Problem Sheet Four (Solution)

(13-17) Estimate the neutral point of the F104 from drawing given.

In order to determine dimensions from the drawing, it is necessary to scale some known dimensions from it. Here we can pick the wing span which is given, and can be measured. I used millimeters since they are done in units of 10 and are relatively small. The scale factor and other measurements taken from the drawing are:

b = 31mm = 21.94 ft ⇒ Scale = 1.413 mm/ft = 0.7077 ft/mm

Then, from measured and calculated values:

Wing:

$$c_t = 7 \text{ mm} = 4.9539 \text{ ft}, \quad c_r = 18 \text{ mm} = 12.7386 \text{ ft}$$

$$\lambda = \frac{c_t}{c_r} = = \frac{4.9539}{12.7386} = 0.3889, \quad AR = \frac{b^2}{S} = \frac{21.94^2}{196.1} = 2.4547$$

Tail:

$$c_{t_{ht}} = 3 \text{ mm} = 2.1231 \text{ ft}, \quad c_{r_{ht}} = 8 \text{ mm} = 5.6616 \text{ ft}$$

$$\lambda_{ht} = \frac{c_{t_{ht}}}{c_{r_{ht}}} = \frac{2.1231}{5.6616} = 0.3750, \quad b_{ht} = 16.5 \text{ mm} = 11.6770 \text{ ft}$$

$$\Lambda_{LE} = 27 \text{ deg}$$

$$S_{ht} = \frac{1}{2}(c_{t_{ht}} + c_{r_{hr}})b = \frac{1}{2}(2.1231 + 5.6616)11.6770 = 45.4510 \text{ ft}^2$$

$$AR_{ht} = \frac{b_{ht}^2}{S_{ht}} = \frac{11.6770^2}{45.4510} = 3.0000, \ \Lambda_{LE_{ht}} = 15 \text{ deg}$$

SUBSONIC

Wing Calculations:

$$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n - m)\frac{1 - \lambda}{1 + \lambda}\right]$$

$$= \tan 27 - \frac{4}{2.4547}\left[(\frac{1}{2} - 0)\frac{1 - 0.3889}{1 + 0.3889}\right]$$

$$= 0.1510 \quad \Rightarrow \quad 8.59 \ deg$$

Lift curve slope of wing:

$$a_{wb} = \frac{2\,\pi\,AR}{2 + \sqrt{\frac{AR^2}{k^2}(1 - M_a^2)\left(1 + \frac{\tan^2\Lambda_{1/2}}{(1 - M_a^2)}\right) + 4}}$$

$$= \frac{2\,\pi\,(2.4547)}{2 + \sqrt{\frac{2.4547^2}{1^2}(1 - 0.257^2)\left(1 = \frac{0.151^2}{(1 - 0.257^2)}\right) + 4}}$$

$$= 3.0095 \;\; /\text{rad}$$

Aerodynamic Center of wing:

$$\tan\Lambda_{1/4} = \tan 27 - \frac{4}{2.4547}\left[(\frac{1}{4} - 0)\,\frac{1 - 0.3889}{1 + 0.3889}\right]$$

$$= 0.3303 \qquad \Rightarrow \qquad \Lambda_{1/4} = 18.28 \;\; \text{deg}$$

Possible Approach

From Fig C.3 (which is for low speed flight (really for M_a= 0) and entering the chart with

$$\lambda = 0.3889, \;\; \Lambda_{1/4} = 18.28, \;\; \text{and} \;\; AR = 2.4547$$

However since there are only charts for $\lambda = 0$ and $\lambda = 0.5$ we must enter the each of these charts and interpolate: We have:

$$\lambda = 0.5 \qquad \lambda = 0.0$$
$$h_{n_w} = 0.24 \qquad h_{n_w} = 0.255$$

$$h_{n_w} = \frac{0.24 - 0.255}{0.5 - 0.0}(0.3889 - 0) + 0.255 = 0.243$$

Alternative Approach

Since we have charts for calculating the wing (or tail) aerodynamic center that includes the effects of Mach number, we should use them.

