Problem Sheet Seven, Answers

26. Assume that the glide path has a small flight path angle, typically 3-6 degrees. Then $L = W\cos\gamma \approx W$. Then we can compute the lift coefficient:

a) $C_L = \dfrac{W}{0.5\rho V^2 S} \equiv \dfrac{38200}{0.5\,(0.002377)\,150^2\,(154.5)} = 2.6187$

b) $\sin\beta = \dfrac{v}{V} = -\dfrac{60}{150} = -0.40 \quad \Rightarrow \quad \beta = -23.58 \text{ deg}$

$$\begin{bmatrix} C_{Y_{\delta_a}} & C_{Y_{\delta_r}} & C_w \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & 0 \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = -\begin{Bmatrix} C_{Y_\beta}\beta \\ C_{l_\beta}\beta \\ C_{n_\beta}\beta \end{Bmatrix}$$

$$\begin{bmatrix} 0 & 0.175 & 2.6187 \\ -0.054 & 0.029 & 0 \\ 0.0075 & -0.063 & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = -\begin{Bmatrix} -0.72 \\ -0.103 \\ 0.137 \end{Bmatrix} \frac{-23.5782\,\pi}{180} = \begin{Bmatrix} -0.2963 \\ -0.0424 \\ 0.0564 \end{Bmatrix}$$

$$\delta_a = 0.3252 = 18.63 \text{ deg}$$

$$\delta_r = -0.8565 = -49.07 \text{ deg}$$

$$\phi = -0.0560 = -3.2086 \text{ deg}$$

Just for information purposes we need to calculate the maximum bank angle to see if it may become a limiting factor. From geometry,

$$\tan\phi = \frac{z_{gear}}{b/2 - y_{gear}} = \frac{4}{53.75/2 - 8} = 0.2119 \quad \Rightarrow \quad \phi_{\max} = 11.965 \text{ deg}$$

iii) Since rudder is limiting factor, set it at -30 degrees (to be consistent with the first part of problem)

$$\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_a}} & C_w \\ C_{l_{\beta_a}} & C_{l_{\delta_a}} & 0 \\ C_{n_{\beta_a}} & C_{n_{\delta_a}} & 0 \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \phi \end{Bmatrix} = -\begin{Bmatrix} C_{Y_{\delta_r}}\delta_r \\ C_{l_{\delta_r}}\delta_r \\ C_{n_{\delta_r}}\delta_r \end{Bmatrix}$$

$$\begin{bmatrix} -0.72 & 0 & 2.6187 \\ -0.103 & -0.054 & 0 \\ 0.137 & 0.0075 & 0 \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \phi \end{Bmatrix} = -\begin{Bmatrix} 0.175 \\ 0.029 \\ -0.063 \end{Bmatrix} \frac{-30.00\,\pi}{180} = \begin{Bmatrix} 00916 \\ 0.0152 \\ 0.0330 \end{Bmatrix}$$

$$\beta = -0.2517 = -14.42 \text{ deg}$$
$$\delta_a = 0.1987 = 11.38 \text{ deg}$$
$$\phi = -0.0342 = -1.96 \text{ deg}$$

Max crosswind:

$$v = V \sin\beta = 150 \sin -14.42 = -37.35 \text{ ft/sec}$$

27. Return to the basic equations:

$$C_{y_\beta}\beta + C_{y_{\delta_r}}\delta_r + C_w\phi = 0$$
$$C_{l_\beta}\beta + C_{l_{delt_r}}\delta_r + C_{l_{\delta_a}}\delta_a = 0$$
$$C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r + C_{n_{\delta_a}}\delta_a = 0$$

where we note that the absence of $C_{Y_{\delta_a}}$ allows us to separate equations so that we can solve for δ_r and δ_r, given β:

$$\begin{bmatrix} C_{l_{\delta_r}} & C_{l_{\delta_a}} \\ C_{n_{\delta_r}} & C_{n_{\delta_a}} \end{bmatrix} \begin{Bmatrix} \delta_r \\ \delta_a \end{Bmatrix} = -\begin{Bmatrix} C_{l_\beta}\beta \\ C_{n_\beta}\beta \end{Bmatrix}$$

or

$$\delta_r = \frac{C_{l_{\delta_a}}C_{n_\beta} - C_{n_{\delta_a}}C_{l_\beta}}{C_{l_{\delta_r}}C_{n_{\delta_a}} - C_{n_{\delta_r}}C_{l_{\delta_a}}}\beta \qquad \text{and} \qquad \delta_a = \frac{C_{n_{\delta_r}}C_{l_\beta} - C_{l_{\delta_r}}C_{n_\beta}}{C_{l_{\delta_r}}C_{n_{\delta_a}} - C_{n_{\delta_r}}C_{l_{\delta_a}}}\beta$$

It is clear that decreasing beta will decrease the elevator and aileron deflections. Since $\sin\beta = \frac{v}{V}$, then for a given *v*, if we increase *V*, we can decrease the size of the sideslip angle, β.

