AOE 5234 Orbital Mechanics Problem Sheet 2

Due 5 February, 2001

Review elliptic orbit properties: Bate Mueller and White Chapters 1,2 4,;Vallado, Chapter 1, 1.2, Chapter 2, 2.2, Chapter 4, 4.2,4.3; Battin, Chapter 3, 3.1, 3.3, 3.4, Chapter 4, 4.1, 4.3.

In class we developed the equation of the orbit

$$r = \frac{\frac{h^2}{\mu}}{1 + e\cos\nu} \qquad \textbf{(1)}$$

This equation is the general equation of a conic section where r is the distance from the focus of the conic section. If e < 1, the conic section is an ellipse. We also know the geometry of an ellipse with the semi-major axis = a, and the semi-minor axis = b. where these axes intersect at the center of the ellipse. In addition we have the energy integral of motion:

$$\frac{V^2}{2} - \frac{\mu}{r} = En \qquad \textbf{(2)}$$

and the angular momentum integral that says the motion is in a plane and that

$$||h|| = \text{const} = r^2\dot{\nu} \qquad \textbf{(3)}$$

Starting with Eq. 1 determine the following properties of the **<u>ellipse.</u>**

1. Show that $h^2/\mu = a(1 - e^2)$
2. The point of closest approach (periapse) is $r_p = a(1 - e)$
3. The point furthest (apoapse) is $r_a = (1 + e)$
4. The distance between the focus and the center is c = ae
5. The distance from the focus to the end of the semi-minor axis (r_b) is a
6. The length of the semi-minor axis is $b = a\sqrt{1 - e^2}$
7. At the end of the semi-minor axis the cos ν = - e
8. Show $\dot{r} = \frac{\mu}{h} e \sin\nu$
9. Evaluate the Energy equation (2) using the orbit equation (2), the results of (8) and and the angular momentum integral (3) and show $e^2 = 1 + \frac{2h^2 En}{\mu^2}$. (Hint: since energy is constant it can be evaluated anywhere on the orbit, pick a point to minimize your efforts)

10. Put the result of (9) into the result of (1) to show $En = -\frac{\mu}{2a}$.

11. Eq. (3) also can be written as $\frac{dA}{dt} = 1/2\, r^2\dot{\nu} = 1/2\, h$, where dA is the area "swept out" by r in the time dt. If we know that the area of an ellipse is πab, show that the period of an orbit is $T_p = 2\pi\sqrt{\frac{a^3}{\mu}}$.

12. Since $\omega_{avg} T_p = 2\pi = n T_p$, where n = ω_{avg} = mean angular rate then

$n = \sqrt{\frac{\mu}{a^3}}$ and we can write $n^2 a^3 = \mu$ a version of Kepler's 3rd law.

13. Define τ as the time of periapse passage so that at t = τ, ν = 0 and show that

$$t - \tau = \frac{h^3}{\mu^2} \int_0^{\nu} \frac{d\beta}{(1 + e\cos\beta)^2}$$

or

$$n(t-\tau) = M = (1-e^2)^{3/2} \int_0^{\nu} \frac{d\beta}{(1+e\cos\beta)^2}$$

where M is the mean anomaly.

14. Show result (13) can be integrated to obtain (for e<1):

$$n(t - \tau) = M = 2\tan^{-1}\left(\sqrt{\left(\frac{1-e}{1+e}\right)} \tan\frac{\nu}{2} \right) - \frac{e\sqrt{1-e^2}\sin\nu}{1 + e\cos\nu}$$