From the handouts. We need the following parameters:

$$AR\sqrt{1 - M_a^2} = 2.4547\,\sqrt{1 - 0.257^2} = 2.3721$$
$$AR\tan\Lambda_{1/2} = 2.547\,(0.1510) = 0.3707$$

From the two charts for $\lambda = 0.25$ and $\lambda = 0.5$ we have:

$\lambda = 0.25 \qquad \lambda = 0.5$
$h_{n_w} = 0.245 \qquad h_{n_w} = 0.235$ We can interpolate between these two values for our $\lambda = 0.3889$

$$h_{n_w} = -\frac{0.245 - 0.235}{0.25 - 0.05}(0.25 - 0.3889) + 0.245 = \underline{0.2394}$$

We will use this one.

Horizontal tail properties:

$$\tan\Lambda_{1/2} = \tan 15 - \frac{4}{3}.000\left[\left(\frac{1}{4} - 0\right)\frac{1 - 0.375}{1 + 0/375}\right]$$

$$= -0.0351 \qquad \Rightarrow \qquad \Lambda_{1/2} = -2.0092 \text{ deg}$$

$$a_{ht} = \frac{2\pi(3.0)}{2 + \sqrt{\frac{3.0^2}{1^2}(1 - 0.257^2)\left(1 + \frac{-0.03512^2}{(1 - 0.257^2)}\right) + 4}}$$

$$= 3.4125 \text{ /rad}$$

We can locate the tail aerodynamic center using the hand out charts (or if desperate, Fig C.3):
$AR\sqrt{1 - M_a^2} = 3.0\sqrt{1 - 0.257^2} = 2.8992$
$AR \tan\Lambda 1/2 = -0.0351$

Entering the two charts for $\lambda = 0.25$ and $\lambda = 0.5$ at these two values, we get very close to the same values:

$h_{n_{ht}} = 0.2350$ (Note that this value is not h_{ht} that we use for the location of the tail ac in wing chord lengths from LE of wing mean aerodynamic chord in C_{m_α} or $C_{m_{0L}}$ calculations)

We now need to locate the mean aerodynamic chords and the aerodynamic centers of the wing and horizontal tail with respect to known positions on the figure. We will locate these with respect to the apex of the wing and horizontal tail surfaces.

Wing:
Location of the leading edge of the mean aerodynamic chord with respect to the apex of the wing (or the leading edge of the root chord)

$$\bar{m} = \frac{b}{2}\,\frac{1 + 2\lambda}{3(1 + \lambda)}\tan\Lambda_{LE}$$

$$= \frac{21.94}{2}\,\frac{(1 + (2)0.3889)}{3(1 + 0.3889)}\tan 27$$

$$= 2.3849 \text{ ft}$$

The aerodynamic center location with respect to the apex of the wing (or LE of wing root chord)

$$d_{ac_{wing}} = \bar{m} =+ h_{n_w}\bar{c}$$

$$= 2.3849 + 0.2394 * 9.55$$

$$= 4.6712 \text{ ft}$$

Tail:
Location of the leading edge of the mean aerodynamic chord with respect to the apex of the horizontal tail (or the leading edge of the root chord)

$$\bar{m}_{ht} = \frac{11.68}{2}\,\frac{1 + (2)\,0.375}{3(1 + 0.375)}\tan 15$$

$$= 0.6639 \text{ ft}$$

$$\bar{c}_{ht} = \frac{2}{3}C_{r_{ht}}\left(\frac{1 + \lambda_{ht} + \lambda_{ht}^2}{1 + \lambda_{ht}}\right)$$

$$= \frac{2}{3}(5.6616)\left(\frac{1 + 0.375 + 0.375^2}{1 + 0.375}\right)$$

$$= 4.1604 \text{ ft}$$

The aerodynamic center location with respect to the apex of the horizontal tail is:

$$d_{ac_{ht}} = 0.6624 + 0.235(4.1604) = 1.6416 \text{ ft}$$

cg location:
Distance from wing apex:

$$d_{cg} = \bar{m} + h\bar{c} = 2.3849 + 0.07(9.55) = 0.3053 \text{ ft}$$

Estimate $\frac{\partial \epsilon}{\partial \alpha}$:

$$\frac{\partial \epsilon}{\partial \alpha} = 4.44\left[K_A K_\lambda K_H (\cos\Lambda_{1/4})^{1/2}\right]^{1.19}$$

$$K_A = \frac{1}{AR} - \frac{1}{1 + AR^{1.7}} = \frac{1}{2.4547} - \frac{1}{1 + 2.4547^{1.7}} = 0.2289$$

$$K_\lambda = \frac{10 - 3\lambda}{7} = \frac{10 - 3(0.3889)}{7} = 1.2619$$

In order to determine K_H we need to find $\bar{l}_{ht}$, the distance between the aerodynamic centers of the wing and the tail. This quantity can be determined by first measuring the distance between the wing and the tail apex points.

