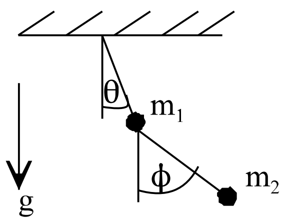
AOE 5234 Orbital Mechanics Problem Sheet 1

Due 29 Jan, 2001

1. Write the differential equations of motion for the double pendulum shown below. Use the generalized coordinates θ and ϕ as shown. Find the generalized force associated with each coordinate.



2. A bead of mass m moves on a frictionless horizontal table and is attached to a string which passes through a hole at the center of the table where it is attached to a linear spring, with spring constant k, and is attached rigidly to the floor. When the spring is un-stretched, R = 0.

A) Polar coordinates

a) Determine the kinetic and potential energy of the system.
b) Determine the Langrangian of the system.
c) Determine the equations of motion of the system
d) Determine obvious integrals of motion of the system

B) Cartesian coordinates

a) Determine the equations of motion of the system
b) Determine obvious integrals of motion of the system
c) Solve the differential equations of motion to get a solution.

3. Find the potential of a spherical homogeneous spherical shell

a) at any point r external to the shell
b) at any point internal to the shell

4. From the results obtained in question 3, for the case of a spherically symmetric Earth, the potential (and hence force) on a particle internal to the surface depends only on the mass internal to a sphere of the radius at which the particle is located. Assuming this fact is true (it is) then

a) write the differential equations of motion for a particle moving in a tube from the North to the South pole
b) Find the equation for the motion in time assuming the particle was held and then dropped from the North Pole. (See if you can estimate some numbers to find the time to reach the South pole and return)

5. A bead of mass m is constrained to move along a smooth rigid wire having the shape of $y = -k/x$ where the y axis is vertical in a uniform gravitational field. Assume that the wire is free to rotate about the y axis. (Use ϕ to represent the second generalized coordinate). Assume that $\phi = \omega t$ and that ω = CONSTANT. Find T_i, $i = 0,1,2$, and find T, the differential equations of motion, and any integrals fo motion that you can.

Find the generalized force associated with each generalized coordinate.