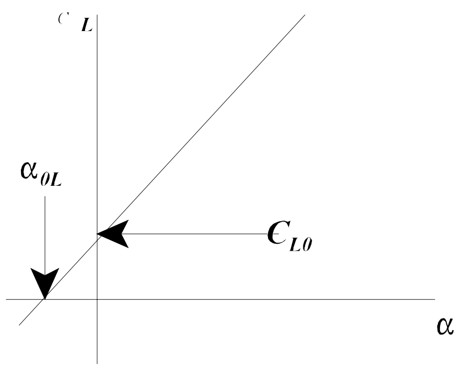
where

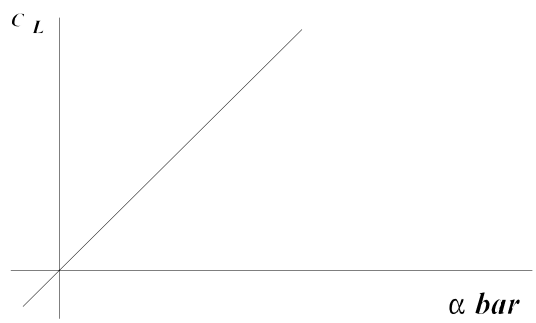
$\bar{\alpha} = \alpha - \alpha_{0L}$

α_{0L} = angle-of-attack when lift = 0

C_{L_0} = Lift coefficient when angle-of-attack = 0

$= -a\,\alpha_{0L}$





These two graphs illustrate α_{0L}, C_{L_0}, and $\bar{\alpha}$.

Example

Consider a wind tunnel experiment where the wing is mounted at the 1/3 chord location. For a given airspeed, lift and pitch moment coefficients were recorded at several angles-of-attack. The data obtained were as follows:

α	C_L	$C_{m\,(1/3)}$
0.5	0.2	-0.02
3.0	0.4	0.0
5.5	0.6	0.02
8.0	0.8	0.04

we can calculate the lift-curve-slope and the pitch-moment-curve slope by differencing two points on the curve (In general, you would plot the points and "fit" aa straight line through the points. Here we can observe that it is a straight line). For the best accuracy pick the points on the straight line that are furthest apart.

$$a_w = \frac{\partial C_L}{\partial C_D} = \frac{\Delta C_L}{\Delta C_D} = \frac{0.8 - 0.2}{8.0 - 0.5} = \frac{0.6}{7.5} = 0.08 \,/\mathrm{deg} = 4.58 \,/\, \mathrm{rad}$$

and

$$C_{m_{\alpha_w}} = \frac{\partial C_m}{\partial \alpha} = \frac{\Delta C_m}{\Delta \alpha} = \frac{0.04 - (-0.02)}{8.0 - 0.5} = \frac{0.06}{7.5} = 0.008 \,/\,\mathrm{deg} + 0.458 \,/\, \mathrm{rad}$$

Then we can estimate the aerodynamic center of the wing as follows:

$$h_{n_w} = h_B - \frac{C_{m_\alpha}}{a} = \frac{1}{3} - \frac{0.008}{0.08} = 0.233 \qquad \text{or} \qquad 23.3\% \text{ mac}$$

We can determine the zero lift angle of attack by picking a line in the data table and inserting it in the lift coefficient equation:

$$C_L = a_w(\alpha - \alpha_{0L}) = 0.4 = 0.08(3.0 - \alpha_{0L}) \quad \Rightarrow \quad \underline{\alpha_{0L} = -2.0 \text{ deg}}$$

alternatively

$$C_L = C_{L_0} + a_w \alpha = 0.4 = C_{L_0} + 0.08(3.0) \quad \Rightarrow \quad \underline{C_{L_0} = 0.16}$$

Find the zero lift pitch-moment (also the pitch-moment about the aerodynamic center) from the basic moment equation:

$$C_m = C_{m_{0L}} + C_L(h - h_{n_w}) = 0.04 = C_{m_{0L}} + 0.8\left(\frac{1}{3} - 0.233\right) \quad \Rightarrow \quad \underline{C_{m_{0L}} = -0.04}$$

We could have selected another line in the data set, for example the line where $C_m = 0$.

$$0 = C_{m_{0L}} + 0.4\left(\frac{1}{3} - 0.2333\right) \quad \Rightarrow \quad \underline{C_{m_{0L}} = -0.04}$$

For balance (equilibrium) flight, $C_m = 0$. So for our example problem the wing would be balanced if the center-og-gravity were at the 1/3 chord location (1/3 $\bar{c}$) when $\alpha = 3$, and $C_L = 0.4$. If, for the sake of discussion, the wing weighed 200 lbs and had S = 50 ft^2. Then we could determine that for sea level conditions ($\rho = 0.00238$ slugs/ft^3), V would have to be 91.67 ft/sec (from the definition of the lift coefficient, $C_L = L / (\frac{1}{2} \rho V^2 S)$).

Suppose the cg were shifted to 0.4 $\bar{c}$, what would the speed at which equilibrium flight could be obtained? We need to determine the pitch-moment about the 0.4 chord location and set it equal to zero. To do this we can use the general pitch moment equation:

$$C_{m_A} = C_{m_B} + C_L(h_A - h_B)$$
$$C_{m_{0.4}} = C_{m_{1/3}} + C_L(0.4 - \frac{1}{3}) = 0$$

Presumably we could pick an α and evaluate $C_{m_{1/3}}$ and C_L at that α until the above equation is satisfied. However if we use the basic form of the moment equation using the aerodynamic center the result is a bit more useful:

$$C_{m_A} = C_{m_{0L}} + C_L(h_A - h_{n_w})$$
$$0 = -0.04 + C_L(0.4 - 0.2333) \qquad \Rightarrow \qquad C_L = 0.2395$$

Then,

$$V = \sqrt{\frac{W}{1/2\,\rho\, S\, C_L}} = \sqrt{\frac{200}{0.00238(50)0.2395}} = 118.47 \text{ ft/sec}$$

Note that for equilibrium flight, we must balance the lift with the weight, and keep the moment equal to zero. If we shift the cg around, the requirement to balance the forces and the moments makes it necessary to adjust our speed. In this case, as the cg is moved aft, the speed increases. Be advised that this case is not the usual one as we will see in the next section.

This concludes the section on pitch-moments.