$$d_{apex} = 29 \text{ mm} * 0.7077 = 20.5233 \text{ ft}$$

The distance between the aerodynamic centers is then given by:

$$\bar{l}_{ht} = d_{apex} - d_{ac_{wing}} + d_{ac_{ht}} = 20.5233 - 4.6712 + 1.6416 = 17.4937 \text{ft}$$

While we are at it we can compute the tail length, l_{ht}, the distance between the cg and the tail aerodynamic center.

$$l_{ht} = d_{apex} - d_{cg} + d_{ac_{ht}} = 20.5233 - 3.0534 + 1.6416 = 19.1115 \text{ ft}$$

$$K_H = \frac{1 - \left|\frac{H_{ht}}{b}\right|}{\sqrt[3]{\frac{2\bar{l}_{ht}}{b}}}$$

From measurement: $H_{ht} = 11 \text{ mm}(0.7077) = 7.7847 \text{ ft}$

$$K_H = \frac{1 - \frac{7.7847}{21.94}}{\sqrt[3]{\frac{2(17.4937)}{21.94}}} = 0.5522$$

$$\left.\frac{\partial \epsilon}{\partial \alpha}\right|_{inc} = 4.44\,[\,0.2289\,(1.2619)\,0.5522\,(\cos 18.28°)^{1/2}\,]^{1.19} = 0.4844$$

This value is the incompressible (low speed) value. It should be corrected for compressibility effects. They won't be large for this problem since the Mach number is only $M_a = 0.257$. The correction is given by (See Appendix B.5):

$$\frac{\partial \epsilon}{\partial \alpha} = \left.\frac{\partial \epsilon}{\partial \alpha}\right|_{inc} \frac{\left.C_{L_\alpha}\right|_M}{\left.C_{m_\alpha}\right|_{inc}}$$ where $(\,\cdot\,)|_M$ means the value at Mach number of interest

We can use one of two methods to make this correction. We have the lift curve slope of the wing corrected for Mach number (also sweep and AR) and we can use the same equation with $M_a = 0$ to get the incompressible value:

$$a_{wing_{inc}} = \frac{2\,\pi\,AR}{2 + \sqrt{\dfrac{AR^2}{k^2}\,(1 + \tan^2\Lambda_{1/2}) + 4}}$$ where here $k \doteq 1$

$$a_{w_{inc}} = \frac{2\,\pi\,(2.4547)}{2 + \sqrt{\dfrac{2.4547^2}{1^2}\,(1 + 0.1510^2) + 4}} = 2.9729\ /\text{rad}$$

The corrected value is:

$$\frac{\partial \epsilon}{\partial \alpha} = 0.4844\,\frac{3.0090}{2.9729} = \underline{0.4903}$$ We will use this one.

Alternatively we could have applied the Prandtl-Glauert correction for compressibility that is given by:

$$(\,\cdot\,)_M = \frac{(\,\cdot\,)_{inc}}{\sqrt{1 - M_a^2}}$$ where $(\,\cdot\,)$ could be a_w, $\dfrac{\partial \epsilon}{\partial \alpha}$, or some other aerodynamic property

Here we have: $\dfrac{\partial \epsilon}{\partial \alpha} = \dfrac{0.4844}{\sqrt{1 - 0.257^2}} = 0.5012$ Not much difference here.

Aircraft Lift-Curve-Slope, $C_{L_\alpha} = a$,

$$a = a_{wb}\left[1 + \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$= 3.0095\left[1 + \frac{3.4125}{3.0095}(1)\frac{45.451}{196.1}(1 - 0.4903)\right]$$

here we will assume $\eta_{ht} = 1$ (why?)

$$= 3.4126 \text{ /rad}$$

Aircraft Pitch-Curve-Slope C_{m_α}

$$C_{m_\alpha} = a_{wb}\left[(h - h_{n_{wb}}) - \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$= 3.0095\left[0.07 - 0.2394 - \frac{3.4125}{3.0095}\frac{45.451}{196.1}\frac{19.1115}{9.55}(1 - 0.4903)\right]$$

$$= -1.3166 \text{ /rad} = -0.0230 \text{ /deg}$$

Aircraft Neutral Point

$$C_{m_\alpha} = a(h - h_n) \quad \Rightarrow \quad h_n = h - \frac{C_{m_\alpha}}{a}$$

$$\boxed{h_n = 0.07 - \frac{-1.3166}{3.4126} = 0.4558}$$

For completion, the lift coefficient:

$$C_L = \frac{W}{1/2\,\rho V^2 S} = \frac{16300}{(1/2)\,0.002378\,(0.257*1116.45)^2\,196.1} = 0.8481$$

