AOE 3134 Problem Sheet 3 (answers)

All the data pertained to the following aircraft: h = 0.2.

Given: W= 10,000 lbs	$h_{n_{wb}} = 0.25$	$\dfrac{\partial \epsilon}{\partial \alpha} = 0.4$	$a_{wb} = 0.09$ /deg
$S = 250\ \mathrm{ft}^2$	$S_{ht} = 50\ \mathrm{ft}^2$	$i_w = 0$ deg $\quad \bar{c} = 8$ ft	$a_{ht} = 0.06$ /deg
$C_{m_{ac_{wb}}} = -0.03$	$h = 0.20$	$i_{ht} = -2$ deg	$\epsilon_0 = \alpha_{0L_{wb}} = 0$
$b = 40$ ft	$l_{ht} = 20$ ft	$a_e = 0.04$ /deg	$\lvert \delta_e \rvert = 30$ deg

The governing equations for this problem are given by:

$$\frac{W}{1/2\, \rho\, V^2 S} = C_L = C_{L_{wb}} + C_{L_{ht}} \eta_{ht} \frac{S_{ht}}{S}$$

$$0 = C_m = C_{m_{0L_{wb}}} + C_{L_{wb}} (h - h_{n_{wb}}) - C_{L_{ht}} \eta_{ht} \frac{S_{ht}}{S} \frac{l_{ht}}{\bar{c}}$$

We can define $C_{L_{ht}} \eta_{ht} \dfrac{S_{ht}}{S} = \left(C_L\right)_{ht}$ as the contribution to the total aircraft lift from the tail. We now have two equations in the two unknowns $C_{L_{wb}}$ and $\left(C_L\right)_{ht}$. If we write the two equations in matrix form, we can solve by inverting the coefficient matrix. The matrix from of the equations is

$$\begin{bmatrix} 1 & 1 \\ (h - h_{n_{wb}}) & -\dfrac{l_{ht}}{\bar{c}} \end{bmatrix} \begin{Bmatrix} C_{L_{wb}} \\ \left(C_L\right)_{ht} \end{Bmatrix} = \begin{Bmatrix} C_L \\ -C_{m_{0L_{wb}}} \end{Bmatrix}$$

Note the determinant of this matrix is $\Delta_1 = -\left[\dfrac{l_{ht}}{\bar{c}} + h - h_{n_{wb}}\right] = -(h_{ht} - h_{n_{wb}})$. We can also note that $\dfrac{l_{ht}}{\bar{c}} = h_{ht} - h$. The solution becomes:

$$\begin{Bmatrix} C_{L_{wb}} \\ \left(C_L\right)_{ht} \end{Bmatrix} = \frac{1}{\Delta_1} \begin{bmatrix} \dfrac{l_{ht}}{\bar{c}} & 1 \\ (h - h_{n_{wb}} & -1 \end{bmatrix} \begin{Bmatrix} C_L \\ -C_{m_{0L_{wb}}} \end{Bmatrix}$$

Rewriting and carrying out the operations indicated we get the equations for the lift coefficients:

$$C_{L_{wb}} = \frac{h_{ht} - h}{h_{ht} - h_{n_{wb}}} C_L - \frac{C_{m_{0L_{wb}}}}{h_{ht} - h_{n_{wb}}}$$

$$\left(C_L\right)_{ht} = \frac{h - h_{n_{wb}}}{h_{ht} - h_{n_{wb}}} C_L + \frac{C_{m_{0L_{wb}}}}{h_{ht} - h_{n_{wb}}}$$

We can evaluate some terms:

$h_{ht} = \frac{l_{ht}}{\bar{c}} + h = \frac{20}{8} + 0.2 = 2.70$ $\qquad\qquad \Delta_1 = h)ht - h)n_{wb} = 2.70 - 0.25 = 2.45$

Then

$$C_{L_{wb}} = \frac{2.7 - h}{2.45} C_L + 0.01224$$

$$\left(C_L\right)_{ht} = \frac{h - 0.25}{2.45} C_L - 0.01224$$

We can relate the lift coefficient to the velocity by:

$$C_L = \frac{W}{1/2\, \rho\, V^2 S} = \frac{10000}{1/2(0.00238)\, 250} \frac{1}{V^2} = \frac{33613.4453}{V^2}$$

