Problem Sheet Six (ans)

21. M_a= 0.2 V = 0.2*1116.4 = 223.28 ft/sec

$L = W\cos\gamma \approx W$ (Need this assumption since we are using the level flight drag coefficient)

$D = C_D\, 1/2\, \rho\, V^2 S = 0.095\,(1/2)\, 0.002377\, (223.28)^2\, 545.5 = 3070.56 \text{ lbs}$

$T = D + W\sin\gamma = 3070.56 + 38200\sin(10) = 9703.92 \text{ lbs}$ (Total thrust)

On one engine:

$T = 4851.92 \text{ lbs}$ Left engine good so it causes a yaw right (+) yaw moment

$$C_{n_T} = -\frac{T y_p}{1/2\, \rho\, V^2 S b} = -\frac{4851.96\,(-11)}{(1/2)\, 0.002377\, (223.38)^2\, 545.5\, (53.75)} = 0.0307$$

$$C_n = C_{n_{\delta_r}} \delta_r + C_{n_\beta} \beta + C_{n_T} = 0 \qquad (\beta = 0)$$

$$0 = -0.063\, \delta_r + 0.0307 \qquad \Rightarrow \qquad \delta_r = 0.4873 \text{ rad} = 27.92 \text{ deg}$$

22. $C_n = C_{n_{\delta_r}} \delta_r + C_{n_T} = 0 \qquad (\beta = 0)$

$$-0.063\,(30\,\frac{\pi}{180}) = -C_{n_T} \qquad \Rightarrow \qquad C_{n_T} = 0.0330 \quad (\text{Max}\, C_{n_T} \text{ available})$$

$$|\, C_{n_T}| = \frac{T\,|y_p|}{1/2\, \rho\, S\, b\, V^2} = 0.0330 = \frac{4851.96\,(11)}{(1/2)\, 0.002377\, (545.5)\, 53.75\, V^2}$$

$$V = 215.43 \text{ ft/sec}$$

23. Landing, we can assume that we have symmetric thrust, or that at landing T = 0. In either case,

$$C_n = 0 = C_{n_\beta} \beta + C_{n_{\delta_r}} \delta_r = 0.137\, \beta + -0.063\,(30)\,\frac{\pi}{180} \quad \Rightarrow \quad \beta = 0.2408 \text{ rad} = 13.796 \text{ deg}$$

$$\sin\beta = \frac{v}{V} \qquad \Rightarrow \qquad v = V\sin\beta = 120\sin 13.796 = 28.62 \text{ ft/sec}$$

24,25. Much of the information is from the last problem sheet as indicated

$$C_{n_\beta} = C_{n_{\beta_{fuselage}}} + C_{n_{\beta_{wing}}} + C_{n_{\beta_{vt}}} + \Delta C_{n_{\beta_{wing\ position}}}$$

$$C_{n_{\beta_{fuse}}} = -0.96\, K_B\, \frac{S_S}{S}\, \frac{L_f}{b} \left(\frac{h_1}{h_2}\right)^{1/2} \left(\frac{W_2}{W_1}\right)^{1/3} \text{/rad}$$

measured from figure: $L_f = 68mm = 48.12 \text{ ft}, \quad h = 7.5mm, \quad d = 40mm$

$h_1 = 7.5mm = 5.3077 \text{ ft} \quad h_2 = 7.0mm = 4.9539 \text{ ft} \quad l_1 = 12.03 \text{ ft} \quad l_2 = 36.093 \text{ ft}$

$W_1 = 6mm \quad W_2 = 6.5mm \quad h_f = 2.1231 \text{ ft}$ (Height of end of fuselage)

