Some Astrodynamic Equations

1. **Equations Applicable to All Orbits**

a) Orbit Equation: $$r(\nu) = \frac{\frac{h^2}{\mu}}{1 + e\cos\nu} = \frac{p}{1 + e\cos\nu}$$

$$\frac{h^2}{\mu} = p = r_{(\nu = \pi/2)}$$ (Orbit parameter, or semi-latus rectum)

b) Energy Equation: $$\frac{V^2}{2} - \frac{\mu}{r} = En$$

c) Angular Momentum: $$\vec{h} = \vec{r} \ x \ \vec{V} = co\vec{n}st$$

$$|\vec{h}| = r^2\dot{\nu} = rV_\theta = rV\cos\phi = h = const$$

d) Velocity Components:

Radial component: $V_r = \dot{r} = V\sin\phi$

Transverse component: $V_\theta = r\dot{\nu} = V\cos\phi$

e) Eccentricity: $$e^2 = 1 + \frac{2h^2 En}{\mu^2}$$

f) Flight path angle (ϕ): $$\tan\phi = \frac{V_r}{V_\theta} = \frac{\vec{r}\cdot\vec{V}}{h} = \frac{e\sin\nu}{1 + e\cos\nu}$$

2. Parabolic Orbits (E_n = 0, e = 1)

a) Orbit equation: $$r(\nu) = \frac{\frac{h^2}{\mu}}{1 + \cos\nu} = \frac{p}{1 + \cos\nu} = \frac{p}{2\cos^2\frac{\nu}{2}}$$

b) Energy Equation: $$\frac{V^2}{2} - \frac{\mu}{r} = 0, \quad \Rightarrow \quad V = V_{esc} = \sqrt{\frac{2\mu}{r}}$$

c) Flight path angle: $$\tan\phi = \frac{\sin\nu}{1 + \cos\nu} = \tan\frac{\nu}{2} \quad \Rightarrow \quad \phi = \frac{\nu}{2}$$

Parabolic Orbits (cont)

$$\cos\phi = \left(\frac{r_p}{r}\right)^{1/2}$$

d) Other relations

$$r_p = \left.\frac{\frac{h^2}{\mu}}{1 + \cos\nu}\right|_{\nu = 0} = \frac{h^2}{2\mu} = \frac{p}{2}$$

3. Elliptic Orbits (En < 0, e < 1)

a) Orbit Equation: $r(\nu) = \dfrac{a(1 - e^2)}{1 + e\cos\nu}$, $\quad \dfrac{h^2}{\mu} = a(1 - e^2) \quad \Leftrightarrow \quad h = \sqrt{\mu a}\sqrt{(1 - e^2)}$

b) Energy Equation: $\dfrac{V^2}{2} - \dfrac{\mu}{r} = -\dfrac{\mu}{2a} = En \quad \Rightarrow \quad a = -\dfrac{\mu}{2En}$

c) Angular momentum: $h = \sqrt{\mu a}\sqrt{(1 - e^2)} = r_p V_p = r_a V_a = r V \cos\phi$

d) Flight Path Angle:

$$\tan\phi = \frac{e\sin\nu}{1 + e\cos\nu}, \qquad \cos\phi = \left[\frac{a(1 - e^2)}{r\left(2 - \frac{r}{a}\right)}\right]^{1/2}$$

e) Period of Elliptic Orbit

$$T_p = 2\pi\sqrt{\frac{a^3}{\mu}} \qquad n = \frac{2\pi}{T_p} \quad \text{(Mean angular rate)}$$

$$n^2 a^3 = \mu \qquad \text{(Kepler's 3rd law)}$$

f) Peri- and apo- center relations:

$$r_p = a(1 - e)$$

$$r_a = a(1 + e)$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{r_a - r_p}{2a}$$

$$h = \sqrt{\frac{2\mu r_p r_a}{r_a + r_p}} = \sqrt{2\mu}\sqrt{\frac{r_a}{1 + \frac{r_a}{r_p}}} = r_p V_p = r_a V_a = \frac{2\mu}{V_a + V_p}$$

velocity at pericenter and apocenter

$$V_p = \sqrt{\frac{\mu}{r_p}}\sqrt{\frac{2\frac{r_a}{r_p}}{1 + \frac{r_a}{r_p}}} \qquad V_a = \sqrt{\frac{\mu}{r_a}}\sqrt{\frac{2}{1 + \frac{r_a}{r_p}}}$$

$$En = -\frac{\mu}{r_a + r_p} = -\frac{V_p V_a}{2}, \qquad b = a\sqrt{1 - e^2}$$

Special Case - Circular Orbit (e = 0)

$$r = \frac{\frac{h^2}{\mu}}{1 + e\cos\nu} = \frac{h^2}{\mu} = a(1 - e^2) = a = r_c$$

$$\left.\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}\right|_{r=a} \quad \Rightarrow \quad V_c = \sqrt{\frac{\mu}{r}}$$

4. **Hyperbolic Orbits (En > 0, e > 1)**

a) Orbit equation: $r(\nu) = \frac{\frac{h^2}{\mu}}{1 + e\cos\nu} = \frac{a(e^2 - 1)}{1 + e\cos\nu}$

b) Energy Equation: $\frac{V^2}{2} - \frac{\mu}{r} = \frac{\mu}{2a} = En$

At r = ∞, $V_\infty = \sqrt{\frac{\mu}{a}}$ (Hyperbolic excess velocity)

c) Turning Angle : $\sin\frac{\delta}{2} = \frac{1}{e}$

Many relations are the same as the elliptic relations with (a) in the elliptic equations replaced with (-a)