Stability and Control
Estimating Aerodynamic Properties

A necessary ingredient for determining the aerodynamic properties of an aircraft is to be able to determine the aerodynamic properties of parts of the aircraft. If we look at the expression for the pitch-moment curve slope, we can see some of the parameters that need to be estimated:

$$C_{m_\alpha} = a_{wb}\left[(h - h_{n_{wb}}) - \frac{a_{ht}}{a_{wb}} \eta_{ht} \frac{S_{ht}}{S} (h_{ht} - h) \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] \qquad (1)$$

Here we see that in order to evaluate this expression we need to estimate a_{wb}, a_{ht}, $h_{n_{wb}}$, and $\frac{\partial \epsilon}{\partial \alpha}$. Furthermore, these same parameters show up in the other coefficients as well. If we look at the control effectiveness and control power terms, the additional parameter, a_e, must be estimated. For the zero-lift pitch-moment and the zero lift angle-of-attack, additional parameters such as the wing-body zero-lift pitch-moment, and the wing-body zero-lift angle-of-attack are needed. Here we will examine some ways of estimating some of these quantities. The primary focus will be on the first four parameters identified above.

The discussion will be limited to straight tapered wings (and tails). A straight tapered surface is one where the leading and trailing edges are straight. Most wings fall into this category.

Wing Section Properties

If we slice a wing parallel to the free stream velocity the cross section that is visible is the wing stream-wise airfoil shape. We can also slice a wing perpendicular to the leading edge or to the quarter cord line or in fact perpendicular to any constant chord line. The resulting cross section would be a different airfoil shape and would be referred to the airfoil perpendicular to x chord position. In most cases we are interested in the stream-wise airfoil, also called the wing section. If this section is then extended out to infinity, we can discuss the properties of the wing section as if it were a two-dimensional (2-D) wing, and hence these are called 2-D airfoil properties or 2-D wing section properties. We need the following three 2-D wing section properties,

a_0 = 2-D lift curve slope

$\alpha_{0L_{2D}}$ = 2-D zero-lift angle-of-attack

$c_{m_{0L_{2D}}}$ = 2-D pitch moment at zero lift = $c_{m_{ac_{2D}}}$

This information is available for all NACA and some other standard airfoils. If we know some of the geometry of an airfoil, we can estimate the 2-D lift curve slope in the following way:

1) Estimate (measure is better) the trailing edge angle, ϕ_{TE}

$$\phi_{TE}{}^{\circ} = 119\,\frac{t}{c}\;\; x \begin{cases} 2 & NACA\ 4\ digit \\ 1 & Nominal \\ 1/2 & high\ speed \end{cases}$$

where $\frac{t}{c}$ is the thickness to chord ratio of the airfoil. Note that this estimate can be off by 300%!

2. Then calculate the theoretical 2-D lift-curve slope:

$$c_{l_{\alpha_{theory}}} = 6.28 + 4.7\left(\frac{t}{c}\right)\left(1 + 0.00375\,\phi_{TE}{}^{\circ}\right)\ \text{/rad}$$

3. Modify the value in (2) to account for Reynolds number

$$\left(\frac{c_{l_\alpha}}{c_{l_{\alpha_{theory}}}}\right) = 0.9 - 1.077\tan\frac{\phi_{TE}}{2} \qquad \text{Re} = 10^6$$

$$\left(\frac{c_{l_\alpha}}{c_{l_{\alpha_{theory}}}}\right) = 0.9498 - 0.05933\tan\frac{\phi_{TE}}{2} - 1.2836\left(\tan\frac{\phi_{TE}}{2}\right)^2 \qquad \text{Re} = 10^7$$

4. Calculate the final 2-D lift-curve slope

$$c_{l_\alpha} = a_0 = \frac{1.05}{\sqrt{1 - M_a^2}}\left(\frac{c_{l_\alpha}}{c_{l_{\alpha_{theory}}}}\right) c_{l_{\alpha_{theory}}} \qquad (2)$$

where the term in parentheses is obtained from (3) and the last term from (2).

