Problem Sheet Five (answers) 18. Atmospheric Properties: Sea level  $\rho = 0.002377 \text{ slugs/ft}^3$  a = 1116.4 ft/sec  $40 \text{ k ft.} - \rho = 0.000587 \text{ slugs/ft}^3$  a = 968.08 ft/secSea level:  $V = M_a a = 0.2 (1116.4) = 223.28 \text{ ft/sec}$   $C_L = \frac{W}{1/2\rho V^2 S} = \frac{38200}{1/2(0.002377) 223.28^2(545.5)} = 1.1819$ a)  $\frac{L}{D} = \frac{C_L}{C_D} = \frac{1.1819}{0.095} = 12.4410$  b)  $D = \frac{W}{L/D} = \frac{38200}{12.4410} = \frac{3070.48 \text{ lbs}}{12.4410}$ c)  $C_{m_a} = a (h - h_n) \rightarrow h_n = h - \frac{C_{m_a}}{a} = 0.25 - \frac{-0.8}{5.0} = 0.4100$ d)  $\frac{\Delta \delta_e}{n-1} = -\frac{C_w \left[ C_{m_a} + (C_{L_a} C_{L_q} - C_{m_a} C_{L_q}) \frac{\rho S \overline{c}}{4m} \right]}{C_{L_a} C_{m_b} - C_{m_a} C_{L_b}}$   $= -\frac{1.1819 \left[ -0.8 + [5.0(-8.0) - (0.8)(0)] \frac{-.002377(545.5)10.93 (32.174)}{4 (38200)} \right]}{5.0(-0.81) - 0.8(0.4)}$  = -0.2913 rad/g = -16.69 °/gNote:  $\Delta = -3.7300 / \text{rad}^2$   $\frac{\rho S \overline{c}}{4m} = 0.0029842$ 

e)

$$h_{mp} = h_n - \frac{C_{m_q}}{\frac{4 m}{\rho S \overline{c}} - C_{L_q}} = 0.4100 - \frac{-0.8}{335.0982 - 0} = 0.04339$$

40000 ft.

Equations are the same as above. The results of the calculations are:

a) 
$$V = 774.4640$$
 ft/sec  $C_L = 0.3978$   $\frac{L}{D} = 9.9448$   
b)  $T = \frac{W}{L/D} = 3841.20$  lbs  
c)  $h_n = 0.3608$   
d)  $\frac{\Delta \delta_e}{n-1} = -0.5300$  rad/g = -3.1110 deg/g  $\Delta = -5.4032$   $\frac{2m}{\rho S \overline{c}} = 1357.0362$   
e)  $h_{mp} = 0.3613$ 

19. This problem deals with level flight so that n = 1 and the equation for the previous problem

does not apply. We have to go back to the level flight equations. Solving for the elevator deflection for balance we get (the now familiar): For the second case:

For the first case:

$$\delta_{e_1} = -\frac{C_{L_{\alpha}}C_{m_{0L}} + C_{m_{\alpha}}C_{L_1}}{C_{L_{\alpha}}C_{m_{\delta_e}} - C_{m_{\alpha}}C_{L_{\delta_e}}} \qquad \qquad \delta_{e_2} = -\frac{C_{L_{\alpha}}C_{m_{0L}} + C_{m_{\alpha}}C_{L_2}}{C_{L_{\alpha}}C_{m_{\delta_e}} - C_{m_{\alpha}}C_{L_{\delta_e}}}$$

Subtracting the first from the second we get:

$$\delta_{e_2} - \delta_{e_1} = \Delta \,\delta_e = -\frac{C_{m_a}}{\Delta} (C_{L_2} - C_{L_1})$$

$$C_{L_2} = C_{L_1} \frac{V_1^2}{V_2^2} = C_{L_1} \frac{M_{a_1}^2}{M_{a_2}^2} = 1.1884 \frac{0.2^2}{0.4^2} = 0.2971$$

$$\Delta \,\delta_e = \frac{-0.8}{-3.7300} (0.2971 - 1.1884) = 0.1912 \text{ rad} = 10.9550 \text{ deg}$$

20. We can resolve the forces into components along and perpendicular to the center line of the aircraft (as we did in class) Now, however we have to consider the moment arm in the z direction as well as the moment arm in the x direction (the only one we considered before). We will look at the contribution from the wing only (as asked in the question).



