AOE 5234 Orbital Mechanics, Problem Sheet 4

Due 19 March, 2001

Read Battin, Chapter 4, sections 4.5-4.7,
Chapter 1, sections 1.1, 1.2, 1.4

1. Develop an algorithm to evaluate a continuing fraction.
The input should include the following:

a) argument x,
b) some parameter α,
c) the term a_0
d) the term b_0,
e) a function fa, for evaluating a_i, $i \neq 0$,
f) a function fb, for evaluating b_i, $i \neq 0$
g) a tolerance to which you want the result to satisfy.

The output should include the following:

a) the value of the fraction
b) the number of iterations, the value of i in $u = u_0+u_1+u_2+\cdots+u_i$

2. a) Apply your algorithm, developed in (1) to evaluating the continued fraction for

$$\tan x = \cfrac{x}{1-\cfrac{x^2}{3-\cfrac{x^2}{5-\cfrac{x^2}{7-\ddots}}}}$$

Evaluate of angles from 10 to 80 degrees in 10 degree increments (note that x must be in radians) and count the number of iterations necessary to reach a tolerance of 10^{-15}

b) Use the identity, $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$ to reduce the number of iterations required to converge by using the half angle in the fraction and then calculating the final value of the tangent of the complete angle using the identity.

c) Repeat the evaluation of the tangent function using the quarter angle in the continuing fraction, and applying the identity twice. Again, note the number of iterations required.

3. The continuing fraction for the inverse tangent is given by:

$$\tan^{-1} x = \cfrac{x}{1 + \cfrac{x^2}{3 + \cfrac{(2x)^2}{5 + \cfrac{(3x)^2}{7 + \ddots}}}}$$

a) For the case where x = Tan 80 deg, calculate the angle using your algorithm and note the number of iteration required.
b) Determine a way to decrease the number of terms required to satisfy the tolerance for the continued fraction. It is similar, in idea, to the method in problem 2. Apply your method twice to part a and note the number of terms needed to satisfy the tolerance for evaluating the fraction. Again additional calculations are needed to obtain the desired result, but the number of terms necessary to evaluate the fraction is reduced.

4. Provide the necessary functions fa and fb to your algorithm, to enable you to evaluate the universal functions U_0, U_1, U_2, and also for evaluating the function U_3.
For all subsequent work, let $C = 1/\sqrt{\mu}$, hence $\alpha = 1/a$. Also we will us canonic units with $\mu=1$, and distances and speeds measured in DU and TU, where 1 DU is the Earth's radius, and 1 SU is the speed of satellite in an Earth radius (Earth grazing) circular orbit. Write generic codes that will solve the following problem, then solve the problem.

Given $\vec{r}_0 = 1\hat{i} + 1\hat{j} + 0\hat{k}$, and $\vec{V}_0 = 0\hat{i} + \frac{\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}$, find the following:

a) a, e, i, Ω, ω, and true anomaly ν.
b) If given ψ = 1.5, find $\vec{r}$, and $\vec{V}$. (use the universal functions, evaluated using continued fractions)
c) Given that the time of flight is 8 TU (time units), find $\vec{r}$, and $\vec{V}$.

5) We have shown that $M = (1-e^2)^{\frac{3}{2}} \int_0^{\nu} \frac{d\beta}{(1+e\cos\beta)^2}$.
a) By assuming e <<1, and by using the binomial expansion for $(1-e^2)^{3/2}$ and for $(1+e\cos\beta)^{-2}$, determine a series to $O(e^3)$ for M in terms of powers of e and sines of ν, 2ν, 3ν, etc. to get M = ν + f(ν)

b) Using Lagrange's expansion theorem, or series reversion algorithm, determine a series of the true anomaly ν in terms of M.