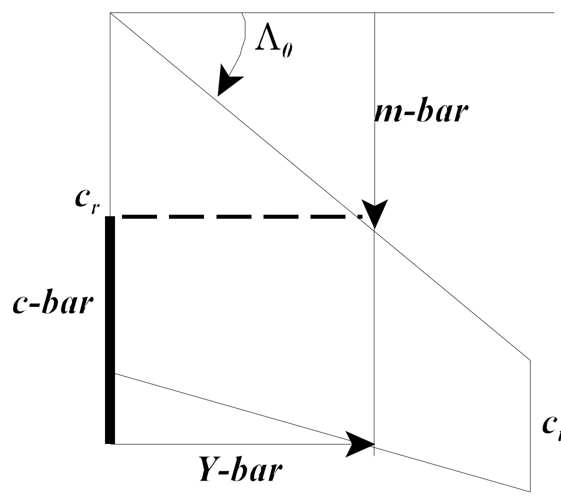
If no information is available, and the ***wing is thin***, an approximate value of the 2-D lift curve slope is

$$a_0 = 2\,\pi$$

$\alpha_{0L_{2D}}$ and $c_{m_{0L_{2D}}}$ must be given or obtained by other means (eg. CFD)

Wing Geometry

The straight tapered wing can be represented by a few geometric properties that describe the wing. We can define some geometric properties of the wing that we will use throughout our discussion. Again you should be reminded that these are characteristics of lifting surfaces. Hence similar properties can be assigned to the horizontal tail but they will have different values.



Taper Ratio, λ

$$\lambda = \frac{c_t}{c_r}$$

Mean Geometric Chord, c′

$$c' = \frac{2}{b}\int_0^{\frac{b}{2}} c(y)\,dy = \frac{S}{b} = \frac{c_r}{2}(1 + \lambda) \qquad (3)$$

Aspect Ratio, AR

$$\boxed{AR = \frac{b}{c'} = \frac{b^2}{S} \qquad (4)}$$

where b is the complete wing span, and S is the wing area.

Mean Aerodynamic Chord, $\bar{c}$

$$\bar{c} = \frac{2}{S}\int_0^{\frac{b}{2}} [c(y)]^2\, dy = \frac{2}{3} c_r \frac{1 + \lambda + \lambda^2}{1 + \lambda} \tag{5}$$

Half-Wing Lateral Mean Aerodynamic Chord Location, $\bar{Y}$

$$\bar{Y} = \frac{b}{2}\left[\frac{1 + 2\lambda}{3(1 + \lambda)}\right] \tag{6}$$

Location of Leading Edge of Mean Aerodynamic Chord With Respect to Wing Apex (Leading Edge of Root Chord), $\bar{m}$

$$\begin{aligned} \bar{m} &= \frac{1 + 2\lambda}{12} c_r \cdot (AR) \cdot Tan\Lambda_0 \\ &= \frac{b}{2}\left[\frac{(1 + 2\lambda)}{3(1 + \lambda)}\right]\tan\Lambda_0 \end{aligned} \tag{7}$$

Sweep of Any Fraction (%) Chord Line

In some of the estimation procedures to follow, the sweep angle of various chord lines are required. We can define the n^{th} fraction chord location, e.g. n = 1/4 for the quarter chord location. Then if one fraction is represented by *n*, and another by *m*, then we can relate the sweep angle at the n^{th} location to the one at the m^{th} location by:

$$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n - m)\frac{1 - \lambda}{1 + \lambda}\right] \tag{8}$$

