AOE 3134 Problem Sheet Two (ans)

5. Lift-curve slope changes with Mach number. Data given: S = 2433 ft^2, b = 131 ft, c_r = 28.6 ft λ = 0.3, $\Lambda_{1/4}$ = 35 deg, M_a = 0.0 , 0.2, 0.4, 0.8. For sample calculation pick M_a = 0.4. First we need to calculate the half chord sweep angle and the aspect ratio for the lift-curve slope equation.

$$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n-m)\,\frac{1-\lambda}{1+\lambda}\right] \qquad \text{For } n = \frac{1}{2} \text{ and } m = \frac{1}{4}$$

$$\tan\Lambda_{1/2} = \tan 30 - \frac{4}{7.0534}\left[\left(\frac{1}{2}-\frac{1}{4}\right)\frac{1-0.3}{1+0.3}\right] = 0.6239$$

$$AR = \frac{b^2}{S} = \frac{131^2}{2433} = 7.0534$$

Then the lift curve slope is given by: (note: $k -= \dfrac{a_0}{2\pi} = \dfrac{2\pi}{2\pi} = 1$)

$$a_w = \frac{2\pi AR}{2 + \sqrt{\dfrac{AR^2(1-M_a^2)}{k^2}\left(1 + \dfrac{\tan^2\Lambda_{1/2}}{(1-M_a^2)}\right) + 4}}$$

$$a_w = \frac{2\pi 4}{2 + \sqrt{\dfrac{4^2}{1^2}(1-0.4^2)\left(1 + \dfrac{0.6239^2}{(1-0.4^2)}\right) + 4}} = 4.4002 \text{ /rad} = 0.0768 \text{ /deg}$$

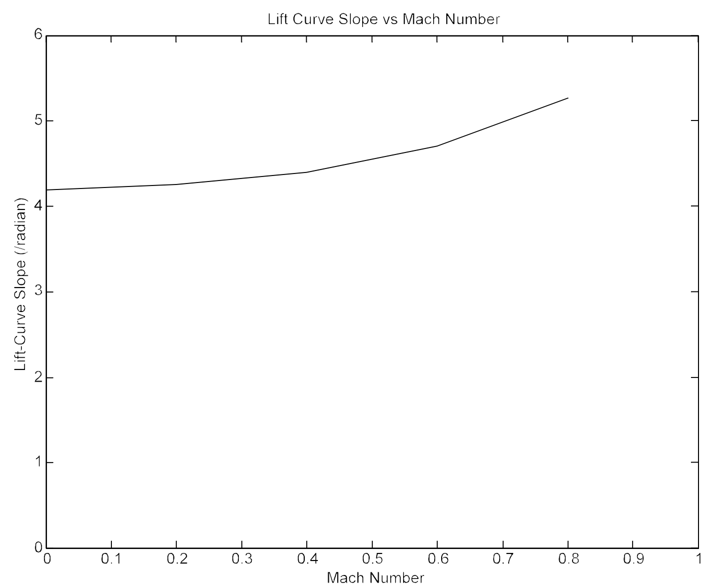
A simple MATLAB code to perform the Mach number sweep is displayed next:

```
%Wing lift-curve calculations
%Vary Mach number and calculate effect on lift-curve slope
s=2433;
b=131;
cr=28.6;
lambda=0.3;
Lambda14 = 35*pi/180;
ar=b^2/s;
k=1;
```

```
tanLambda12 = tan(Lambda14)-4/ar*((1/2-1/4)*(1-lambda)/(1+lambda))
fid=fopen('table1.txt','w');
fprintf(fid,'    ma            a\n'     );
for i = 1:5
    ma(i)=(i-1)*0.2;
    beta=sqrt(1-ma(i)^2);
    a(i)=(2*pi*ar)/(2+sqrt((ar*beta)^2/k^2*(1+tanLambda12^2/beta^2)+4));
    fprintf(fid,'%8.4f     %8.4f\n',ma(i),a(i));
end
plot(ma,a);
xlabel('Mach Number');
ylabel('Lift-Curve Slope (/radian)');
title('Lift Curve Slope vs Mach Number');
axis([0,1,0,6]);
fclose(fid);
```

It gives the following table:

