Lateral Directional Flight Considerations

This section discusses the lateral-directional control requirements for various flight conditions including cross-wind landings, asymmetric thrust, turning flight, and others. The controls for longitudinal flight maneuvers were presented earlier. Hence to compute the full control requirements for a particular flight condition is necessary to do both the longitudinal and lateral-directional control calculations. This section is just for the lateral-directional controls.

Steady State Flight

The equations for steady state flight (time derivatives = 0) for the lateral- directional control considerations are given by:

$$\begin{aligned} Y_{aero} + W\sin\phi &= m\,V\,r \\ L_{aero} &= 0 \\ N_{aero} &= 0 \end{aligned} \tag{1}$$

For straight, nearly level flight, the yaw rate will be zero and the bank angle will be small, $r = 0$, and $\sin\phi \approx \phi$. If we divide the Y equation by $\bar{q}\,S$, and the two moment equations by $\bar{q}\,S\,b$, and assume a certain dependence of these aerodynamic force and moments on the sideslip angle and aileron and rudder deflections, Eq. (1) can be rewritten as:

$$\begin{aligned} C_{Y_\beta}\beta + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r + C_w\,\phi &= 0 \\ C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r &= 0 \\ C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r &= 0 \end{aligned} \tag{2}$$

where C_w is the "weight" coefficient $W/\bar{q}S$. Here we see that there are three equations in the four unknowns, $\beta,\ \delta_a,\ \delta_r,$ and ϕ. Therefore we must pick one, and solve for the remaining three. For example if we wanted to fly in a straight line with a steady bank angle (say for taking pictures or something) then we could specify the desired bank angle and solve for the resulting sideslip angle and control deflections:

$$\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_a}} & C_{Y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = \begin{Bmatrix} -C_w\,\phi \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

A typical problem would be to determine the maximum bank angle at which the aircraft could fly before one of the controls reaches its limits.

Another problem that could be solve is to determine the controls and bank angle required to maintain a particular sideslip angle. Then the equations could be rearranged in the following manner:

$$\begin{bmatrix} C_{Y_{\delta_a}} & C_{Y_{\delta_r}} & C_w \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & 0 \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = \begin{Bmatrix} -C_{Y_\beta}\beta \\ -C_{l_\beta}\beta \\ -C_{n_\beta}\beta \end{Bmatrix} \tag{4}$$

Such an occasion might occur in a cross-wind landing. For a given landing speed, and a given cross-wind the sideslip angle can be found from $\sin\beta = \text{cross-wind}/\text{airspeed}$.

Asymmetric Thrust

Although we looked at asymmetric thrust previously, we can now look at it including the effects of controls on the roll and sideforce as well as the yaw due to asymmetric thrust. In some case considering all the effects in sideforce, roll, and yaw is more restrictive then considering only the yaw equation as did before. The equations for asymmetric thrust must include an additional yaw moment term due to the asymmetric thrust. Hence the equations become:

$$\begin{aligned} C_{Y_\beta}\beta + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r + C_w\phi &= 0 \\ C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r &= 0 \\ C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + C_{n_T} &= 0 \end{aligned} \tag{5}$$

However the asymmetric thrust requires that the sideslip angle, $\beta = 0$. Rearranging the equations with the $\beta = 0$ constraint leads to:

$$\begin{bmatrix} C_{Y_{\delta_a}} & C_{Y_{\delta_r}} & C_w \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & 0 \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -C_{n_T} \end{Bmatrix} \tag{6}$$

As an example, consider the left engine out, with the right engine thrusting at 3000 lbs, ft from the center line. The vehicle has the properties W = 13,000 lbs, S = 230 ft^2 , b = 34 ft and $\bar{c}$ = 7 ft. The speed is 250 ft/sec at sea level. We can calculate the thust yaw moment coefficient and the weight coefficient:

$$C_{n_T} = \frac{-T y_p}{1/2\, \rho\, V^2 S\, b} = \frac{-3000(5)}{1/2\ (0.00238)\, 250^2 (230)\, 34} = -0.0258$$

and

$$C_w = \frac{W}{1/2\ \rho\ V^2\ S} = \frac{13000}{1/2\ (0.00238)\ 250^2\ (230)} = 0.760$$

The aerodynamic properties are given in the following table:

	C_Y	C_l	C_n
β	-0.73	0.173	0.15
p	0	-0.39	-0.13
r	-.40	0.45	-0.26
δ_a	0	-0.149	0.05
δ_r	0.140	0.014	-0.074

All units are non-dimensional

We can use these values to complete our matrix in order to solve the problem.

$$\begin{bmatrix} 0 & 0.140 & 0.760 \\ -0.149 & 0.014 & 0 \\ 0.050 & -0.074 & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.0258 \end{Bmatrix}$$

The solution is

$$\begin{aligned} \delta_a &= -0.0350 \quad &\Rightarrow \quad &-2.005\ \text{deg} \\ \delta_r &= -0.3723 \quad &\Rightarrow \quad &-21.33\ \text{deg} \\ \phi &= 0.0686 \quad &\Rightarrow \quad &3.930\ \text{deg} \end{aligned}$$

The interesting variation of this problem is to find the minimum controllable airspeed at which one of the controls saturates (hits its maximum value), or the bank angle exceeds 5 degrees, while maintaining zero sideslip angle.

