12. Satellite Look Angle

The satellite look angle refers to the angle that one would look for a satellite at a given time from a specified position on the Earth. For example, if you had a satellite dish and wanted to point it at a specific communications satellite, you would need to know the look angle, generally described by the azimuth and elevation angles. The azimuth angle is an angle measured from North direction in the local horizontal plane, and the elevation angle is the angle measured perpendicular to the horizontal plane (in the vertical plane) to the line-of-sight to the satellite. These angles can best be described by introducing the topocentric-horizontal coordinate system. This system is one with the origin located a some point on the surface of the Earth (say Blacksburg), with its x-z plane being in the local horizontal plane. There are (at least) two versions of such a system that are often used, the South, East, Zenith (SEZ) system, and the North, East, Down (or Nadir) (NED) system. These two systems are described next.

Topocentric-Horizontal System

The origin of the topocentric system is located with respect to the Earth-Centered Inertial (ECI) system by the right ascension angle, θ, and the declination (or in this case the latitude) angle, ϕ. At a given instant of time, these angles will specify the location of a point on the surface of a spherical Earth. (Note: If accuracy is needed, it is necessary to include the fact that the Earth is elliptic in shape and that in locating a point on the surface of the Earth, this shape must be accounted for - see Bate, Mueller, and White for more details. Here we will assume a spherical Earth). At that point we will locate the origin of the coordinate system with its axes oriented according to the following rules:

Axis	SEZ	NED
x	Points South	Points North
y	Points East	Points East
z	Points vertically “up”	Points vertically “down”

The NED system can be transformed to the SEZ system (or vice versa) by rotating about the y axis an angle π.

The position of the satellite can be represented by the some of the position vector from the center of the Earth to the observation point, plus the position vector from the observation point to the satellite. Thus we have:

$$\vec{r} = \vec{R}_e + \vec{\rho} \tag{1}$$

where $\vec{r}$ = position vector from origin of ECI system (center of Earth) to satellite of interest

$\vec{R}_e$ = position vector from origin of ECI system to origin of topocentric-horizontal system

$\vec{\rho}$ = position vector from origin of topocentric-horizontal system to the satellite of interest

It would be convenient to represent all these vectors in the topocentric-horizontal system. Then, from the components of $\vec{\rho}$ in that system, we could determine the azimuth and elevation angles. Hence if we know the time and orbital elements, we can determine the position vector represented in the ECI coordinate system. We can then apply a transformation, and represent the position vector in the topocentric system. We can easily determine the position of the origin in the topcentric-horizontal system, and can easily determine $\vec{\rho}^{NED}$ from:

$$\vec{\rho}^{NED} = T_{ECI2NED}\vec{r}^{ECI} - \vec{R}_e^{NED} \tag{2}$$

We now need to determine the transformation matrix that takes us from ECI to NED.

We can determine this transformation matrix by finding out what sequence of rotations will take the ECI axes system and align it with the NED system. Making a sketch and noting the sequence gives us the following sequence of rotations:

1. Rotate an angle θ about the z^{ECI} axis to some intermediate x'y'z' system
2. Rotate an angle $-(\phi + \pi/2)$ about the y' axis to the NED system

(Note that the last rotation would have been $-(\phi - \pi/2)$ to go to the SEZ system)

The transformation is then given by:

$$T_{ECI2NED} = \left[-(\phi + \pi/2)_y\right]\left[\theta_z\right] = \begin{bmatrix} \cos(\phi+\pi/2) & 0 & \sin(\phi+\pi/2) \\ 0 & 1 & 0 \\ -\sin(\phi+\pi/2) & 0 & \cos(\phi+\pi/2) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{NED} = \left[-(\phi + \pi/2)_y\right]\left[\theta_z\right]\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{ECI}$$

$$= \begin{bmatrix} \cos(\phi+\pi/2) & 0 & \sin(\phi+\pi/2) \\ 0 & 1 & 0 \\ -\sin(\phi+\pi/2) & 0 & \cos(\phi+\pi/2) \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{ECI} \tag{3}$$

$$= \begin{bmatrix} -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \\ -\sin\theta & \cos\theta & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & -\sin\phi \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{ECI}$$

While we are here, we can calculate the transformation to the SEZ system by noting:

$$\vec{A}^{SEZ} = \left[\pi_y\right]A^{NED} \tag{4}$$

The final required ECI to SEZ transformation is given by:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{SEZ} = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{ECI} \tag{5}$$

In the local system, we can represent the local position vector to the satellite using the magnitude of the vector, ρ, and the orientation angles of the vector. For our purposes it is convenient to use the Azimuth angle measured from the North direction clockwise about the down (z) axis to the projection of the vector on the local horizontal plane, and the Elevation angle measured in a vertical plane from the horizontal plane to the vector itself. Hence we can write the components of the vector in the NED or SEZ system.