Summing the moments about the cg due to the wing, we have:

$$M_{cg_{wb}} = M_{ac} + (L\cos\alpha + D\sin\alpha)(h - h_{n_{wb}})\overline{c} + (L\sin\alpha - D\cos\alpha)z_{w}$$

The first terms appear in the moment equation with the wing on the center line so only the last term on the right contributes to the change in the moment due to the high or low wing. We will concentrate on those terms.

 $\Delta M_{cg} = (L \sin \alpha - D \cos \alpha) z_w$ 

If we divide by  $1/2 \rho V^2 S \overline{c}$  we get the coefficient form:

$$\Delta C_m = (C_L \sin \alpha - C_D \cos \alpha) \frac{z_w}{\overline{c}}$$

Note that although  $C_L >> C_D$ , we can't neglect the second term compared to the first because  $C_L \sin \alpha \approx C_D \cos \alpha$ , that is they are of the same order of magnitude. We are not interested in the moment caused by the high or low wing, but are interested in the contribution to stability. Hence we need to determine the contribution of this term as angle of attack changes, or

$$\frac{d\Delta C_m}{d\alpha} = \Delta C_{m_{\alpha}} = (C_{L_{\alpha}}\sin\alpha + C_L\cos\alpha - C_{D_{\alpha}}\cos\alpha + C_D\sin\alpha)\frac{z_w}{\overline{c}}$$

Now here we see the terms  $C_L \cos \alpha + C_D \sin \alpha$  and can note that the product of two small numbers,  $C_D$  and  $\sin \alpha$  (for small angles of attack) is much smaller than  $C_L \cos \alpha$ , and can be neglected, leaving us with three terms. If we further make the small angle assumption for the angle of attack,  $\sin \alpha \approx \alpha$  and  $\cos \alpha = 1$ , the contribution to the longitudinal stability parameter from the offset wing is:

$$\Delta C_{m_{\alpha}} = (C_L + C_{L_{\alpha}} \alpha - C_{D_{\alpha}}) \frac{z_w}{\overline{c}}$$

Note that all quantities in this equation or wing (or wing-body) values and should have the subscript w or wb on them. Leaving these off for now we have:

 $C_L = a \overline{\alpha}, \quad C_{L_{\alpha}} = a$ , and assuming a parabolic drag polar,  $C_D = C_{D_{0L}} + K C_L^2$ 

We can perform the indicated operation on  $C_D$  to get  $C_{D_{\alpha}} = 2 K C_L C_{L_{\alpha}}$ . Noting that  $C_{L_{\alpha}} \alpha = C_{L_{\alpha}} \overline{\alpha} + C_{L_{\alpha}} \alpha_{0L}$ , we can combine all this into the expression of interest to get:

$$\Delta C_{m_{\alpha}} = a \left[ 2 \left( 1 - Ka \right) \overline{\alpha} + \alpha_{0L} \right] \frac{z_{w}}{\overline{c}}$$

Generally the  $\alpha_{0L}$  is negative which is a stabilizing contribution ( $z_w$  positive), however,

 $K = \frac{1}{\pi ARe}$  which times the lift-curve slope may be less than one (but not necessarily). If it is, the first term is positive, which is destabilizing ( $z_w$  positive). Hence the contribution from the

offset wing depends on the AR and lift-curve slope.

The tail contribution is similar except when taking the derivative with respect to angle of attack, that particular term leads to:  $\frac{\partial \alpha_{vt}}{\partial \alpha} = (1 - \frac{\partial \epsilon}{\partial \alpha})$  How this affects the result is left as an exercise for the student.