Lifting Surface Lift Curve Slope

The objective of all this activity is to predict the lift curve slope of the wing (wb), horizontal tail, and later the vertical tail surfaces. Hence the lift curve slope estimation presented next must be applied several times, once for the wing using wing properties, once for the horizontal tail using horizontal tail properties, and once for each other lifting surface using the properties of that surface. The results given below are valid for all ranges of subsonic flight up until the critical Mach number (that Mach number where somewhere on the aircraft the flow becomes supersonic). The estimate is given by:

$$a_x = \frac{2\,\pi\,AR_x}{2 + \sqrt{\dfrac{AR_x^2\,(1 - M_a^2)}{k_x^2}\left(1 + \dfrac{\tan^2\Lambda_{1/2_x}}{(1 - M_a^2)}\right) + 4}} \qquad (9)$$

where x indicates the surface of interest, wb, ht, etc.

The value of k_x is the ratio of the actual 2-D theoretical lift-curve slope with corrections for compressibility (Mach number) effects divided by the flat plate theoretical lift curve slope corrected for compressibility effects, $2\,\pi\,/\sqrt{1 - M_a^2}$. Consequently it becomes:

$$k = \frac{a_{0_{comp}}}{\dfrac{2\,\pi}{\sqrt{1 - M_a^2}}} \qquad (10)$$

However, if we assume that the compressibility correction is the same for the actual as the theoretical, e.g. $a_{0_{comp}} = a_0 / \sqrt{1 - M_a^2}$, then we can write Eq. (10) in the following way:

$$k \approx \frac{a_0}{2\,\pi} \qquad (11)$$

where a_0 is the incompressible 2-D lift curve slope.

If a_0 is not given, and the wing is thin, then the theoretical 2-D lift curve slope can be assume to be 2π and the value of $k = 1$!

Again, this estimate equation is valid for a wide range of aspect ratios and sweep angles, and for the full range of subsonic Mach numbers up to the critical Mach number.

Supersonic Lift Curve Slope

For super sonic flight, the lift curve slope behaves differently. It can be given approximately for thin wings as:

$$a = \frac{4}{\sqrt{M^2 - 1}} \qquad (12)$$

Wing Zero-Lift Angle-of-Attack, $\alpha_{0L_{wb}}$

Here it is assumed that we are given the zero-lift angle-of-attack of the wing section. Then the zero-lift angle-of-attack for the complete wing can be determined for the following cases:

1) Constant wing section (across the span), no sweep,

$$\alpha_{0L} = \alpha_{0L_{2D}}$$

2) Constant wing section, sweep, wing section taken normal to x% chord line

$$\tan \alpha_{0L} = \frac{\tan \alpha_{0L_{2D}}}{\cos \Lambda_x} \tag{13}$$

where Λ_x is the sweep angle of the x% chord line.

3) Constant wing section with twist. Reference all angles-of-attack to the root chord, (think of $\alpha_{wing} = \alpha_r$) for this calculation. Then,

$$\alpha(y) = \alpha_r + \epsilon(y)$$

where $\epsilon(y)$ is the wing twist,(if linear it would be $\epsilon(y) = k_{twist} \cdot y$)

$$\alpha_{0L} = \frac{2}{S} \int_0^{\frac{b}{2}} \left[\alpha_{0L_{2D}} - \epsilon(y) \right] c(y)\, dy \tag{14}$$

Wing $C_{m_{0L}}$

For this calculation we define the airfoil as that parallel to the free stream. It is assumed we know the zero-lift pitch-moment for this 2D wing section. Then we can calculate the complete wing zero-lift pitch-moment for the following cases:

1) Untwisted, constant section wings:

$$C_{m_{0L_w}} = \frac{AR \cos^2 \Lambda_{1/4}}{AR + 2 \cos \Lambda_{1/4}} C_{m_{0L_{2D}}} \tag{15}$$

2) Untwisted, varying wing section:

$$C_{m_{0L_w}} = \frac{AR\cos^2\Lambda_{1/4}}{AR + 2\cos\Lambda_{1/4}}\left(\frac{C_{m_{0L_{2D}root}} + C_{m_{0L_{2D}tip}}}{2}\right) \tag{16}$$

Wing Aerodynamic Center, $h_{n_{wb}}$

The location of the aerodynamic center on the mean aerodynamic chord can be best obtained from charts developed for this purpose. These charts can present the results in different ways. Typically the aerodynamic center position is referenced to the leading edge of the mean aerodynamic chord, but sometimes its given with respect to the leading edge of the root chord or wing apex. Etkin and Reid, Appendix C, Fig C.3 gives the aerodynamic center location for various aspect ratios and wing taper ratios. Values that fall between the graphs or lines on the graphs can be interpolated.