The local position vector takes the form (for the NED system)

$$\vec{\rho} = \begin{Bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{Bmatrix}^{NED} = \rho \begin{Bmatrix} \cos EL \cos AZ \\ \cos EL \sin AZ \\ -\sin EL \end{Bmatrix}^{NED} \tag{6}$$

or (for the SEZ system)

$$\vec{\rho} = \begin{Bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{Bmatrix}^{SEZ} = \rho \begin{Bmatrix} -\cos EL \cos AZ \\ \cos EL \sin AZ \\ \sin EL \end{Bmatrix}^{SEZ} \tag{7}$$

The position vector to the site location can be represented easily in either the NED or the SEZ system as:

$$\vec{R}_e = \begin{Bmatrix} 0 \\ 0 \\ -R_e \end{Bmatrix}^{NED} = \begin{Bmatrix} 0 \\ 0 \\ R_e \end{Bmatrix}^{SEZ} \tag{8}$$

<u>Determining the Look Angle</u>

In order to determine the look angle, all we need to do is to find the local position vector represented in the local horizontal coordinates.

$$\vec{\rho} = \vec{r} - \vec{R}_e \big|^{NED} \tag{9}$$

This equation becomes:

$$\vec{\rho} = \begin{Bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{Bmatrix}^{NED} = \rho \begin{Bmatrix} \cos EL \cos AZ \\ \cos EL \sin AZ \\ -\sin EL \end{Bmatrix} = T_{ECI2NED} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{ECI} - \begin{Bmatrix} 0 \\ 0 \\ -R_e \end{Bmatrix}^{NED} \tag{10}$$

The look angles can be easily calculated from the components of the local position vector in the following way:

$$\tan AZ = \frac{\rho_y}{\rho_x} \tag{11}$$

Note that by using a two argument arctangent code the result will be assigned to the correct quadrant. Otherwise you will have to make the quadrant assignment using the relations:

$$\frac{+}{+} \Rightarrow \text{first}; \qquad \frac{+}{-} \Rightarrow \text{second}; \qquad \frac{-}{-} \Rightarrow \text{third}; \qquad \frac{-}{+} \Rightarrow \text{fourth}$$

The elevation angle can be calculated from:

$$\sin EL = \frac{-\rho_z}{\rho} \quad \text{or}$$

$$\tan EL = \frac{-\rho_z}{\sqrt{\rho_x^2 + \rho_y^2}} \tag{12}$$

Example:
Consider a geosynchronous satellite in the equatorial plane, and an observation site in Blacksburg, VA. The following information is given: At the instant of interest, the right ascension of Blacksburg is 90 deg. The latitude of Blacksburg is 37 deg 12.8 min. and the inertial position of the satellite is $\vec{r} = \begin{Bmatrix} 4.6669 \\ 4.6669 \\ 0 \end{Bmatrix}^{ECI}$. We can convert the latitude to $37 + \frac{12.8}{60}\ \text{deg} = 37.2133\ \text{deg}$

Then the position vector represented in the NED coordinate system is given by: ($\theta = 90$, $\phi = 37.2133$)

$$\vec{r} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{NED} = \begin{bmatrix} -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \\ -\sin\theta & \cos\theta\, 0 & \\ -\cos\pi\cos\theta & -\cos\phi\sin\theta & -\sin\phi \end{bmatrix} \begin{Bmatrix} 4.6669 \\ 4.6669 \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} 0 & -0.6048 & 0.7964 \\ -1 & 0 & 0 \\ 0 & -0.7964 & -0.6048 \end{bmatrix} \begin{Bmatrix} 4.6669 \\ 4.6669 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2.8225 \\ -4.6669 \\ -3.7167 \end{Bmatrix}$$

Now we can compute the local position vector from $\vec{\rho} = \vec{r} - \vec{R}_e$:

$$\vec{\rho} = \begin{Bmatrix} -2.8225 \\ -4.6669 \\ -3.7167 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix} = \begin{Bmatrix} -0.8225 \\ -4.6669 \\ -2.7167 \end{Bmatrix} \text{ DU}$$

The angles then can be calculated from:

$$\tan AZ = \frac{\rho_y}{\rho_x} = \frac{-4.6669}{-2.8225} = 1.6535 \quad \Rightarrow \quad \underline{AZ = 238.83 \text{ deg}}$$

(Note the quadrant!)

and

$$\sin EL = \frac{-\rho_z}{\sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}} = \frac{-(-2.7167}{6.0932} = 0.4459$$

$$\underline{EL = 26.48 \text{ deg}}$$

The only question now is how do we get the right ascension angle θ ? See the section on Sidereal Time.