Stability and Control
Longitudinal Static Stability

We now examine the concept of longitudinal static stability and the contributors to it. As indicated in previous sections, the concept of static stability is concerned with the forces and moments caused by a disturbance away from an equilibrium flight condition. If we apply this idea to the pitch motion of an aircraft (or any other vehicle) we arrive at the following sequence of events: If the vehicle is in an equilibrium reference flight condition, all the moments and forces sum to zero. If we assume that the vehicle is disturbed in pitch (angle-of-attack), then this disturbance will cause the system to no longer be in equilibrium and in fact cause unbalance forces and moments. For this particular study, we are interested in the pitch moment. This unbalanced pitch moment is caused by the disturbance in angle-of-attack. If the pitch moment is in the opposite direction of the displacement, then the vehicle is said to be statically stable in pitch. Consequently we can characterize pitch static stability in the following way: if the disturbance in angle-of-attack ($\Delta\alpha$) is positive, then the moment generated must be negative (for static stability), and vice-versa. This description of static stability can be summarized by the simple expression:

$$\left.\frac{\Delta C_m}{\Delta \alpha}\right|_{RFC} < 0$$

where $|_{RFC}$ means the derivative is evaluated at the reference flight condition.

In the limit we can require

$$\boxed{\left.\frac{\partial C_m}{\partial \alpha}\right|_{RFC} = C_{m_\alpha} < 0 \qquad (1)}$$

where the partial derivative is used since C_m can be a function of other variables besides α.

The derivative C_{m_α} is sometimes called the ***longitudinal stability parameter***, and is an indicator of static stability in pitch at the particular reference flight condition at which it is evaluated.

The most useful form of the pitch-moment equation is the form that uses the aerodynamic center as one of the reference points. This point is convenient because the pitch-moment about that point is constant with angle-of-attack. Thus the pitch-moment equation about any point (usually the center of mass) on the wing (or vehicle for that matter) can be written as:

$$\boxed{C_{m_w} = C_{m_{0L}} + C_{L_w}\left(h - h_{n_w}\right) \qquad (2)}$$

For equilibrium (balanced) flight, $C_m = 0.0$. Furthermore for aerodynamic static stability in pitch, we require Eq. (1) to be satisfied. Taking the derivative of Eq. (2), we get:

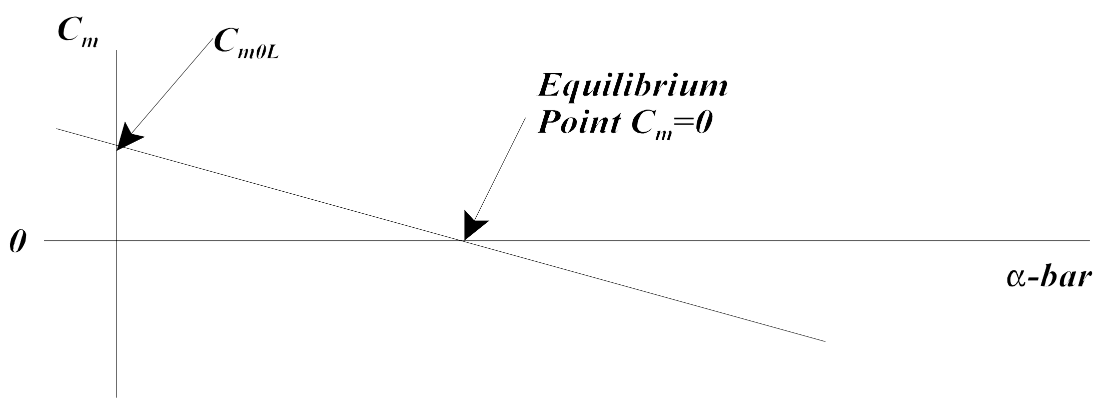
$$\frac{\partial C_m}{\partial \alpha} = \frac{\partial C_L}{\partial \alpha}\left(h - h_{n_w}\right) = a_w\left(h - h_{n_w}\right) < 0$$

Since the lift-curve slope is generally greater than zero, the requirement for stability of this flying wing (and later for the complete aircraft with $h_{n_w} \Rightarrow h_{n_{aircraft}}$) is that

$$\boxed{h - h_{n_w} < 0 \qquad (3)}$$

where h is the center-of-mass (cg) location and h_{n_w} is the aerodynamic center of the wing (or aircraft). ***Hence for static stability of a flying wing or vehicle, the center-of-mass (or center of gravity) must be in front of the aerodynamic center of the wing or vehicle.***

This result has some additional consequences regarding requirements for equilibrium flight. In particular, at the equilibrium flight condition, the pitch-moment must be zero, and the slope of the pitch-moment must be negative for static stability. If we look at a graph of pitch-moment vs angle-of-attack, it is clear that the intercept of the (straight line) graph with the moment axis (zero lift axis, $\bar{\alpha} = 0$) must be positive. Or equivalently, looking at Eq. (2) subject to the requirement of Eq. (3), the only way we can have a zero pitch-moment is if $C_{m_{0L}} > 0$.



Consequently we have:

For Pitch Static Stability:

The pitch moment curve slope must be negative at the reference flight condition

$$\boxed{\left. C_{m_\alpha} \right|_{\text{ref}} < 0}$$

For acceptable balanced (stable) flight (where lift is not zero):

$$\boxed{C_{m_{0L}} > 0}$$

The stability condition can be met with the center-of-mass forward of the aerodynamic center, and the balance flight condition can be met by making the forward lifting surface have a positive angle-of-attack relative to the rear lifting surface. (For a single lifting surface, the front part of the lifting surface must have a positive angle-of-attack relative to the aft part of the lifting surface, eg. a concave up wing).

The End