Downwash Parameter, $\frac{\partial\epsilon}{\partial\alpha}$

There are several ways to estimate this parameter. For straight tapered wings, a quasi-empirical, analytic method has been devised that includes the effects of taper ratio, aspect ratio, and horizontal tail vertical position. The method is presented in Etkin and Reid, Appendix B.5, and is reproduced here with an example.

$$\boxed{\frac{\partial\epsilon}{\partial\alpha} = 4.44\left[K_A K_\lambda K_H\left(\cos\Lambda_{1/4}\right)^{1/2}\right]^{1.19}} \tag{17}$$

where: K_A is a correction factor for the aspect ratio and is given by,

$$K_A = \frac{1}{AR} - \frac{1}{1 + AR^{1.7}} \tag{18}$$

and K_λ is a correction factor for the taper ratio and is given by,

$$K_\lambda = \frac{10 - 3\lambda}{7} \tag{19}$$

and K_H is the correction factor for the location of the horizontal tail given by,

$$K_H = \frac{1 - \left|\dfrac{H_{ht}}{b}\right|}{\sqrt[3]{\dfrac{2\,\bar{l}_{ht}}{b}}} \tag{20}$$

where: b is the wing span,

$\bar{l}_{ht} = (h_{ht} - h_{n_{wb}})\,\bar{c}$ (modified tail length)

H_{ht} is the height above or below the plane formed by the root chord and the body fixed y axis, or the distance measured in the plane of symmetry from the root chord line to the horizontal tail aerodynamic center. (Not shown correctly in Etkin and Reid, at least in my edition)

The result of evaluating Eq. (17) is the incompressible downwash parameter. It can be corrected for compressibility effects using the standard compressibility correction (Prandtl-Glauert):

$$\left.\frac{\partial\epsilon}{\partial\alpha}\right|_{comp} = \left.\frac{\partial\epsilon}{\partial\alpha}\right|_{inc} \frac{C_{L_{\alpha_{comp}}}}{C_{L_{\alpha_{incomp}}}} \approx \left.\frac{\partial\epsilon}{\partial\alpha}\right|_{inc} \frac{1}{\sqrt{1 - M_a^2}} \tag{21}$$

Again, these estimations are only good for subsonic flight speeds through the critical Mach number.

Example: AR = 2.31, b = 36.5 ft $\lambda = 0$, H_{ht}=15.88 ft $\Lambda_{1/4}$ = 52.4 deg,

$\bar{l}_{ht}$ = 31.57 ft

$$K_A = \frac{1}{2.31} - \frac{1}{1 + (2.31)^{1.7}} = 0.239 \qquad K_\lambda = \frac{10 - 3\lambda}{7} = \frac{10-3\cdot 0}{7} = 1.428$$

$$K_H = \frac{1 - \left|\dfrac{H_{ht}}{b}\right|}{\dfrac{\sqrt[3]{2\,\bar{l}_{ht}}}{b}} = \frac{1 - \dfrac{15.88}{36.5}}{\sqrt[3]{\dfrac{2\,(31.57)}{36.5}}} = 0.471$$

$$\frac{\partial\epsilon}{\partial\alpha} = 4.44\,[\,0.239(1.428)\,0.471\,(\cos 52.4)^{1/2}\,]^{1.19} = 1.44\,(\,1.256\,)^{1.19} = 0.376$$

This result is for the incompressible (low speed) case.