Supersonic

The supersonic calculations are more table or chart look-ups. We need to calculate the parameters with which to enter the charts.

$$C_L = \frac{W}{1/2\,\rho V^2 S} \qquad V = M_a a = 1.8\,(968.08) = 1742.54 \text{ ft/sec}$$

$$C_L = \frac{16300}{1/2\,(0.000286)\,1742.54^2\,(196.1)} = 0.1914$$

Wing properties: ($\lambda = 0.3889$)

$$AR\sqrt{M_a^2 - 1} = 2.4547\sqrt{1.8^2 - 1} = 3.6738$$

$$AR\tan\Lambda_{1/2} = 2.4547\,(0.151) = 0.3706$$

From Fig C.5 $\frac{1}{AR}\frac{dC_L}{d\alpha} =$

$\lambda = 0.25$	$\lambda = 0.5$
1.05	1.00

$$\frac{1}{AR}\frac{dC_L}{d\alpha} = \frac{1.05 - 1.00}{0.25 - 0.5}(0.25 - 0.3889) + 1.05 = 1.03$$

$$C_{L_\alpha} = 1.03\,AR = 1.03\,(2.4547) = 2.5283 \text{ /rad}$$

Wing Aerodynamic Center

$$AR\sqrt{M_a^2 - 1} = 2.4547\sqrt{1.8^2 - 1} = 3.6738$$

$$AR\tan\Lambda_{1/2} = 2.4547\,(0.151) = 0.3706$$

For :

$\lambda = 0.25$ and $\lambda = 0.5$ $\underline{h_{n_w} = 0.48}$

Tail properties: ($\lambda = 0.375$)

$$AR\sqrt{M_a^2 - 1} = 3.0\sqrt{1.8^2 - 1} = 4.4900$$

$$AR\tan\Lambda_{1/2} = 3.0\,(-0.351) = -0.1053$$

For $\lambda = 0.25$ and $\lambda = 0.5$

$$\frac{1}{AR}\frac{dC_L}{d\alpha} = 0.82$$

$$\frac{dC_L}{d\alpha} = 3.0\,(0.82) = 2.46 \text{ /rad}$$

Tail Aerodynamic Center:

$$AR\sqrt{M_a^2 - 1} = 3.0\sqrt{1.8^2 - 1} = 4.4900$$

$$AR\tan\Lambda_{1/2} = 3.0\,(-0.351) = -0.1053$$

For:

$\lambda = 0.25$ and $\lambda = 0.5$ $\underline{h_{n_{ht}} = 0.485}$

Tail length, l_{ht}:

l_{ht} = distance from wing to tail apex - distance to cg from wing apex + distance to tail ac from tail apex

$$l_{ht} = d_a - (\bar{m}_w + h_{n_w}\bar{c}) + (\bar{m}_{ht} + h_{n_{ht}}\bar{c}_{ht})$$

$$l_{ht} = 21.8725 - (2.3849 + 0.07\,(9.55)) + 0.6639 + 0.485\,(4.1604) = 21.5008 \text{ ft}$$

Given: $\frac{\partial\epsilon}{\partial\alpha} = 0.18$

2)

$$a = a_{wb}\left[1 + \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$= 2.5283\left[1 + \frac{2.46}{2.5283}(1)\frac{45.451}{196.1}(1 - 0.18)\right]$$

$$= 2.996 \text{ /rad}$$

3)

$$C_{m_\alpha} = a_{wb}\left[h - h_{n_{wb}} - \frac{a_{ht}}{a_{wb}}\eta_{ht}\frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$= 2.5283\left[0.07 - 0.48 - \frac{2.46}{2.5283}\,\frac{45.451}{196.1}\,\frac{21.5008}{9.55}(1 - 0.18)\right]$$

$$= -2.0892 \text{ /rad}$$

4)

$$h_n = h - \frac{C_{m_\alpha}}{a}$$

$$= 0.07 - \frac{-2.0892}{2.996}$$

$$= 0.7673$$

$$K_n = 0.7673 - 0.07 = 0.697$$