Side area can be determined by using the 1/4, 3/4 and end fuselage height measurements assuming a leading triangle, and two trapezoids:

$$S_s = 1/2\, l_1 h_1 + 1/2(h_1 + h_2)(l_2 - l_1) + 1/2\,(h_2 + h_f)(L_f - l_2)$$

$$S_f = 31.92 + 123.46 + 42.57 = 197.95 \text{ ft}^2$$

$$\frac{L_f}{h} = \frac{68\,mm}{7.5\,mm} = 9.067 \quad \Rightarrow \quad k_B^{/} = 0.015$$

$$K_B = (k_B^{/} - 0.02857) + 0.2857\,\frac{d}{L_f}$$

$$= (0.015 - 0.02857) + 0.2857\, 40\frac{mm}{68}mm$$

$$= 0.1545$$

$$C_{n_{\beta_{fuse}}} = -0.96\,(0.1545)\,\frac{197.95}{196.1}\left(\frac{48.1236}{21.94}\right)\left(\frac{7.5\,mm}{7.0\,mm}\right)^{1/2}\left(\frac{6.5\,mm}{6.0\,mm}\right)^{1/3}$$

$$= -0.3491 \text{ /rad} = -0.0061 \text{ /deg}$$

$$\frac{C_{n_{\beta_{wing}}}}{C_L^2} = \frac{1}{4\,\pi\,AR} - \frac{\tan\Lambda_{1/4}}{\pi AR(AR + 4\cos\Lambda_{1/4})}\left(\cos\lambda_{1/4} - \frac{AR}{2} - \frac{AR^2}{8\cos\Lambda_{1/4}} + 6(h_{nw} - h)\,\frac{\sin\Lambda_{1/4}}{AR}\right)$$

From problem set 4 we have:

$$\Lambda_{1/4} = 6.64 \text{ deg} \qquad h_{n_w} = 0.242 \qquad AR = 2.4547 \qquad h = 0.07$$

$$C_L = \frac{W}{1/2\,\rho\,V^2 S} = \frac{16300}{(1/2)\,0.002377\,(223.28)^2\,196.1} = 1.4028$$

$$\frac{C_{n_\beta}}{C_L^2} = \frac{1}{4\pi 2.4547} - \frac{\tan(6.64)}{\pi(2.4547)(2.4547 + 4\cos 6.64)}\,\cdot$$

$$\left(\cos(6.64) - \frac{2.4547}{2} - \frac{2.4547^2}{8\cos(6.64)} + 6\,(0.242 - 0.07)\,\frac{\sin(6.64)}{2/4547}\right)$$

$$= 0.0346 \text{ /rad} = 0.0006 \text{ /deg}$$

$$C_{n_{\beta_{wing}}} = \frac{C_{n_{\beta_{wing}}}}{C_L^2}(C_L^2) = 0.0346\,(1.4028)^2 = 0.0681 \text{ /rad} = 0.00119 \text{ deg}$$

$$C_{n_{\beta_{vt}}} = a_{vt}\,\eta_{vt}\,\frac{S_{vt}\,l_{vt}}{S\,b}\left(1 - \frac{\partial\sigma}{\partial\beta}\right)$$

Measured from drawing (root chord at centerline of fuselage)

$c_{r_{vt}} = 15mm = 10.62\ \text{ft}$ $\quad c_t = 4mm = 2.8308\ \text{ft}$ $\quad b_{vt} = 12.5mm = 8.8462\ \text{ft}$

$\Lambda_{\text{LE}} = 38\ \text{deg}$ $\quad \lambda = c_t / c_r = 2.8308/10.6155 = 0.267$

$$S_{vt} = 1/2\,(c_{t_{vt}} + c_{t_{vt}})\,b_{vt} = \frac{1}{2}(10.6155 + 2.8308)\,8.8462 = 59.4743\ \text{ft}^2$$

$$AR_G = \frac{(2\,(8.8402))^2}{2\,(59.4743)} = 2.6316 \quad \text{Geometric aspect ratio}$$

$$AR_{eff} = 1.55\frac{b_{vt}^2}{S_{vt}} = 1.55\,\frac{8.8462^2}{59.4743} = 2.0395$$

$$\tan\Lambda_{1/2} = \tan\Lambda_{\text{LE}} - \frac{4}{AR_G}(1/2 - 0)\,\frac{1-\lambda}{1+\lambda}$$