Putting in the numbers:

$$\delta_r = \frac{-0.054\,(0.137) - 0.0075\,(-0.108)}{0.029\,(0.0075) - (-0.063)\,(-0.054)}\beta$$
$$= 2.0805\,\beta$$

Similarly we have $\delta_a = -0.7901\,\beta$

From the previous work, the rudder is the limiting factor, so in the above equation set the rudder equal to 30 deg.

$$30 = 2.0805\,\beta \quad \Rightarrow \quad \beta = 14.4193 \text{ deg}$$

Then

$$\sin\beta = \frac{v}{V} = \frac{60}{V} \quad \Rightarrow \quad \underline{V = 240.95 \text{ ft/sec}}$$

The value of C_w is determined from

$$C_w = \frac{W}{0.5\,\rho\,V^2\,S} = \frac{38200}{0.5\,(0.002377)\,240.95^2\,(545.5} = 1.0149$$

Now, from above:

$$\begin{bmatrix} C_{Y_{\delta_a}} & C_{Y_{\delta_r}} & C_w \\ C_{l_{\delta_a}} & C_{l_{\delta_r}} & 0 \\ C_{n_{\delta_a}} & C_{n_{\delta_r}} & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = -\begin{Bmatrix} C_{Y_\beta}\,\beta \\ C_{l_\beta}\,\beta \\ C_{n_\beta}\,\beta \end{Bmatrix}$$

$$\begin{bmatrix} 0 & 0.175 & 1.0149 \\ -0.054 & 0.029 & 0 \\ 0.0075 & -0.063 & 0 \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \\ \phi \end{Bmatrix} = -\begin{Bmatrix} -0.72 \\ -0.103 \\ 0.137 \end{Bmatrix} \frac{-14.4193\,\pi}{180} = \begin{Bmatrix} -0.1812 \\ -0.0259 \\ 0.0345 \end{Bmatrix}$$

$$\delta_a = 0.1982 = 11.36 \text{ deg}$$

$$\delta_r = -0.5240 = -30.02 \text{ deg}$$

$$\phi = -0.0881 = -5.05 \text{ deg}$$

28. $C_{l_p}\hat{p} + C_{l_{\delta_a}}\delta_a = 0$ $\qquad\qquad$ $\delta_a = 30 \text{ deg} = \pi/6 \text{ rad}$

$$\hat{p} = -\frac{C_{l_{\delta_a}}\delta_a}{C_{l_p}} = -\frac{-0.054}{-0.37}\,\frac{\pi}{6} = -0.0764$$

$$p = \frac{2\,V}{b}\hat{p} = \frac{2\,(0.2)\,1116.4)}{53.75}\,(0.0764) = 0.6349 \text{ rad/sec} = 36.38 \text{ deg/sec}$$

29.

$$C_{y_\beta}\,\beta + C_{y_{\delta_r}}\,\delta_r + C_w\,\phi = 0$$

$$C_{l_\beta}\,\beta + C_{l_{delt_r}}\,\delta_r + C_{l_{\delta_a}}\,\delta_a = 0$$

$$C_{n_\beta}\,\beta + C_{n_{\delta_r}}\,\delta_r + C_{n_{\delta_a}}\,\delta_a + C_{n_T} = 0$$

For minimum control speed, we set $\beta = 0$. Again, since $C_{Y_{\delta_a}} = 0$, we can solve these equations sequentially if we assume the limiting factor is the rudder. If the engine-out case causes a positive yaw moment, we will require a positive rudder deflection to counter it.

From the second equation we have:

$$\delta_a = -\frac{C_{l_{\delta_r}}\delta_r}{C_{l_{\delta_a}}} = -\frac{(0.029)(\pi/6)}{-0.054} = 0.2812 = 16.11 \text{ deg}$$

From the last equation we have:

$$-C_{n_T} = C_{n_{\delta_r}}\delta_r + C_{n_{\delta_a}}$$

$$= (-0.063)(\pi/6) + 0.0075\,(0.2812)$$

$$= -0.03088$$

$$C_{n_T} = 0.03088 = -\frac{T\,Y_p}{1/2\,\rho\,S\,b\,V^2} = -\frac{4000\,(-10)}{1/2\,(0.002377)\,545.5\,(53.75)\,V^2}$$

$$V = 192.80 \text{ ft/sec}$$

At this speed: $C_w = \frac{38200}{(1/2)\,0.002377\,(192.80)^2\,545.5} = 1.5851$

From the first equation:

$$\phi = -\frac{C_{Y_{\delta_r}}\delta_r}{C_w} = -\frac{0.175\,(\pi/6)}{1.5851} = -0.0565 \text{ rad} = -3.2363 \text{ deg}$$

30. $L\sin\phi = m\frac{V^2}{2} = m\,V\,\Omega$ $\qquad$ $L\cos\phi = mg = W$

Also $V = a\,M_a = 0.2\,(1116.4) = 223.28$ ft/sec

Then:

$$\tan\phi = \frac{V}{g}\Omega \quad \Rightarrow \quad \Omega = \frac{g}{V}\tan\phi \equiv \frac{32.174}{223.28}\tan 60 = 0.2496 \text{ rad/sec}$$

Sine the flight path angle is zero, $\sin\gamma = 0$, and we have

$$\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_a}} & C_{Y_{\delta_r}} \\ C_{l_{\beta_a}} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_{\beta_a}} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} = -\begin{Bmatrix} C_{Y_r} \\ C_{l_r} \\ C_{n_r} \end{Bmatrix} \frac{\Omega\,b}{2\,V}\cos\phi$$

$$\frac{\Omega\, b}{2\, V}\cos\phi = \frac{0.2496\,(53.75)}{2\,(223.28)}\cos 60 = 0.0150$$

Then:

$$\begin{bmatrix} -0.72 & 0 & 0.175 \\ -0.103 & -0.054 & 0.029 \\ 0.137 & 0.0075 & -0.063 \end{bmatrix}\begin{Bmatrix} \beta \\ \delta_a \\ \phi \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 0.11 \\ -0.16 \end{Bmatrix} 0.0150 = \begin{Bmatrix} 0 \\ -0.00165 \\ 0.0024 \end{Bmatrix}$$

$$\beta = -0.0181 = -1.037 \text{ deg}$$

$$\delta_a = 0.0251 = 1.438 \text{ deg}$$

$$\delta_r = -0.0745 = -4.268 \text{ deg}$$