AOE 3134 Stability and Control Problem Sheet One (Ans)

1. @ $C_L = 0.2 \quad C_{m_{\alpha(1/4)}} = -0.040$ And @ $C_L = 0.6 \quad C_{m_{(1/4)}} = -0.036$

Basic Equation:

$$C_m = C_{m_{0L}} + C_L(h - h_{n_w})$$

a)

$$C_{m_{1/4}} = C_{m_{ac}} + 0.2(1/4 - h_{n_w}) = -0.040$$
$$= C_{m_{ac}} + 0.6(1/4 - h_{n_w}) = -0.036$$

Subtract the top equation from the bottom equation to get:

$$0.004 = 0.4(1/4 - h_{n_w})$$

or

$$\boxed{h_{n_w} = 0.24}$$

b) Use bottom (or top) eq in (a)

$$C_{m_{ac}} + 0.6(.25 - 0.24) - -0.036 \qquad \Rightarrow \qquad \underline{C_{m_{ac}} = =0.042}$$

c)

$$C_{m_{cp}} \equiv 0 = C_{m_{0L}} + C_L(h_{cp} - h_{n_w})$$
$$= -0.042 + 0.5(h_{cp} - 0.24)$$

$$\boxed{h_{cp} = 0.324}$$

d) $C_{m_{0L}} < 0 \qquad C_{m_\alpha} > 0.$ No, its not a good candidate for a flying wing - unstable but It is able to balance with positive angle of attack! Why?

2. a) The wing is in a vertical wind tunnel, and is supported by a rod pinned at one end with the other end attached to the leading edge of the wing. We can write the moments about the pin. We also can assume the angle of attack is small if necessary, and check our assumption afer we obtain a solution. The moment about the pinned joint can be broken into two parts, the moment due to aerodynamics, and the moment due to gravit;y:

$$M_{total} = M_{aero} + M_g = M_{aero} + W(\bar{c} + 0.3\bar{c})\sin\alpha$$ Here we note that leading edge up is +.

If we divide this equation by $\bar{q}S\bar{c}$, we can write it in coefficient form. Further we can define the "weight coefficient" as $C_W = \dfrac{W}{\bar{q}S}$, leading to the moment equation in coefficient form:

$$C_m = C_{m_{aero}} + C_w(1 + 0.3)\sin\alpha = C_{m_{aero}} + C_w(1.3)\sin\alpha$$

But, for small angles, the aerodynamic pitch-moment coefficient is given by (the now familiar!):

$$C_m = C_{m_{0L}} + C_L(h - h_{n_w})$$

Hence the total moment coefficient

$$C_{m_{total}} = C_{m_{0L}} + C_{L_w}(h - h_{n_w}) + C_w(1.3)\sin\alpha$$

Also:

$$C_{L_w} = C_{L_\alpha}(\alpha - \alpha_{0L})$$

Putting it all together and substituting in the numbers yields:

$$C_{m_{total}} = 0 = -0.04 + 0.08(5 - (-2))(-1.0 - 0.25) + \frac{50}{1/2\,(0.00238)\,5\,V^2}(1.3)\sin 5$$

$$= -0.04 + 0.08(7.0)1(-1.25) + \frac{952.1216}{V^2}$$

$$= = -.74 + \frac{952.1216}{V^2}$$

$$\boxed{V = 35.8699 \text{ ft/sec}}$$

b) For the case $\alpha > 0$ $\left(-0.1 + \frac{8403.3613\,(l)}{V^2}\frac{\pi}{180}\right)\alpha° = 0.24$

At low speeds, the angle-of-attack increases with V (and decreases with l). At high speeds (above about 43 ft/sec, the angle of attack is negative and gets less negative with increasing speed until the angle of attack = -2.4 degrees is obtained. Why does this problem behave so strangely?

3. V = 300 ft/sec, W = 3000 lbs,, $h_{cp} = \frac{2.4\text{ ft}}{\bar{c}} = \frac{2.4}{6} = 0.4$, b = 36 ft. $\bar{c} = 6$ ft.

$$C_L = \frac{w}{1/2\,\rho V^2 S} = \frac{3000}{1/2\,(0.00238)\,300^2\,(6\cdot 36)} = 0.1297$$

Note that the area may not be $b\cdot\bar{c}$, but we have no alternative. Once we define our area to be that value, our answers are correct, based on that value. The quickest way to do this problem is to relate the moment at the cp ($C_{m_{cp}} = 0$) to the moment at the leading edge:

$$C_{m_{LE}} = C_{m_{cp}} + C_L(h_0 - h_{cp}) = 0 + 0.1297(0 - 0.4) = \underline{-0.0519}$$

4. $C_{m_{0L}} = 0.04$, $C_{m_\alpha} = -0.0064$ /deg, $\frac{W}{S} = 75$ lbs/ft $h = 0.3$, $C_{L_\alpha} = 0.08$ /deg

a)

$$C_m = C_{m_{0L}} + C_L(h - h_{n_w})$$

$$\frac{\partial C_m}{\partial \alpha} = C_{L_\alpha}(h - h_{n_w}) = -0.0064 = 0.08(h - h_{n_w})$$

$$h - h_{n_w} = \frac{-0.0064}{0.08} = -0.08$$

$$C_m = C_{m_{0L}} + C_L(h - h_{n_w}) = 0.04 + C_L(-0.08)$$

$$\boxed{C_L = 0.5}$$

b) $h - h_{n_w} = 0.3 - h_{n_w} \quad \Rightarrow \quad h_{n_w} = 0.38$

$$C_m = 0 = 0.04 + C_L(0.5 - 0.38) \quad \Rightarrow \quad C_L = -0.3333$$

Therefore it can not be balance for normal level flight (but it is stable for inverted flight at that lift coefficient).

Note: there are other ways to do some of these problems. These are the ways I did them, you may have found an easier (or more difficult way!).