For the case of $h = 0.2,\ 0.25,$ and 0.3 we have,

cg location h = 0.2000

V	CL	CLwb	CLht`	CLwfrac	CLhtfrac
100.00	3.3613	3.4422	-0.0808	1.0241	-0.0241
150.00	1.4939	1.5367	-0.0427	1.0286	-0.0286
200.00	0.8403	0.8697	-0.0294	1.0350	-0.0350
250.00	0.5378	0.5610	-0.0232	1.0432	-0.0432
300.00	0.3735	0.3933	-0.0199	1.0532	-0.0532
350.00	0.2744	0.2922	-0.0178	1.0650	-0.0650
400.00	0.2101	0.2266	-0.0165	1.0787	-0.0787
450.00	0.1660	0.1816	-0.0156	1.0942	-0.0942

cg location h = 0.2500

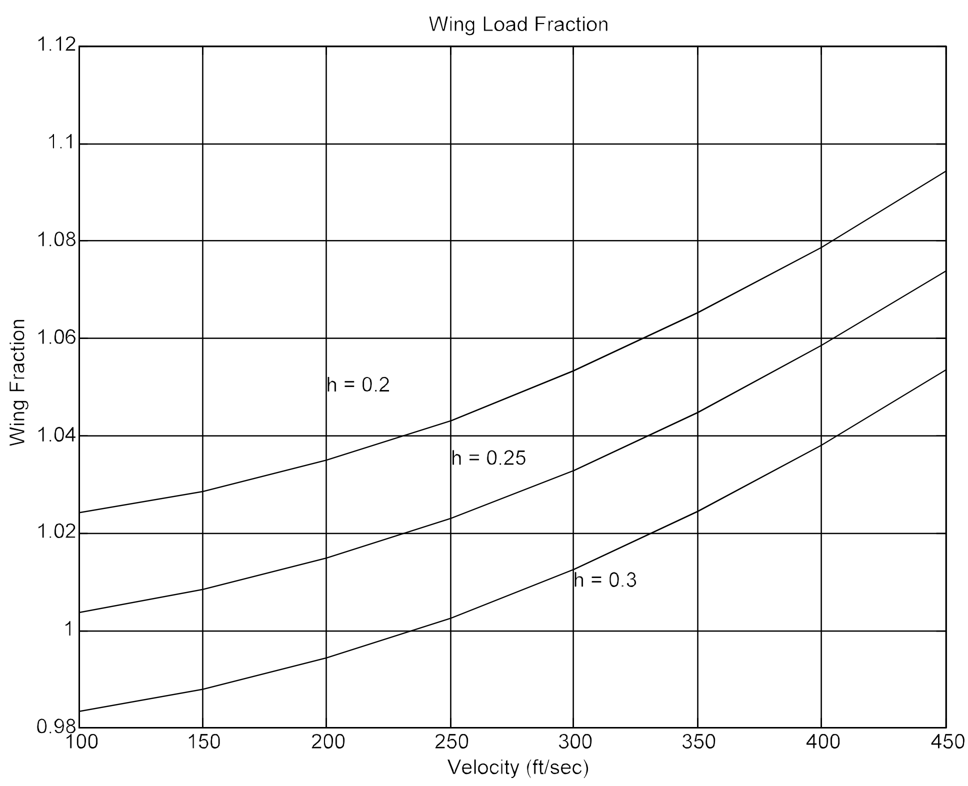
V	CL	CLwb	CLht`	CLwfrac	CLhtfrac
100.00	3.3613	3.3736	-0.0122	1.0036	-0.0036
150.00	1.4939	1.5062	-0.0122	1.0082	-0.0082
200.00	0.8403	0.8526	-0.0122	1.0146	-0.0146
250.00	0.5378	0.5501	-0.0122	1.0228	-0.0228
300.00	0.3735	0.3857	-0.0122	1.0328	-0.0328

350.00	0.2744	0.2866	-0.0122	1.0446	-0.0446
400.00	0.2101	0.2223	-0.0122	1.0583	-0.0583
450.00	0.1660	0.1782	-0.0122	1.0738	-0.0738