ma	a
0.0000	4.2005
0.2000	4.2477
0.4000	4.4002
0.6000	4.6997
0.8000	5.2606



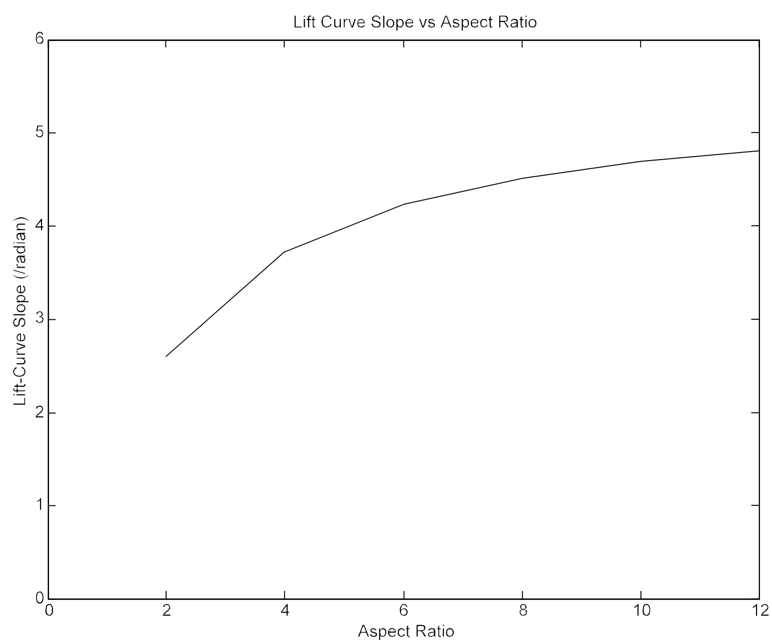
The aspect ratio effect is given by the following MATLAB file.

```
%Determine the effect of aspect ratio on the lift curve slope
%hold Mach nuber constant at 0.4 and vary aspect ratio
%Wing lift-curve calculations
s=2433;
b=131;
cr=28.6;
lambda=0.3;
Lambda14 = 35*pi/180;
ma=0.4;
```

```
k=1;
beta=sqrt(1-ma^2);
fid=fopen('table2.txt','w');
fprintf(fid,'    ar            a\n'     );
for i = 1:6
    ar(i)=i*2;
    tanLambda12 = tan(Lambda14)-4/ar(i)*((1/2-1/4)*(1-lambda)/(1+lambda));
    a(i)=(2*pi*ar(i))/(2+sqrt((ar(i)*beta)^2/k^2*(1+tanLambda12^2/beta^2)+4));
fprintf(fid,'%8.4f      %8.4f\n',ar(i),a(i));
end
plot(ar,a);
xlabel('Aspect Ratio');
ylabel('Lift-Curve Slope (/radian)');
title('Lift Curve Slope vs Aspect Ratio');
axis([0,12,0,6]);
fclose (fid);
```

with the results:

ar	a
2.0000	2.5928
4.0000	3.7236
6.0000	4.2343
8.0000	4.5153
10.0000	4.6917
12.0000	4.8124



7. Etkin and Reid, Problem 2.1 b = 150 ft. c_r = 25 ft, c_t = 12 ft. Λ_0 =26 deg

b) $\lambda = \dfrac{c_t}{c_r} = \dfrac{12}{25} = 0.48, \quad S = \dfrac{1}{2} b\, c_r (1 + \lambda) = \dfrac{1}{2} 150\,(25)\,(1 + 0.48) = 2775 \ \text{ft}^2$

$$AR = \frac{b^2}{S} = \frac{150^2}{2775} = 8.108$$

$$\bar{c} = \frac{2}{3} c_r \left(\frac{1 + \lambda + \lambda^2}{1 + \lambda} \right) = \frac{2}{3}(25)\, \frac{1+0.48+0.48^2}{1 + 0.48} = 19.26 \ \text{ft}$$

c)
$$m = \frac{b}{2}\,\frac{1}{3}\,\frac{1 + 2\lambda}{(1 + \lambda)} \tan \Lambda_0 = \frac{150}{2\cdot 3}\,\frac{1+2(0.48)}{1 + 0.48} = 16.15 \ \text{ft}$$

$$\bar{y} = \frac{b}{2}\,\frac{1 + 2\lambda}{3\,(1 + \lambda)} = \frac{150}{2 \cdot 3}\,\frac{1 + 2(0.48)}{1 + 0.48} = 33.11 \ \text{ft}$$

From Fig C.3, $h_{n_w} = 0.25$

Distance from apex to leading edge of MAC is 16.15 ft.
distance from leading edge of MAC to ac is 0.25(19.26) = 4.815 ft.
distance form apex to ac is 16.25+4.815 = 20.965 ft.
distance from apex to trailing edge of MAC = 16.15+19.26 = 35.41 ft (10.41 ft behind trailing edge of root chord!)

d) We need to calculate the lift curve slope of the wing (= that of the tail), the rearmost cg position and the distance back to the horizontal tail ac. We need the tangent of the sweep angle of the half chord. (See equation above):

$$\tan \Lambda_{1/2} = \tan 30 - \frac{4}{8.103}\left[\left(\frac{1}{2} - 0\right)\frac{1 - 0.48}{1 + 0.48}\right] = 0.401$$

The lift curve slope is given by (see equation above) Note: low speed $M_a \approx 0$

$$a_w = \frac{2\pi\,(8.108)}{2 + \sqrt{\dfrac{8.108^2}{1^2}(1-0^2)\,(1 + \dfrac{0.401^2}{1 - 0^2}) + 4}} = 4.647 \ \text{/rad} \qquad \left[= a_{ht} \qquad \text{(given)} \right]$$