Cross-Wind Landing

The problem here is to find the maximum cross wind in which the aircraft can (theoretically) land. Although we don't know for sure until we try it, we will assume that the rudder is the limiting control, that is it is the first one to hit the stop as the cross wind is increased. So we will assume that the rudder angle is at its maximum deflection value and is a known. The equations take the form:

$$\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_a}} & C_w \\ C_{l_\beta} & C_{l_{\delta_a}} & 0 \\ C_{n_\beta} & C_{n_{\delta_a}} & 0 \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \phi \end{Bmatrix} = \begin{Bmatrix} -C_{Y_{\delta_r}} \delta_r \\ -C_{l_{\delta_r}} \delta_r \\ -C_{n_{\delta_r}} \delta_r \end{Bmatrix}$$

We will assume a landing speed of 170 ft/sec, a maximum rudder deflection of 30 degrees. Under these conditions, C_w = 1.64. and the numbers become:

$$\begin{bmatrix} -0.73 & 0 & 1.64 \\ -0.173 & -0.149 & 0 \\ 0.150 & 0.05 & 0 \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \phi \end{Bmatrix} = \begin{Bmatrix} -0.140 \\ 0.014 \\ 0.074 \end{Bmatrix} 30 \frac{\pi}{180} = \begin{Bmatrix} -0.0733 \\ -0.0073 \\ 0.0387 \end{Bmatrix}$$

The solution:

$$\begin{aligned} \beta &= 0.3943 &\Rightarrow&\quad 22.591 \text{ deg} \\ \delta_a &= -0.4088 &\Rightarrow&\quad -23.42 \text{ deg} \\ \phi &= 0.1308 &\Rightarrow&\quad 7.493 \text{ deg} \end{aligned}$$

Finally, $\sin\beta = \dfrac{v}{V} = \sin 22.59 = \dfrac{v}{170} \quad \Rightarrow \quad v = 65.3 \text{ ft/sec}$ max cross-wind

Turning Flight

We define a coordinated turn as one where the turning lateral acceleration is balanced by the component of gravity. From Eq. (1), we have

$$\begin{aligned} Y_{aero} + W \sin\phi &= m V r \\ L_{aero} &= 0 \\ N_{aero} &= 0 \end{aligned} \tag{7}$$

The requirement for a coordinated turn is that $W \sin\phi = m V r$. Therefore in a coordinated turn, all the aerodynamic moments and the aerodynamic sideforce are zero. In coefficient form, and including the effects of roll and yaw rates, we can write:

$$\begin{aligned} C_{Y_\beta}\beta + C_{Y_p}\hat{p} + C_{Y_r}\hat{r} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r &= 0 \\ C_{l_\beta}\beta + C_{l_p}\hat{p} + C_{l_r}\hat{r} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r &= 0 \\ C_{n_\beta}\beta + C_{n_p}\hat{p} + C_{n_r}\hat{r} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r &= 0 \end{aligned} \tag{8}$$

For a climbing (or diving) turn, the angular rates p and q are related to the turn rate. By drawing a picture of a banked aircraft in a turn it is easy to determine the following relationships:

$$\begin{aligned} \hat{p} &= -\frac{\omega b}{2V}\sin\theta \\ \hat{r} &= \frac{\omega b}{2V}\cos\phi\cos\theta \end{aligned} \tag{9}$$

where theta is the climb angle and phi is the bank angle. For a given coordinated turn condition, we can compute the turn angular rate and hence the vehicle angular rates and therefore these are known quantities and can be put on the right hand side of the equations. In general the equations can be rearranged in the form:

$$\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_a}} & C_{Y_{\delta_r}} \\ C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = \begin{bmatrix} C_{Y_p} & C_{Y_r} \\ C_{l_p} & C_{l_r} \\ C_{n_p} & C_{n_r} \end{bmatrix} \begin{Bmatrix} \sin\theta \\ -\cos\theta\cos\phi \end{Bmatrix} \frac{\omega b}{2V} \tag{10}$$

Hence, given the turn rate, climb angle and roll angle, you can solve for the sideslip angle, the aileron, and rudder deflections. Note that the turn rate, bank angle and climb angle are not independent of each other in a coordinated turn..