$$= \tan 38 - \frac{4}{2.6316}(1/2 - 0)\,\frac{1-0.267}{1+0.267}$$

$$= 0.3416 \quad \Rightarrow \quad \Lambda_{1/2} = 18.86\ \text{deg}$$

$$\tan\Lambda_{1/4} = \tan 38 - \frac{4}{2.6316}(1/4 - 0)\,\frac{1-0.267}{1+0.267} = 0.5614 \quad \Rightarrow \quad \Lambda_{1/4} = 29.31\ \text{deg}$$

$$a_{vt} = \frac{2\,\pi\,AR}{2 + \sqrt{\frac{AR^2}{k^2}(1 - M_a^2)\left(1 + \frac{\tan^2\Lambda_{1/2}}{1 - M_a^2}\right) + 4}}$$

$$a_{vt} = \frac{2\pi(2.0395)}{2 + \sqrt{\frac{2.0395^2}{1^2}(1 - 0.2^2)\left(1 + \frac{0.3416^2}{1 - 0.2^2}\right) + 4}} = 2.6089\ /\text{rad}$$

$$\eta_{vt}\left(1 - \frac{\partial\sigma}{\partial\beta}\right) = 0.724 + 3.06\left(\frac{S_s/S}{1 + \cos\lambda_{1/4}}\right) + 0.4\,\frac{z_w}{d} + 0.009\,AR_w$$

$$\eta_{vt}\left(1 - \frac{\partial\sigma}{\partial\beta}\right) = 0.724 + 3.06\,\frac{59.47}{196.1}\,\frac{1}{1 + \cos 18.28} + 0.4\,\frac{0}{d} + 0.009\,(2.4547) = 1.2221$$

Need to locate the aerodynamic center of the vertical tail. First locate the mean aerodynamic chord of the vertical tail with respect to the apex of the vertical tail.

$$\bar{m}_{vt} = \frac{b}{2}\,\frac{1 + 2\lambda}{3\,(1 + \lambda)}\tan\Lambda_{\mathrm{LE}} = 8.8462\,\frac{1 + 2\,(0.267)}{3\,(1 + 0.267)} = 3.5701 \;\; \mathrm{ft}$$

We can determine the aerodynamic center location from Fig C.3 (or from the handouts) Fig C.3 is for incompressible (low speed) and the handouts can include high subsonic or supersonic. Here we have $M_a = 0.2$ which is low speed, so either should give close to the same result within eyeball accuracy. Enter Fig C.3 with (Note that the MAC is a geometric quantity - see definition)

$\Lambda_{1/4} = 29.31, \quad AR = 2.63 \quad \lambda = 0.0 \text{ and } 0.5 \quad \Rightarrow \quad \lambda = 0.267$ interpolate to get:

$$h_{n_{vt}} = 0.26$$

$$\bar{c}_{vt} = \frac{2}{3}\,c_{r_{vt}}\,\frac{1 + \lambda + \lambda^2}{1 + \lambda} = \frac{2}{3}\,(10.6155)\,\frac{1 + 0.267 + 0.267^2}{1 + 0.267} = 7.4752 \;\; \mathrm{ft}$$

$$\begin{aligned} l_{vt} &= \text{dist to vertical tail apex cg} + \bar{m}_{vt} + h_{n_{vt}}\bar{c}_{vt} \\ &= 15\,mm * 0.7077 + 3.5707 + 0.26\,(7.4752) \\ &= 16.1291 \;\; \mathrm{ft} \end{aligned}$$

$$C_{n_{\beta_{tail}}} = 2.6089\left(\frac{59.47}{196.1}\right)\left(\frac{16.1291}{21.94}\right)(1.2221) = 0.7108 \;\; /\mathrm{rad}$$

$$\begin{aligned} C_{n_\beta} &= C_{n_{\beta_{fuse}}} + C_{n_{\beta_{wing}}} + C_{n_{\beta_{vt}}} + \Delta C_{n_{\beta_{wing\ position}}} \\ &= -0.3491 + 0.0681 + 0.7108 + 0.0057 \\ &= 0.4355 \;\; /\mathrm{rad} = 0.0076 \;\; /\mathrm{deg} \end{aligned}$$