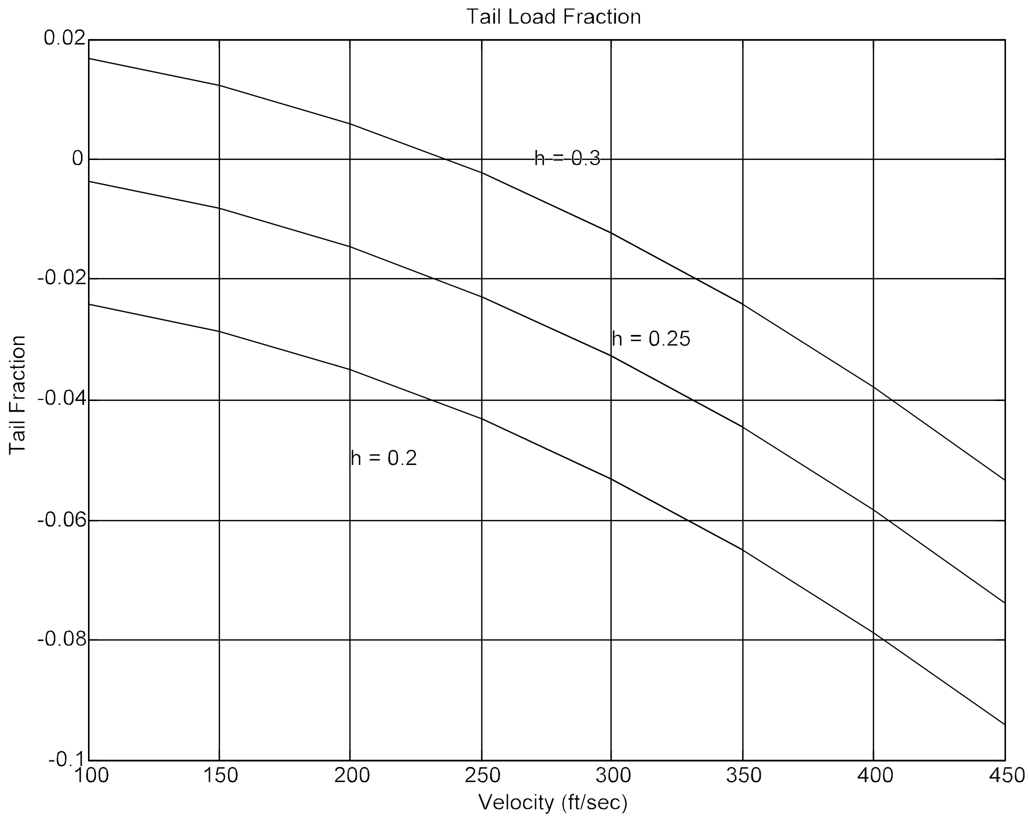
cg location h = 0.3000

V	CL	CLwb	CLht`	CLwfrac	CLhtfrac
100.00	3.3613	3.3050	0.0564	0.9832	0.0168
150.00	1.4939	1.4757	0.0182	0.9878	0.0122
200.00	0.8403	0.8354	0.0049	0.9942	0.0058
250.00	0.5378	0.5391	-0.0013	1.0024	-0.0024
300.00	0.3735	0.3781	-0.0046	1.0124	-0.0124
350.00	0.2744	0.2810	-0.0066	1.0242	-0.0242
400.00	0.2101	0.2180	-0.0080	1.0379	-0.0379
450.00	0.1660	0.1748	-0.0089	1.0534	-0.0534

with the associated graphs:



b)



We can see for aft cg positions, the tail load can be positive, while for forward cg positions, the tail load is generally negative for all speeds.

10. a) We can compute the stick fixed neutral point from our basic neutral point equation:

$$h_n = \frac{h_{n_{wb}} + \dfrac{a_{ht}}{a_{wb}} \eta_{ht} \dfrac{S_{ht}}{S} h_{ht} \left(1 - \dfrac{\partial \epsilon}{\partial \alpha} \right)}{1 + \dfrac{a_{ht}}{a_{wb}} \eta_{ht} \dfrac{S_{ht}}{S} \left(1 - \dfrac{\partial \epsilon}{\partial \alpha} \right)}$$

Everything is known or has been can be calculated

$$h_n = \frac{0.25 + \dfrac{0.06}{0.09} (1) \dfrac{50}{250} 2.7 (1 - 0.4)}{1 + \dfrac{0.06}{0.09} (1) \dfrac{50}{250} (1 - 0.4)} = \underline{0.4315}$$

b)

Static Margin = $K_n = h_n - h = 0.4315 - 0.2 = 0.2315$ or 23.15% of mac

c) The max forward cg position:

$$h_{\min} = -\frac{\Delta}{a\,C_{L_{Land}}}\left[\frac{C_{m_{0L}}\,C_{L_\alpha}}{\Delta} - \left|\delta_e\right|_{\max\ up}\right]$$

We need to determine $\Delta, C_{m_{0L}}$, and a.

$$a = C_{L_\alpha} = a_{wb}\left[1 + \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right] = 0.09\left[1 + \frac{0.06}{0.09}(1)\frac{50}{250}(1 - 0.4)\right]$$

$$a = 0.0972\ \text{/deg}$$

$$C_{m_{\delta_e}} = -a_e\,\eta_{ht}\frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}} = -0.04\,(1)\,\frac{50}{250}\,\frac{20}{8} = -0.02\ \text{/deg}$$

$$C_{L_{\delta_e}} = a_e\,\eta_{ht}\frac{S_{ht}}{S} = 0.04\,(1)\,\frac{50}{250} = 0.008\ \text{/deg}$$