Determine cg position, h

cg position is 25 ft behind apex of the wing, and the leading edge of the MAC is 16.15 fr behind the apex. Hence the cg is 25 - 16.15 = 8.85 ft behind the leading edge of the MAC. Therefor the cg position is given by

$$h = \frac{cg \ \text{distance}}{\bar{c}} = \frac{8.15}{19.26} = 0.46$$

The minimum static margin is given as:

$$K_n = h_n - h = 0.05 = h_n - 0.46 \qquad \Rightarrow \qquad h_n = 0.51$$

We can no locate the tail, h_{ht},

$$h_{ht} - h_{n_{wb}} = \frac{\bar{l}_{ht}}{\bar{c}} \qquad \Rightarrow \qquad h_{ht} = h_{n_{wb}} + \frac{\bar{l}_{ht}}{\bar{c}} = 0.25 + \frac{55}{19.26} = 3.106$$

But we have the neutral point equation:

$$h_n = \frac{h_{n_{wb}} + \frac{a_{ht}}{a_{wb}} \eta_{ht} \frac{S_{ht}}{S} h_{ht} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)}{1 + \frac{a_{ht}}{a_{wb}} \eta_{ht} \frac{S_{ht}}{S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)}$$

Simple algebra will allow us to solve for the horizontal tail size since everything else is known. The solution we get is a approximation to the minimum tail size for maintaining the required static margin at the rearmost cg position.

$$S_{ht} = S\left[\left(\frac{h_n - h)n_{wb}}{h_{ht} - h_n} \right) \frac{a_{wb}}{a_{ht}} \frac{1}{\eta_{ht} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)} \right]$$

With numbers:

$$S_{ht} = 2775\left[\left(\frac{0.51 - 0.25}{3.106 - 0.51} \right) (1) \frac{1}{(1)\,(\,1 - 0.25\,)} \right] = 370.57 \ \text{ft}^2$$

8. Given: S_w= 360 ft^2,$\bar{c}$ = 6 ft, $h_{n_w} = 0.24$, $i_{ht} = -2$ deg,□□$_w = 10$, $\alpha_{0L_w} = -3.0°$, $i_w = 0$, $AR_{ht} = 4$, $\eta_{ht} = 0.9$, $C_{m_{0L_w}} = -0.02$, $\alpha_{0L_{ht}} = 0$, $S_{ht} = 72 \ \text{ft}^2$, $h_{ht} = 3.24$, and $\frac{\partial \epsilon}{\partial \alpha} = 0.264$ Rectangular wing implies $\lambda = 1$, and $\Lambda_n = 0$ for all n

To find the neutral point we need to find the lift-curve slopes of the wing and tail, and the change in downwash with respect to angle of attack (it was given, but it could have been estimated which will be done here, ie verify the value given!) A glider is low speed so we can assume $M_a = 0$

$$a_x = \frac{2\pi AR_x}{2 + \sqrt{\frac{AR_x^2}{k^2}(1 - M_a^2)\left(1 \frac{\tan^2 \Lambda_{1/2_x}}{(1 - M_a^2)}\right) + 4}}$$

Where x = w or ht. For the wing:

$$a_w = \frac{2\pi(10)}{2 + \sqrt{\frac{10^2}{1^2}(1 - 0^2)\left(1 + \frac{0^2}{(1 - 0^2)}\right) + 4}} = 5.151 \ \text{/rad}$$

For the tail:

$$a_{ht} = \frac{2\pi(4)}{2 + \sqrt{\frac{4^2}{1^2}(1 - 0^2)\left(1 + \frac{0^2}{(1 - 0^2)}\right) + 4}} = 3.883 \ \text{/rad}$$

Estimating the downwash parameter:

$$\frac{\partial \epsilon}{\partial \alpha} = 0.44\left[K_a K_\lambda K(\cos\Lambda_{1/4})^{1/2}\right]^{1.19}$$

where:

$$K_a = \frac{1}{AR} - \frac{1}{1 + AR^{1.7}} = \frac{1}{10} - \frac{1}{1 + 10^{1.7}} = 0.08$$

$$K_\lambda = \frac{10 - 3\lambda}{7} = \frac{10 - 3(1)}{7} = 1$$

$$K_H = \frac{1 - \left|\frac{H_{ht}}{b}\right|}{\sqrt[3]{\frac{2\bar{l}_{ht}}{b}}} = \frac{1 - 0}{\sqrt[3]{\frac{2(3.24)6}{60}}} = 1.183$$

$$\frac{\partial \epsilon}{\partial \alpha} = 4.44\left[0.08(1)\,1.183\,(\cos 0)^{1/2}\right]^{1.19} = 0.268 \approx 0.264$$

(We will use 0.264 since it was given!)

$$h_n = \frac{h_{n_w} + \frac{a_{ht}}{a_w}\eta_{ht}\frac{S_{ht}}{S}h_{ht}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)}{1 + \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)} = \frac{0.24 + \frac{3.883}{5.151}(0.9)\,72over360\,(3.24)\,(1 - 0.264)}{1 + \frac{3.883}{5.151}(0.9)\frac{72}{360}(1 - -.264)}$$

$$h_n = 0.5124$$