$$C_{m_\alpha} = a\,(h - h_n) = 0.0972\,(0.2 - 0.4315) = -0.0225\ \text{/deg}$$

$$\Delta = C_{L_\alpha}C_{m_{\delta_e}} - C_{m_\alpha}C_{L_{\delta_e}} = (0.0972)(-0.02) - (-0.0225)(0.008) = -0.001764$$

$$C_{m_{0L}} = C_{m_{0L_{wb}}} + a_{ht}\,\eta_{ht}\frac{S_{ht}}{S}(h_{ht} - h_{n_{wb}})\frac{a_{wb}}{a}(i_w - \alpha_{0L_{wb}} - i_{ht} + \epsilon_{0L_{wb}} + \alpha_{0L_{ht}})$$

$$= -0.03 + 0.06\,(1)\,\frac{50}{250}(2.7 - 0.25)\frac{0.09}{0.0972}(0 - 0 - (-2) + 0 + 0) = \underline{0.0244}$$

$$C_{L_{land}} = \frac{10000}{1/2\,(0.00238)\,(150)^2\,250} = 1.4939$$

$$h_{\min} = 0.4215 - \frac{-0.001764}{0.0972\ 1.4939}\left[\frac{0.0244\ 0.0972}{-0.001764} - 30\right] = 0.0507$$

11. The angle of attack and elevator deflection equations can be written in the following form:

$$\bar{\alpha} = \frac{C_{m_{0L}}C_{L_{\delta_e}}}{\Delta} + \frac{C_{m_{\delta_e}}}{\Delta}C_L = \frac{0.0244\ 0.008}{-0.001764} + \frac{-0.02}{-0.001764}C_L = -0.1109 + 11.3379\,C_L$$

or

$$\bar{\alpha} = -0.1109 + \frac{11.3379\,(33613.4454)}{V^2} = -0.1109 + \frac{381105.88}{V^2}$$

Similarly for the elevator deflection:

$$\delta_e = -\frac{C_{m_{0L}} C_{L_\alpha}}{\Delta} - \frac{C_{m_\alpha}}{\Delta} C_L = -\frac{0.0244\,(0.0972)}{-0.001764} - \frac{-0.0225}{-0.001764} C_L$$

$$= 1.3469 - \frac{428742.8574}{V^2}$$

For these values the following table can be created:

V	alpha	delta_e
100.0000	37.9997	-41.5274,
150.0000	16.8271	-17.7083,
200.0000	9.4167	-9.3717,
250.0000	5.9868	-5.5130,
300.0000	4.1236	-3.4169,
350.0000	3.0002	-2.1530,
400.0000	2.2710	-1.3327,
450.0000	1.7711	-0.7703

Clearly we run out of elevator deflection between 100 and 150 ft/sec. We can use the elevator equation to determine the exact speed (assuming no limit on maximum C_L)

$$-30 = 1.3469 - \frac{428742.8574}{V^2} \quad \Rightarrow \quad V = 116.95 \text{ ft/sec}$$

Note that this value corresponds to a lift coefficient, $C_L = 2.4576$.

12. If we shift the cg. to the aft position, $h = 0.3$, then if we examine the elevator equation, we see that only the term C_{m_α} is affected. The denominator term, Δ, is invariant with the cg movement although at firs glance it appears that is should change since it contains C_{m_α}! However it also contains $C_{m_{\delta_e}}$ that also changes with cg. position is such a way that the denominator is constant with cg. motion. The new value of the pitch stability parameter is

$$C_{m_\alpha} = a\,(h - h_n) = 0.0972\,(0.3 - 0.4315) = -0.0128 \text{ /deg}$$

Then

$$\delta_e = 1.3469 - \frac{-0.0128}{-0.001764} C_L = 1.3469 - 7.2562\, C_L$$

Putting in the limiting value of the elevator (max up = -30 deg) we get

$$-30 = 1.3469 - 7.2562\, C_{L_{\min\ speed}} \quad \Rightarrow \quad C_{L_{\min\ speed}} = 4.3200$$

$$C_L = 4.3200 = \frac{10000}{1/2\,(0.00238)\,250} \frac{1}{V_{\min}^2} \quad \Rightarrow \quad V_{\min} = 88.2092 \text{ ft/sec}$$

This lift coefficient is likely beyond stall. The important thing to note here is that as the cg. moves aft, the range of speeds that the aircraft can fly in equilibrium ($C_m = 0$) increases and the "elevator gradient," the change of elevator angle per change in velocity, decreases (elevator deflection required for a given change in velocity is smaller than when cg is forward)..

b) What really determines the aircraft lift coefficient is when the wing stalls. As we have seen from problem 9, the load on the wing is generally more than the weight of the plane. Hence we might expect the aircraft to stall at a higher speed than we suggest using the wing stall lift coefficient as the aircraft lift coefficient. - Not conservative.